

A.N. Bogomolov, Prof., DrSc.
Volgograd State University of Architecture and Civil Engineering, Russia
Perm National Research Polytechnic University, Russia
O.A. Bogomolova, PhD., associate professor
Volgograd State University of Architecture and Civil Engineering, Russia

MIXED TASK OF THE THEORY OF ELASTICITY AND THEORY OF PLASTICITY OF SOIL FOR THE UNIFORM BASIS

Article contains approximate analytical solution of the mixed task of the theory of elasticity and theory of plasticity of soil: formulas for calculation of tension in points of areas of plastic deformations and expressions, allowing to define position and form of the last are presented. Graphics of areas of plastic deformations are given in the uniform basis of the buried base at various values of physic mechanical properties of soil.

Keywords: mixed task of the theory of elasticity and theory of plasticity of soil, size, position and form of area of plastic deformations, tension components in elastic and plastic areas.

О.М. Богомолов, д.т.н., профессор
Волгоградський державний архітектурно-будівельний університет, Росія
Пермський національний політехнічний дослідний університет, Росія
О.О. Богомолова, к.т.н., доцент
Волгоградський державний архітектурно-будівельний університет, Росія

ЗМІШАНА ЗАДАЧА ТЕОРІЇ ПРУЖНОСТІ ТА ТЕОРІЇ ПЛАСТИЧНОСТІ ҐРУНТУ ДЛЯ ОДНОРІДНОЇ ОСНОВИ

Наведено наближене аналітичне рішення змішаної задачі теорії пружності й теорії пластичності ґрунту: подано формули для обчислення напружень у точках областей пластичних деформацій і вирази, які дозволяють визначити положення та форму останніх. Наведено графічні зображення областей пластичних деформацій в однорідній основі заглибленого фундаменту при різних значеннях фізико-механічних властивостей ґрунту.

Ключові слова: змішана задача теорії пружності та пластичності ґрунту, розмір, положення й форма області пластичних деформацій, компоненти напруження в пружних і пластичних областях.

А.Н. Богомолов, д.т.н., профессор
Волгоградский государственный архитектурно-строительный университет, Россия
Пермский национальный политехнический исследовательский университет, Россия
О.А. Богомолова, к.т.н., доцент
Волгоградский государственный архитектурно-строительный университет, Россия

СМЕШАННАЯ ЗАДАЧА ТЕОРИИ УПРУГОСТИ И ТЕОРИИ ПЛАСТИЧНОСТИ ГРУНТА ДЛЯ ОДНОРОДНОГО ОСНОВАНИЯ

Приведено приближенное аналитическое решение смешанной задачи теории упругости и теории пластичности грунта: представлены формулы для вычисления напряжений в точках областей пластических деформаций и выражения, позволяющие определить положение и форму последних. Приведены графические изображения областей пластических деформаций в однородном основании заглибленного фундамента при различных значениях физико-механических свойств грунта.

Ключевые слова: смешанная задача теории упругости и пластичности грунта, размер, положение и форма области пластических деформаций, компоненты напряжения в упругих и пластических областях.

Problem definition. Among various ways of statement of nonlinear tasks for the soil basis the separate place is taken by the mixed task of the theory of elasticity and plasticity of the soil idea of solving of which was formulated for the first time by Russian scientists, first by D.E. Polshin [1, 2], and then M.I. Gorbunov-Posadov [3].

Problem definition has the following features. Until intensity of external load of the basis is insignificant, areas of plastic deformations are absent. At achievement by intensity of external loading of some limit value q_{lin} in soil arises one or several plastic areas which have a clear boundary. Further growth of loading conducts to increase in the sizes of plastic areas, on internal surfaces of borders and in which soil is in ideally plastic condition.

Part of the soil body located outside the boundaries of areas of plastic deformations, is in linearly-elastic condition. In some time point loading reaches the second limit value q_{lim} , to which there corresponds completion of development and association of plastic areas, the basis passes to the plastic condition described by the theory of limit balance.

Thus, physical model of the mixed task of the theory of elasticity and the theory of plasticity of soil means that:

1. Between elastic and plastic areas there is a clear boundary. During the process of basis loading, the soil being in elastic areas, submits to laws of the linear theory of elasticity, and in the plastic – to laws of the theory of plasticity; both in elastic, and in plastic areas the balance equations are carried out.

2. In each point of border the condition of a continuity of a field of tension is satisfied: normal and tangent tension to border has to be identical on both of its parties.

3. In areas of linear deformability the condition of compatibility of deformations, and in plastic areas – a plasticity condition is satisfied.

