UDC 656.2

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EVALUATION OF STOCHASTIC CHARACTERISTICS OF GOODS DELIVERY SCHEDULES BY RAIL

The issues of statistical evaluation of goods delivery schedule efficiency are considered. Classification and stochastic analysis of the costs associated with delayed delivery in general are set. Stochastic model to assess the deviation of the actual delivery time from the minimum is proposed. The random variable is distributed according to Weibull.

У статті розглянуті питання статистичної оцінки ефективності графіків доставки вантажів залізничним транспортом. Проведено класифікацію та стохастичний аналіз витрат, пов'язаних з несвоєчасною доставкою в загальному випадку. Запропонована стохастична модель для оцінки відхилення фактичного часу доставки від мінімального, в якій дана випадкова величина розподілена за законом Вейбулла.

В статье рассмотрены вопросы статистической оценки эффективности графиков доставки грузов железнодорожным транспортом. Проведена классификация и стохастический анализ расходов, связанных с несвоевременной доставкой в общем случае. Предложенна стохастическая модель для оценки отклонения фактического времени доставки от минимального, в которой данная случайная величина распределена по закону Вейбулла.

Keywords: terms of delivery, Weibull distribution, the actual delivery time, stochastic costs, minimal (technological) delivery time.

Introduction. The terms of cargoes delivery have a direct impact on the performance of the obligations assumed by the contract of carriage. On the duration of cargo transportation depends the efficiency of transport, acceleration of vehicles turn-over time and the safety of cargoes. Timely delivery of goods to the destination is one of the main duties of the carrier, for breach of which comes liability.

Relevance of research. The process of European integration, rising transportation services standards necessitate compliance of the logistics principle "just in time".

Especially important for railway's transport service are accurate forecast of time of approach to transport hub and transfer wagon for unloading at each rail station, driveway, cargo handling facilities. It will enable to plan ahead and ensure the preparedness of the loading / unloading devices, human resources and trucks to deliver goods between customers and railway yards. Prediction of cargo arrival time is also required during transportation of raw materials to large industrial enterprises, the technology of which is based on providing a continuous production cycle and complex interdependent manufacturing processes. Such a prediction will allow enterprises to minimize emergency supply of raw materials at the warehouses without freezing a significant portion of current assets [4].

Trends in delivery of goods by rail through the development of specific technologies of transportation and scheduling delivery of goods, taking into account customer needs and available opportunities of railways, require the development of appropriate techniques and methods.

Analysis of publications. A number of works were devoted to assessing of the schedule delivery efficiency.

In [1] a stochastic model was performed to determine the most probable loss of railway fees associated with the delay of goods delivery time, assuming that the delay time is distributed exponentially.

In [2, 3] the costs associated with premature delivery are taken into account. In this model the deviation of actual time delivery from the sheduled (t_{sh}) subject to the normal distribution law.

The disadvantage of this model is that the normal random variable defined on the whole real axis, and the actual time of delivery shall not be less than a minimum (technological time) $t_{\min} > 0$ and the appropriate rejection $\Delta t > t_{\min} - t_{sh}$.

According to the authors, the two most common laws are not limited to possible statistical characteristics of the process of delivery.

Purpose was to assess the costs associated with delayed delivery of goods by means of a stochastic model, assuming that the deviation from the minimum delivery time is subject to Weibull distribution law.

The main part.

Let's introduce the following notation:

Δt^* is deviation of actual delivery time from minimum (technological time);

t_i is time within which the largest number of goods are expected to be delivered (distribution mode);

$\Delta t_i^* = t_i - t_{\min}$;

t_m is time within which half of goods are expected to be delivered (median);

$\Delta t_m^* = t_m - t_{\min}$

$\tau^* = \dfrac{\Delta t^*}{t_{\min}}$ – relative deviation of the actual delivery time from minimum.

Let the random variable that describes the deviation of actual time delivery from a minimum have the following density function:

$$f(\Delta t^*) = \begin{cases} 0, & \Delta t^* < 0 \\ f(\Delta t^*), & \Delta t^* \geq 0 \end{cases}.$$

Then, when $\Delta t^* < \Delta t_{sh}$ costs associated with servicing prematurely delivered goods are occurred (for example, storage, warehousing etc.), and when $\Delta t^* > \Delta t_{ãð}$ – costs associated with the delay of the delivery time (fines for late delivery).

In [2] the total costs were given as:

$$C = C_{pen} + C_{st} + C_{\text{var}} + \xi$$

where C_{pen} is penalty for late delivery time;

C_{st} – fixed costs associated with servicing prematurely delivered goods;

C_{var}– variable costs, depending on the service duration of prematurely delivered goods;

ξ – «stochastic» costs, caused by the influence of random external factors.

