УДК 624.01

# Design of spatial cable structures for sport arenas

**Valdek Kulbach,** DSc., **Karl Oiger,** DSc.

Tallinn University of Technology, Estonia

**Анотація.** Розглядаються 2 види покриттів: 1) сітки, сформовані усередині контуру двома нахиленими плоскими арками і 2) сітки у вигляді гіпару, оточені замкнутим контуром элептичної форми в плані. Дані рекомендації щодо вибору початкової форми, розмірів сіток і контурного кільця. Наведені методи визначення внутрішніх зусиль і переміщень для сіток і контурного кільця. Наведені приклади покриттів, побудованих в Естонії, для акустичних екранів трибун. Приведені рекомендації щодо використання гіпарів для покриттів спортивних і громадських будівель.

**Аннотация.** Рассматриваются 2 вида покрытий: 1) сети, сформированные внутри контура двумя наклоненными плоскими арками и 2) сети в виде гипара, окруженные замкнутым контуром элептичекогой формы в плане. Даны рекомендации по выбору начальной формы, размеров сетей и контурного кольца. Представлены методы определения внутренних усилий и перемещений для сетей и контурного кольца. Приведены примеры покрытий, построенных в Эстонии, для акустических экранов трибун. Даны рекомендации по использованию гипаров для покрытий спортивных и общественных зданий.

**Abstract.** Two types of roof networks are under investigation in our report: 1) networks, formed inside the contour of two inclined planar arches and 2) hypar-formed networks, surrounded by a closed contour ring, having an elliptical form on plan. Recommendations for choice the initial form and dimensions of the network and the contour ring are given. Methods of determination of inner forces and deflections for the network and the contour ring are presented. Erected in Estonia networks for the acoustic screens of song festival tribunes is presented. Use of hypar-networks as roof structures for sports and spectacle halls is recommended.

**Key words:** Cable structures, suspension roofs, hypar-formed networks.

**Introduction.** The most propagated roof networks are shaped inside the contour of two inclined planar arches. Massive counter-forts are usually supporting the arches. When the zones at arches' abutments have a curved form, the areas neighbouring the keys of arches are very flat. The curvature in these flat zones may even change the direction for stretching cables. As an example of an unsuitable behaviour of the network in these zones the well-known Raleigh Arena may be presented, erected in the USA in 1953. For stabilizing the action of the blast wind, it was necessary to strengthen it by additional inclined cables. Investigation of pre-stressed roof networks started at TUT at the end of 1950s in connection with design of the acoustic screen for the song festival tribune in Tallinn [1]. This acoustic screen is a hanging roof of negative Gaussian curvature. The acoustic factor was dominant in the design of the screen surface, so it is inclined in the direction of the audience. The roof of the screen consists of ribbed wooden panels, resting on the bearing cables. Its pre-stressed network is formed inside the contour case, consisting of two planar arches. The back and the front arches have common main supports in the form of massive counter-forts, which develop considerable horizontal reactions to the arch forces. Vertical columns are supporting the back arch. The front arch has no intermediate supports, therefore it has to resist not only the moments applied in its plane, but the perpendicular forces as well. Experience in design, model testing and field investigation on the acoustic screen in Tallinn served for subsequent investigations on network structures. The main advantages of networks with elliptical contour ring consist in its fluent form and the possibility of renounce to outer horizontal supports. These advances and thorough investigations encouraged us to use the network of this kind in design and construction of the acoustic screen for the song festival tribune in Tartu.

**Initial form of the network. Networks inside the contour of two inclines planar arches.** The initial form of the network depends upon distribution of cables' prestress forces and of mutual connecting cables in the nodes. For the sake of simplicity of building up the network, usually networks with planar cables (orthogonal networks or surfaces of revolution) and networks formed in conditions of free mutual sliding of the cables in all internal nodes (so-called self-forming networks) are preferred. The cables of the former networks are to be connected during pre-stressing the network. In actual cases the network's surface is exclusively determined by the conditions of equilibrium of the nodes [2]. Our main attention in the following will be paid to the self-forming and orthogonal networks. In the first case the co ordinates of a network may be determined by a vector equation of equilibrium for every node $i,k$.

**Orthogonal network** is the simplest case for determination of the initial form of a network. Both the carrying and the stretching cables are located in parallel and orthogonal vertical planes. The spacing between the cables has to be determined in the state of structural design. Only the nodes' co-ordinates, which have to be determined by the conditions of equilibrium, are vertical. Such kind of network is characterized by equal horizontal components in all sections of every cable (the inner force of a section of a cable may be determined as the quotient of the horizontal cable force to the corresponding cosine of the section under investigation). The condition of equilibrium for the node *i,k* (the point of intersection of the *i*th carrying and the *k*th stretching cable) may be presented in the following form:

$$H_{0xi}\left(\frac{z_{i,k+1}-z_{i,k}}{a_{i,k}}+\frac{z_{i,k-1}-z_{i,k}}{a_{i,k-1}}\right)+H_{0yk}\left(\frac{z_{i+1,k}-z_{i,k}}{b_{i,k}}+\frac{z_{i-1,k}-z_{i,k}}{b_{i-1,k}}\right)=0\,, \quad (1)$$

where $H_{0xi}$ and $H_{0yk}$ are horizontal forces of the *i*th carrying and the *k*th stretching cable in the state of pre-stressing, $a_{i,k}$ and $b_{i,k}$ are horizontal projections of the corresponding cable sections. In the case of a network with equidistant cables this condition of equilibrium takes a simpler form

$$z_{i,k} = \frac{\left(z_{i,k-1} + z_{i,k+1}\right) + \lambda\left(z_{i-1,k} + z_{i+1,k}\right)}{2(1+\lambda)}, \tag{2}$$

where $\lambda = H_{0y}a_1/H_{0x}b_1$, $a_1$ and $b_1$ are distances between the stretching and the carrying cables, respectively. It is worth mentioning, that for the case $\lambda = 1$ we have

$$z_{i,k} = \tfrac{1}{4}(z_{i,k-1}+z_{i,k+1}+z_{i-1,k}+z_{i+1,k}). \tag{3}$$

Equation (3) corresponds to the hypar-formed network. Equation (2) may be written for every node of the network with given horizontal components of cable forces. As a result, we obtain a system of linear algebraic equations related to the unknown ordinates $z_{i,k}$. Every equation consists of 5 unknown ordinates. It should be mentioned that the product of the cable force and the quantity in the parentheses represent the initial contact load $F_{0,i,k}$ in the node. The system of equations (2) may be solved by means of standard programs or by means of simple iteration. The ordinates of the contour beam are to be given ahead. For the ordinates of the contour beam located outside the regular nodes we may use the iteration formulas of Collatz for fictitious nodes.

**Hypar-formed networks.** Equation for determination of a hyperbolic-parabolic (hypar) surface (Figure 1) may be presented in the following form

$$z = f_x \frac{x^2}{a^2} - f_y \frac{y^2}{b^2}. \tag{4}$$

The contact load between two families of cables in case of a hypar-network surrounded by an elliptical contour beam represents a uniformly distributed load $p_0$. Therefore the initial cable (pre-stressing) forces have the following horizontal components

$$H_{0x} = \frac{p_0 a^2}{2f_x}; \quad H_{0y} = \frac{p_0 b^2}{2f_y}, \tag{5}$$

where $f_x$ and $f_y$ are the sags of the central cables, $a$ and $b$ are the half-axes of the elliptical contour.

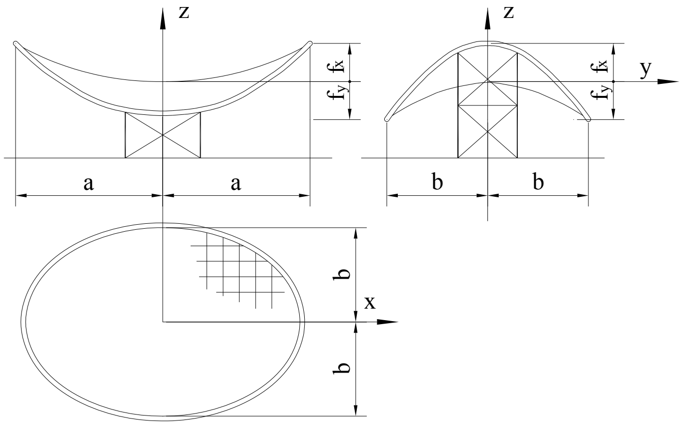


Fig. 1. Hypar-formed roof structure

**Behaviour of a network under the action of external loads. Self-forming networks.** In general, the initial form of a network is characterized by the position vectors of its nodal points $\bar{r}_{i,k}$ or by the corresponding co-ordinates $x_{i,k}$, $y_{i,k}$ and $z_{i,k}$ and by the cable forces $S_{0,i}$ and $T_{0,k}$. The stress-strain state of the network under the action of nodal forces of any kind is determined by vector equations of equilibrium of nodal points and equations of deformation compatibility for the cable sections neighbouring the node. The unknowns for actual calculation cases include 3 displacement components and 2 inner forces of the cable's sections for every nodal point; these unknowns are covered by 3 conditions of equilibrium and 2 equations of deformation compatibility of cable sections for every nodal point. Detailed method for analysis of self-forming networks under the action of outer loads is left aside in that report and in following the main attention will be paid to the orthogonal networks.