In general case:

$$C_{pen} = l \int_{\Delta t^*_{sh}}^{+\infty} \gamma(\Delta t^*) f(\Delta t^*) d\Delta t^* \quad (1)$$

$$C_{st} = l \int_{0}^{\Delta t^*_{sh}} f(\Delta t^*) d\Delta t^* \quad (2)$$

$$C_{\text{var}} = l \int_{0}^{\Delta t^*_{sh}} k(\Delta t^*) f(\Delta t^*) d\Delta t^* \quad (3)$$

$$\xi = l \int_{0}^{+\infty} \xi(\Delta t^*) f(\Delta t^*) d\Delta t^* , \quad (4)$$

where l is total number of transported goods in a certain period of time (e.g. one year), $k(\Delta t^*)$ та $\gamma(\Delta t^*)$ – rates for cargo unit for servicing prematurely delivered goods and the delay time of delivery (for example, the fee for use).

Similar formulas can be written for the relative deviation of the actual delivery time from the minimum (technological), replacing in (1) – (4) Δt^* to τ^* (in accordance $\Delta t^*_{ãð}$ to $\tau_{ãð}{}^*$).

Considering that fines are fixed in a certain interval of delay time, that is $\gamma(\Delta t^*) = \gamma_i$ when $\Delta t^* \in [t_i ; t_{i+1}]\, i = \overline{0, n}$, than (1) can be written as a sum of integrals:

$$C_{pen} = l \sum_{з=1}^{n} \gamma_i \int_{\Delta t^*_i}^{\Delta t^*_{i+1}} f(\Delta t^*) d\Delta t^* ,$$

where Δt^*_0 – minimum time delivery delay, after which the fines come $\Delta t^*_{n+1} = +\infty$ (for example, according to [5] for internal transport Δt^*_0 = 2 days).

Considering that C_{var} is linearly depends on the service time $k(\Delta t^*_i) = k \cdot \Delta t^*$, (3) can be rewritten as:

$$C_{\text{var}} = k \int_{0}^{\Delta t^*_{sh}} \Delta t^* f(\Delta t^*) d\Delta t^* .$$

To assess the deviation of the actual delivery time from minimum the stochastic model is proposed in which this random variable is distributed according to Weibull distribution with density:

$$f(\Delta t^*) = \begin{cases} 0, & \Delta t^* < 0 \\ \alpha\beta(\Delta t^*)^{\beta-1} e^{-\alpha(\Delta t^*)^\beta}, & \Delta t^* \geq 0, \quad \alpha > 0, \beta > 1 \end{cases}.$$

To determine the parameters α, β it's necessary to have empirical data on Δt_i^* and Δt_m^*. After statistical data processing with respect to t_i, for which most of goods are delivered to a certain route section and t_m, for which half of goods are delivered, we find point estimates of these terms and deviation $\overline{\Delta t_i^*}$ and $\overline{\Delta t_m^*}$. Differentiating (5) and equating the resulting derivative to zero, we find:

$$\Delta t_i^* = \alpha^{-\frac{1}{\beta}} \left(\frac{\beta - 1}{\beta} \right)^{\frac{1}{\beta}} \tag{6}$$

By solving the equation

$$\int_0^{\Delta t_m^*} \alpha\beta(\Delta t)^{\beta-1} e^{-\alpha(\Delta t^*)^\beta} d\Delta t^* = \frac{1}{2}$$

we obtain

$$\Delta t_m^* = \alpha^{-\frac{1}{\beta}} (\ln 2)^{\frac{1}{\beta}}. \tag{7}$$

Replace such deviations to the empirical ($\overline{\Delta t_i^*}$ and $\overline{\Delta t_m^*}$) and solve the system (6), (7). β is determined from the equation: $P = \frac{\overline{\Delta t_m^*}}{\overline{\Delta t_i^*}} = \left(\frac{\beta \ln 2}{\beta - 1} \right)^{\frac{1}{\beta}}$ or equivalent to it $P^\beta = \ln 2 \left(1 + \frac{1}{\beta - 1} \right)$.

When $P > 1$ ($\Delta t_m > \Delta t_i$) this equation has one root: $\beta < (1 - \ln 2)^{-1} \approx 3{,}259$. Figure 1 graphically shows that this equation has a root $\beta > 1$.

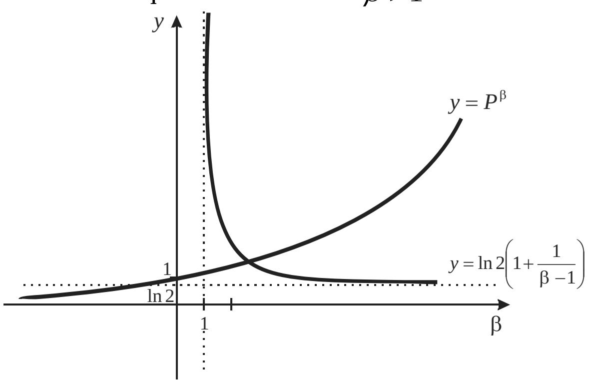

Fig. 1. **Graphical determination of the parameter β**

β can be find by approximate methods with the required accuracy. Then:

$$\alpha = \frac{\ln 2}{(\Delta t_m^*)^\beta}.$$

This model accurately describes the distribution of deviations of actual time delivery from the minimum, but it is not universal. For each route section $t_{\min}$ is different, so every time it is necessary to assess the empirical data. If instead of the absolute take a relative deviations, all calculations while replacing Δt^* to τ^* won't change.