**Orthogal networks under the action of vertical loads applied in the nodes.** For an orthogonal network loaded by vertical loads a simplified solution can be obtained in the form of a system of non-linear equations [3], including one unknown displacement for every node and one unknown inner force component for every cable. 0n the base of our former works the transversal horizontal displacements of the networks nodes have been left aside in these equations. In the case of a network with equidistant cables the condition of equilibrium for the node *i,k* may be presented in the form

$$w_{i,k} = \Big\{ \left( w_{i,k-1} + w_{i,k+1} \right) + \left( z_{i,k-1} - 2z_{i,k} + z_{i,k+1} \right) + \\ + \frac{H_{y,k} a_1}{H_{x,i} b_1} \Big[ \left( w_{i-1,k} + w_{i+1,k} \right) + \left( z_{i-1,k} - 2z_{i,k} + z_{i+1,k} \right) \Big] - \\ - \frac{F_{i,k} a_1}{H_{xi}} \Big\} / \left[ 2 \left( 1 + \frac{H_{yk} a_1}{H_{xi} b_1} \right) \right] \tag{6}$$

and the equations of deformation compatibility for the i-th carrying and the k-th stretching cable as follows

$$\frac{H_{xi} - H_{0,xi}}{EA_i} \sum \left[ 1 + \left( \frac{z_{i,,k+1} - z_{i,k}}{a_1} \right)^2 \right]^{3/2} = \frac{u_{i,m} - u_{i,0}}{b_1} + \\ + \sum \left[ \frac{w_{i,k+1} - w_{i,k}}{a_1} \left( \frac{z_{i+1,k} - z_{i,k}}{a_1} + \frac{w_{i+1,k} - w_{i,k}}{2a_1} \right) \right]; \tag{7}$$

$$\frac{H_{yk} - H_{0,yk}}{EA_k} \sum \left[ 1 + \left( \frac{z_{i+1,k} - z_{i,k}}{b_1} \right)^2 \right]^{3/2} = \frac{v_{k,n} - v_{k,0}}{b_1} + \\ + \sum \left[ \frac{w_{i+1,k} - w_{i,k}}{b_1} \left( \frac{z_{i+1,k} - z_{i,k}}{b_1} + \frac{w_{i+1,k} - w_{i,k}}{2b_1} \right) \right], \tag{8}$$

where $u_{i,m}$ and $u_{i,o}$ are displacements of contour nodes for the *i*th carrying cable and $v_{k,n}$ and $v_{k,o}$ for the *k*th stretching cable. The displacements may be determined by means of influence lines composed for the displacements of the contour beam.

**Hypar-network under the action of uniform vertical loads.** In the following the analytical model of calculation for a hypar-network is presented. As the most influential factors in presented approximate model of analysis may be considered the following ones: 1) casting aside lateral displacements in conditions of equilibrium and equations of deformation compatibility; 2) use of simplified approximation of deflection functions for the network; 3) simplified approximation of deformed form of the contour ring and its linear dependence upon cable forces; 4) casting aside deformation of the contour ring under the action of pressure forces (bending deformations are taken into account as the main factors influencing onto the form transformation); 5) linear dependence of network's deflections upon the cable forces; 6) substitution of the real mesh of the network by a fictitious differential one (Fig. 2).

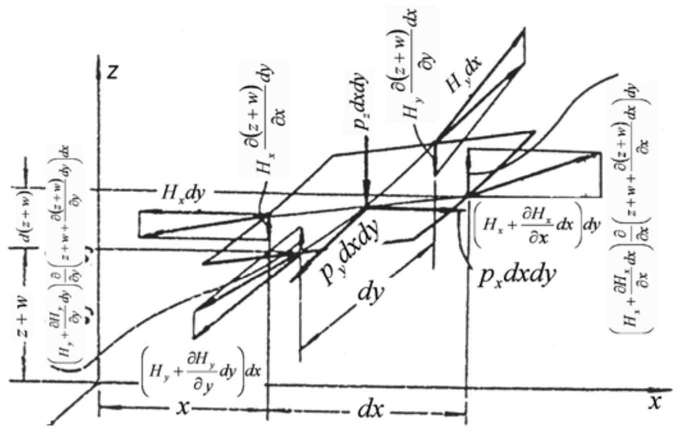


Fig. 2. Equilibrium of differential element for a fictitious network

The conditions of equilibrium for a differential element of an orthogonal network (Fig. 2), in the case of application only vertical loads may be presented as follows

$$H_x \frac{\partial^2 (z+w)}{\partial x^2} + H_y \frac{\partial^2 (z+w)}{\partial y^2} = p. \tag{9}$$