In this case for the function $f(\tau^*)$ β remain the same, and $\alpha = \frac{\ln 2}{\tau_m^{*\,\beta}}$.

The figures 2, 3 presented graphs of $f(\tau^*)$ and $F(\tau^*)$ for $\alpha = 0{,}5; 1; 2$, $\beta = 2{,}5$.

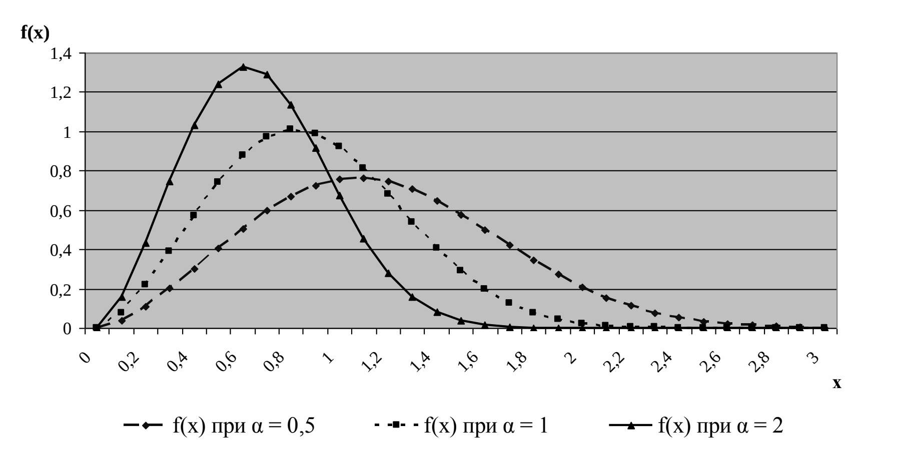

Fig. 2. **Density of Weibull distribution at** $\beta = 2{,}5$, $\alpha = 0{,}5; 1; 2$

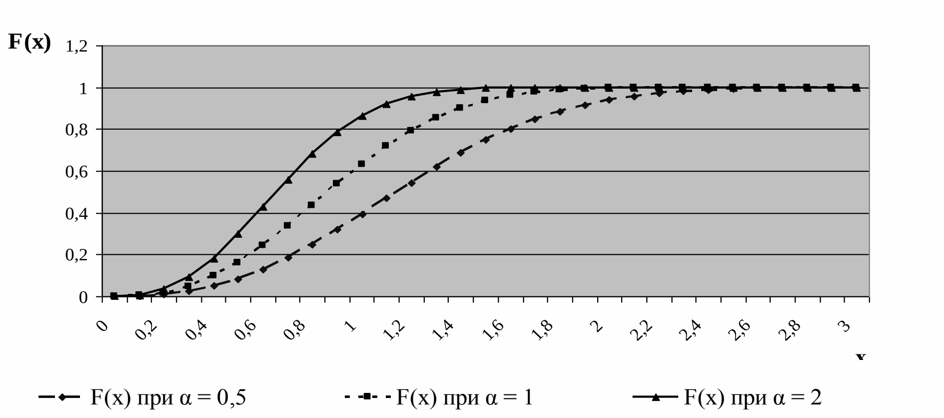

Fig. 3. **Integrated Weibull distribution function at** $\beta = 2{,}5$, $\alpha = 0{,}5; 1; 2$

The graphs show that while increasing α (decreasing τ_m^*) the portion of goods with significant deviations from scheduled delivery time, which is closely associated with τ_m^* and τ_M^*, decreases. This makes it possible the significant reduction of the costs associated with delayed delivery. In addition, this model takes into account the asymmetry of scheduled delivery time that suggests it is more realistic than the model based on the assumption of normal distribution of deviations.

It should be noted that by increasing β costs are reduced.

Conclusions. To apply the proposed model it is necessary to carry out selective research of t_i and t_m in different parts of the railway line between node stations, then find the point estimates of $\overline{\tau_i^*}$ and $\overline{\tau_m^*}$. Such research can be based on statistical data processing of shipping documents on goods delivery in different kinds of links.

Then it is necessary to check the hypothesis of Weibull distribution of deviations from the minimum delivery time. Confirmation of this hypothesis requires correction of scheduled delivery time to increase the rate of α and β, that will significantly improve the efficiency of transport service.

In further work it is planned to develop an appropriate analysis of statistical data and consider a similar model based on the gamma distribution.

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