Conditions of deformation compatibility for a network's differential element connect the cable forces with deflections of the network. For determination of the stress-strain state two following equations of deformation compatibility are to be used

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial x}\left(\frac{\partial z}{\partial x} + \frac{1}{2}\frac{\partial w}{\partial x}\right) = \frac{H_x - H_{0x}}{Et_x}\left[1 + \left(\frac{\partial z}{\partial x}\right)^2\right]^{\frac{3}{2}}; \tag{10}$$

$$\frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\left(\frac{\partial z}{\partial y} + \frac{1}{2}\frac{\partial w}{\partial y}\right) = \frac{H_y - H_{0y}}{Et_y}\left[1 + \left(\frac{\partial z}{\partial y}\right)^2\right]^{\frac{3}{2}}, \tag{11}$$

where $H_x$ and $H_y$ are forces of carrying and stretching cables correspondingly, $u$, $v$ and $w$ are the components of displacements in direction of axes $x$, $y$ and $z$ respectively, $Et_x$ and $Et_y$ are the rigidities in tension of the families of carrying and stretching cables reduced to a width unit. For elimination of horizontal displacements $u$, $v$ we have to integrate Eqs. (10) and (11). Integrals $u = \int_{x_1}^{x_2} \frac{\partial u}{\partial x} dx$ and $v = \int_{y_1}^{y_2} \frac{\partial v}{\partial y} dy$ are to be equalized to corresponding displacements of the contour structure; the integrating boundaries corresponding to elliptical contour structure, have the following values $x_2 = -x_1 = a(1 - x^2/a^2)^{1/2}$ and $y_2 = -y_1 = b(1 - y^2/b^2)^{1/2}$.

Horizontal displacements of an elliptical contour ring under action of cable forces may be approximated in the following form

$$u_1 = \frac{5b^2\sqrt{\frac{a}{b}}}{72E_cI_c}\left[-\left(H_x - H_{0x}\right)b^2 + \left(H_y - H_{0y}\right)a^2\right]\left(1 - \frac{y^2}{b^2}\right)^{\frac{3}{2}}; \tag{12}$$

$$v_1 = \frac{5a^2\sqrt{\frac{a}{b}}}{72E_cI_c}\left[\left(H_x - H_{0x}\right)b^2 - \left(H_y - H_{0y}\right)a^2\right]\left(1 - \frac{x^2}{a^2}\right)^{\frac{3}{2}}, \tag{13}$$

where $E_cI_c$ is the bending rigidity of the contour beam. Approximation of displacements $u_1$ and $v_1$ brings us to values, very near to exact ones. For solution of Eqs (11), (12), (13) we have to approximate the deflection function of the network's surface. Assuming the form of deflected cables square parabolas, we may use the following approximation

$$w = w_0\left(\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1\right). \tag{14}$$

Taking into account Eqs. (10) and (11), determining their second and third integrals and presenting the cable forces as function of displacements from the equations of deformation compatibility, we obtain a system of two equations

$$A_1\Delta H_x + B_1\Delta H_y = C_1\ ; \tag{15}$$

$$A_2\Delta H_x + B_2\Delta H_y = C_2\ , \tag{16}$$

where coefficients $A_1, A_2, B_1, B_2$, $C_1$, $C_2$ depend from the deflection $w_0$ and coordinates $x, y$.

Cable forces $\Delta H_x$ and $\Delta H_y$, determined from Eqs. 6-16 as functions of the network's deflection, are to be inserted into the condition of equilibrium 3.4; as result we obtain a cubic equation regard to $\zeta_0 = w_0/f_x$ containing functions of powers of independent parameters $x$ and $y$. To eliminate these functions we have to apply a certain method for approximate analysis of mathematical physics. As the main method we have preferred method of Galyorkin (known also as Bubnoff-Galyorkin method); the latter consists in inserting the basis function (in our case $(1 - x^2/a^2 - y^2/b^2)$) as multiplier into the equation to be solved, and integrating the obtained equation in boundaries of elliptical contour ring (in case of doubly symmetrical structure in boundaries of a quarter of it). The functions to be integrated may be written as follows:

$$\int_0^b dy \int_0^{a\sqrt{1-y^2/b^2}} \left(\frac{x}{a}\right)^{2m} \left(\frac{y}{b}\right)^{2n} \left(1-\frac{x^2}{a^2}\right)^q \left(1-\frac{y^2}{b^2}\right)^q \left(1-\frac{x^2}{a^2}-\frac{y_2}{b^2}\right) dx. \tag{17}$$

After integrating and taking into use non-dimensional parameters we obtain the key formula as a cubic equation for determination of the relative deflection of the network $\zeta_0 = \frac{w_0}{f_x}$ [5]

$$\begin{aligned}&\left(1+\psi+4\xi\right)\zeta_0^{\,3} + 3\left[(1-\alpha\psi)+2(1-\alpha)\xi\right]\zeta_0^{\,2} + \\ &+\left\{2\left[\left(1+\alpha^2\psi\right)+\left(1-\alpha\right)^2\xi\right]+\left(1+\frac{1}{\alpha}\right)p_0^*\right\}\zeta_0 = p^*,\end{aligned} \tag{18}$$

where $\alpha = \dfrac{f_y}{f_x}$; $\psi = \dfrac{a^4 t_y\left(1+\kappa_x\right)}{b^4 t_x\left(1+\kappa_y\right)}$; $\kappa_x = \dfrac{5}{3}\dfrac{f_x^2}{a^2}$; $\kappa_y = \dfrac{5}{3}\dfrac{f_y^2}{b^2}$; $\mu = 1+1/\psi$ are geometrical factors, $\Phi = \dfrac{5}{9}\dfrac{Et_x f_x^{\,2}}{a^2(1+\kappa_x)(1+\mu\xi)}$ - is the rigidity parameter of the network (dimension kN/m); $\xi = \dfrac{5}{72}\dfrac{Et_y a^3\sqrt{\dfrac{a}{b}}}{E_c I_c\left(1+\kappa_x\right)}$ - is the relative stiffness of the contour ring $p_0^{\,*} = \dfrac{p_0 a^2}{2 f_x \Phi}$, $p^* = \dfrac{pa^2}{2 f_x \Phi}$ are the parameters of the initial and the additional loading correspondingly.

For the cable forces we may write

$$H_x = H_{0x} + \Phi\zeta_0\left[\left(2+\zeta_0\right)-2\left(1-\alpha+\zeta_0\right)\xi\right]; \tag{19}$$

$$H_y = H_{0y} - \beta^2\Phi\zeta_0\left[\left(2\alpha-\zeta_0\right)\psi-2\left(1-\alpha+\zeta_0\right)\xi\right]. \tag{20}$$

After determination of the relative deflection and cable forces we can calculate the extreme bending moments of the contour beam by the equation

$$M_{x=0} = \frac{1}{2}\left(H_x b^2 - H_y a^2\right)\left[\frac{1+k^2}{3k^2} - \frac{\left(1-k^2\right)\mathrm{K}\left(k\right)}{3k^2 E\left(k\right)} - 1\right], \tag{21}$$

where $k = (1 - \beta^2)^{1/2}$, K($k$) and E($k$) are the full elliptical integrals of the first and the second grade accordingly. For the cross section at $x = a$ the equation (21) is to be taken without the member 1 inside the square brackets.

Effectiveness at use of hypar-network roof structures with deformable elliptical contour ring depends greatly upon their structural parameters. The latter are connected with successful collaboration between the network and the surrounding contour ring. Limitations at evaluation the behaviour of the structure are usually determined by excessive tensile and bending deformations of cables (together with the roof covering) and resistance of the contour ring to bending moments (with consideration of its pressure). Self-evidently the economy of materials is also to be taken into account.

The layout dimensions $a$, $b$ (simultaneously their relationship $\beta = b/a$) and the overall sag of carrying and stretching cables ($f = f_x+f_y$) are usually given beforehand. The main factors, having remarkable influence upon behaviour of the structure under the action of network's external loads, are the following ones:

1. ratio between the cable sags $\alpha = f_y/f_x$;
2. ratio between the effective thickness of the fictitious shell layers $\tau = t_y / t_x$
3. factor of rigidity for the network layers $\Phi$;
4. relative stiffness of the contour ring $\xi$;
5. network's pre-stressing parameter $p_0$*;
6. parameter of the external load $p$*.

Dependence of network's deflection upon the loading factor by different values of the factor $\xi$ is presented in Fig. 3. It may be stated, that smaller deflections correspond to relatively small curvature of stretching cables; for actual structures the value $\alpha$ is to be chosen in bounds of 0.3–0.5. Limitations are connected mainly with bending moments in the stage of pre-stressing the network and action of interrupting the stretching cables. The other considerations, in the first place estimation of the contour ring's bending, constrain renunciation from the values $\beta > 1$. The influence of ratio of cables' cross sectional areas $\tau$ onto the network's deflection may be evaluated as moderate one Variation of this factor in bounds from 0,7 to 1,5 brings about moderate changes of the network's relative deflection. Its main influence onto the structural behaviour is connected with reduction of deformation of the contour ring – that phenomena is expressed by the value of the parameter $\xi$. The relative stiffness (more punctually the relative yielding) of the contour ring is probably the most influential factor in determination of network's deflection. Usually it is suitable to choose the values of the contour rings cross section of minimal dimensions; at need of restriction of the network's deflection, it is more effective to increase the cross section area of cables (at first of stretching ones) than increase of the rigidity of the contour ring.

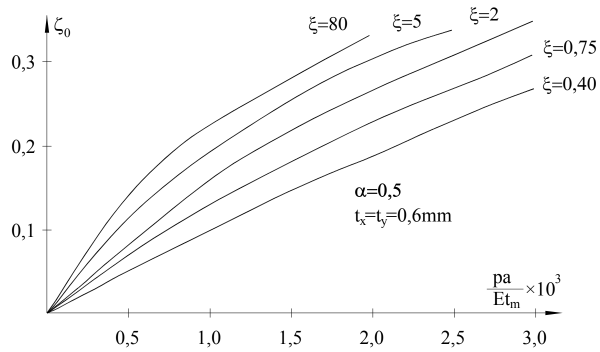


Fig. 3. Dependence of the relative deflection upon the loading parameter

The influence of the network's pre-stressing factor $p_o$* in case of suitable structure parameters is moderate, it may be limited by very small values (for example about 10 % - 20 % from the maximum loading parameter $p$*). Nevertheless, when it is not contradicted by conditions of prestressing realization or by need of minimizing the contour ring's bending moments, the increase of pre-stressing forces may be useful in consideration the network's rigidity.

**Conceptual design of the possible roof structure for a football stadium.** Let us have as calculation example the roof structure in the form of a hypar-network surrounded by an elliptical contour ring; the structure parameters are chosen with consideration of accommodation the stadium with a football playground and tribunes for the spectators. Let us proceed from the main dimensions $L = 2a = 160$ m, $B = 2b = 128$ m (corresponding $\beta = 0{,}8$), $f = f_x + f_y = 28{,}0$ m. The dead load $p_1 = 0{,}80$ kN/m$^2$, the live load $p_2 = 1{,}0$ kN/m$^2$, the initial contact load together with the own weight of the cable net $p_{0x} = 0{,}40 + 0{,}28 =$ $= 0{,}68$ kN/m$^2$, $p_{0y} = 0{,}40 – 0{,}28 = 0{,}12$ kN/m$^2$. Taking for the ratio of cable sags $\alpha = 0{,}4$, we have $f_x = 20{,}0$ m, $f_y = 8{,}0$ m.
Let us have the cables as steel ropes $d = 60$ mm for carrying cables doubled ropes with the distance $b_1 = 2{,}0$ m, for stretching cables single ropes with the same distance $a_1 = 2{,}00$ m; then we have the effective thickness $t_x = 2{,}390$ mm, $t_y = 1{,}195$ mm and their ratio $\tau = 0{,}5$. Now we may calculate the values of geometrical parameters $1+\kappa_x = 1+\dfrac{5f_x^2}{3a^2} = 1{,}104,..1+\dfrac{5f_y^2}{3b^2} = 1{,}026$, $\psi$ = 1.3135 and $\mu = 1 + {}^{1}\!/\!_{\psi} = 1{,}7613$.
For the following we have to choose the contour ring's cross section; let us have a tubular bar $D_c \times t_c = 3600\times80$ mm; it's geometrical parameters $A_c = 0{,}8847$ m$^2$, $I_c = 1{,}3709$ m$^4$, $W = 0{,}7616$ m$^3$. Now we may calculate $\xi = 22{,}419$ and $\Phi = 389{,}89$ kN/m.

For the key equation (18) we have the network's pre-stress forces $H_{0x}$ = 108,8 kN/m,. $H_{0y}$ = 30,72 kN/m; pre-stress and loading parameters for cases of loading the network by total or dead load have correspondingly values $p_0* = 0.2260$; $p_1*$ = = 0,3064 for dead load and $p* = 0,7150$ for total load.

After calculating the coefficients we have the cubic key equation as follows

$$91{,}99\, z_0^3 + 82{,}132\, z_0^2 + 89{,}676\, z_0 = 0{,}7387 \text{ or } 0{,}3283.$$

Corresponding values for the relative and absolute deflection are $\zeta_0$ =0,03306, $w_0$ =0,661 m and $\zeta_0$ =0,01574, $w_0$ =0,315 m.

Change of the curvature may be characterized by the ratio of the roof centre deflection under the action of the live load to the shorter span (diameter); for our actual case we have the ratio $\dfrac{0,661-0,315}{128}=\dfrac{1}{370}$, what may be taken as normal for suspension roof structures. The cable forces for the state of pre-stressing are $H_{0x}$=108,80 kN/m and $H_{0y}$=30,72 kN/m.

For their increments we have $\Delta H_x$ = 392,1 kN/m and $\Delta H_y$ = 225,84 kN/m.

The maximum cable force $N = \dfrac{108,8+392,1}{\cos\left(\operatorname{arctg}\dfrac{2f_x}{a}\right)}$ = 560,0 kN/m and corresponding tensile stress $\sigma$ = 560 / 2,390 = 234,3 N/mm².

For determination of the bending moment for the contour ring we have to calculate its ellipticity $k = \sqrt{1-\beta^2}$ = 0,6; to this value correspond the full elliptical integrals $K(k)$ = 1,7508, $E(k)$ = 1,4181; calculation of coefficients $\dfrac{1+k^2}{3k^2}-\dfrac{\left(1-k^2\right)K(k)}{3k^2E(k)}$ and $\dfrac{1+k^2}{3k^2}-\dfrac{\left(1-k^2\right)K(k)}{3k^2E(k)}-1$ brings us to the values 0,5277 at $x = a$ and –0,4723 at $x$ = 0; these coefficients are to be multiplied to the value $\dfrac{1}{2}\left(H_x b^2 - H_y a^2\right)$ in cases both of maximum loading and pre-stressing. Taking into account corresponding forces and contour ring's dimensions, we have for the state of loading max $M$ = 108100 kN/m the maximum compression stress we obtain at the normal force $N = H_y a$ and the bending moment max $M$; so we have

$$\max\sigma = \left(\frac{108100}{761,6}+\frac{256,56\times 80}{884,7}\right)10^{-3} = 141,9+23,2 = 165,1 \text{ N/mm}^2 .$$

**Examples of construction of networks in Estonia.** For Estonia song festivals are events of important meaning. Therefore we have special song festival courts both in our city Tallinn and in the second town Tartu, where the first song festival was organized more than 140 years ago. On the both courts acoustic screens for song festival tribunes were constructed as inclined network structures. The first of them [1] was built in 1960 with the contour structure of two planar arches and massive counter-forts, the latter, erected in 1994 [4], represents an inclined hypar-network surrounded by an elliptical contour ring and covered by a timber shell (Fig. 4).

Fig. 4. Acoustic screen for the song festival tribune in Tartu

Both of them differ from roof structures for usual buildings by inclined form and absence of supporting columns on the front side. In Fig. 5 is presented the model 1:10 of the acoustic screen for the song festival tribune in Tartu.

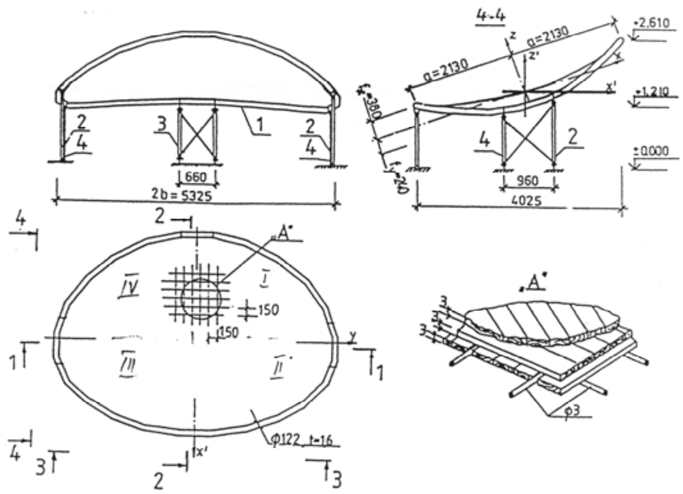

Fig. 5. Experimental model for the acoustic screen of the song festival tribune in Tartu

The possible roof structure for a spectacle or sports building may be illustrated by an experimental model, investigated at TTU (Fig. 6).

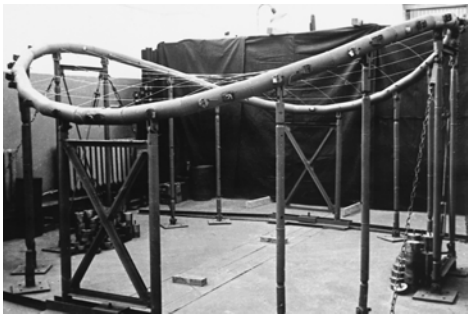

Fig. 6. Model for a roof network for spectacle or sports buildings

## Conclusion

**Methods of calculation.** For analysis of roof cable networks both discrete and continual models of calculation may be used. Continual models are suitable for hypar-formed networks. One of the most important preferences of the method, described in the subsection 3.3, consists in use of the cubic equation as the key equation for analysis. In that equation the linear member is the most importance. Therefore it may be solved by simple iteration process. Our experience affirms possibility to extend the results of analysis, obtained for hypar-networks, without remarkable inaccuracy also onto self-forming networks. Proceeding from a number of approximations in the continual model of analysis, presented in the subsection 3.3, we have compared corresponding results of analysis with results of more precise calculation for cases of symmetrical and one-sided live loading (Fig. 7). Our experience demonstrates appropriateness of using the method of calculation, given in the subsection 3.2, not only for orthogonal, but also for other form of networks (for example to self-forming ones). It is worth mentioning, that continual and discrete methods of analysis are eminently analogous.

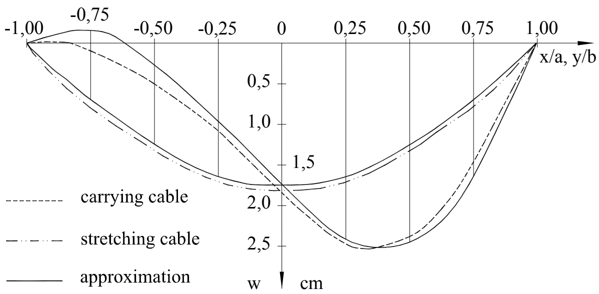


Fig. 7. Comparison of results of approximate and more precise analysis

**Comparison of networks with the contour structures of inclined arches and elliptical ring.** The contour structure of two inclined planar arches is usually supported by massive counter-forts and due to its great stiffness, the network may be characterized by relatively small deflections under the outer loads. But this advantage is contracted by abrupt change of cables' curvature. Relatively flat regions at the arch crowns are especially unfavourable as they tend to loose contact load and are susceptible to the action of fluctuating wind. The unfavourable distribution may be observed as for deflections so for cable forces. It is to be mentioned, that the comparison was made with equal areas of under-roof room; the stiffness of the contour bars was also equal (corresponding to the parameter $\xi$ = 80). The additional material consumption for the massive counter-forts was not taken into account by comparison in spite of its remarkable cost.

**Reference**

**[1]** Laul, H. and Kulbach, V. Aus den Erfahrungen beim Modellversuch und beim Prüfen des Hängedachs des Musikpavillons in Reval. *Hängedächer / I.M. Rabinovitš, ed.)* Bauverlag GmbH, Wiesbaden, Belin, 1966, pp. 202 – 206.

**[2]** Kulbach, V. and Oiger, K. About the initial form of the pre-stressed cable network. *Trans. Tallinn Polytechn. Inst. ,*1969, No 278, pp. 105 – 118 (in Russian).

**[3]** Kulbach, V. and Oiger, K. Investigation and erection of the saddle-shaped suspended, cable-tent and wooden shell roofs. *Proc. of the 1-st Intern. Conf. On Light-weight Structures.* Sydney, 1986, pp. 131 – 138.

**[4]** Kulbach, V. and Oiger, K. Rational form of saddle-shaped shells and suspension roofs. *Abstracts of the 2-nd Conf. "Spatial Roof Structures".* Tbilisi, 1984, pp. 47 – 48 (in Russian).

**[5]** Kulbach, V. Cable structures. Design and Static analysis. *Estonian Academy publishers.* Tallinn, 2007, 224 p-s.

*Надійшла до редколегії 03.06.2012 р.*