Thus, the mixed task allows to connect the solution of a task of the linear theory of elasticity for an initial stage of loading and the solution of a task of the theory of limit balance for a stage of destruction of the basis that is very important from the point of view of development of the uniform theory describing behavior of the basis in a full interval of loadings.

Its analytical decision is connected with permission of system of the differential equations of balance, the equation of continuity of deformations and plasticity conditions which for a flat task register as follows (an axis X directed to the right, Z – vertically down):

$$\left. \begin{aligned} \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} + X &= 0; \\ \frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{zx}}{\partial x} + Z &= 0; \end{aligned} \right\}; \quad (1)$$

$$\nabla^2(\sigma_x + \sigma_z) = -\frac{1}{1-\mu} \left(\frac{\partial X}{\partial x} + \frac{\partial Z}{\partial z} \right); \quad (2)$$

$$(\sigma_x - \sigma_z)^2 + 4\tau_{xz} = (\sigma_x + \sigma_z + 2\sigma_{c\%})^2 \sin^2 \varphi, \quad (3)$$

where σ_z ; σ_x ; τ_{xz} – vertical, horizontal and tangent components of full tension in a considered point; X ; Z – projections of total (including a body weight of soil) a vector of loads of the corresponding axes of coordinates; $\sigma_{coh}=Cctg\varphi$ – connectivity pressure; C and φ – respectively specific cohesion and a corner of internal friction of soil; ∇^2 – Laplace's operator; μ – soil Poisson's coefficient.

It is obvious that these equations have to be added with boundary conditions on an external contour of the basis and border of elastic and plastic areas, and also dependences characterizing mode of deformation of the soil in plastic area. It is natural that the solution of the mixed task has to be constructed so that the equations (1) were carried out in all soil thickness, the equation (2) – only in elastic, and the equation (3) – only in plastic area.

For component definition of full tension in the points located in areas of plastic deformations, we will use the assumptions made in work [4], that at the time of "transition" of a point of the soil body in the course of loading from elastic into plastic condition the corner of orientation of a platform of shift α and size of normal vertical tension of σ_z practically don't change.

The correctness of these assumptions is obvious. According to analytical solutions of tasks of the theory of plasticity in limit strained condition in soil there is an uncountable set of surfaces of sliding which are absolutely equivalent [5, 6]. Besides, as a result of the experiment made by us on destruction of the bases on models from equivalent materials it is established that real surfaces of destruction are formed near NVPR constructed on tensions, found on the basis of the solution of a task of the theory of elasticity (see fig. 1) [7]. Also it is established that with transition of a slope to a limit condition vertical tension in the corresponding points of the soil body practically doesn't change [4].

Determination of tensions in plastic areas. For formulas derivation defining numerical values of tension in plastic areas, we will receive some ratios.

Let's consider balance of an infinitesimal prism of the single height (fig. 2a) and work out the equation of projections to the directions \bar{n} and $\bar{\tau}$, from which we will express normal σ_n and tangent τ_n components of the full tension operating on an inclined platform with a corner α through components σ_z ; σ_x ; τ_{xz} tension in considered point.

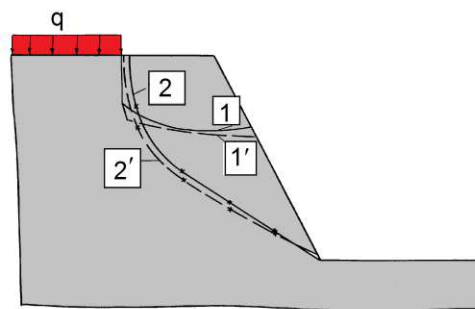


Fig. 1. The most probable surfaces of destruction in models from igdantin (1) and sand-oil mix (2), constructed on "elastic" tension; corresponding real surfaces of destruction (1') and (2'), received on models in the course of carrying out experiment (It is quoted on work [7])

Will receive

$$\left. \begin{aligned} \sigma_n &= \sigma_z \cos^2 \alpha + \sigma_x \sin^2 \alpha + 2\tau_{xz} \sin \alpha \cos \alpha ; \\ \tau_n &= (\sigma_x - \sigma_z) \sin \alpha \cos \alpha + \tau_{xz} (\cos^2 \alpha - \sin^2 \alpha) . \end{aligned} \right\} \quad (4)$$

Let's write down expression

$$K\tau_n = \sigma' \operatorname{tg} \varphi, \quad (5)$$

where K – some function of a tension, physic mechanical properties and tilt angle of a platform of possible shift of soil in a considered point of the soil body; $\sigma' = \sigma_n + \sigma_{\text{coh}}$ – the specified normal tension; σ_{coh} – pressure of connectivity of $\sigma_{\text{coh}} = C \operatorname{ctg} \varphi$ (C – specific coupling; φ – corner of internal friction of soil) [15].

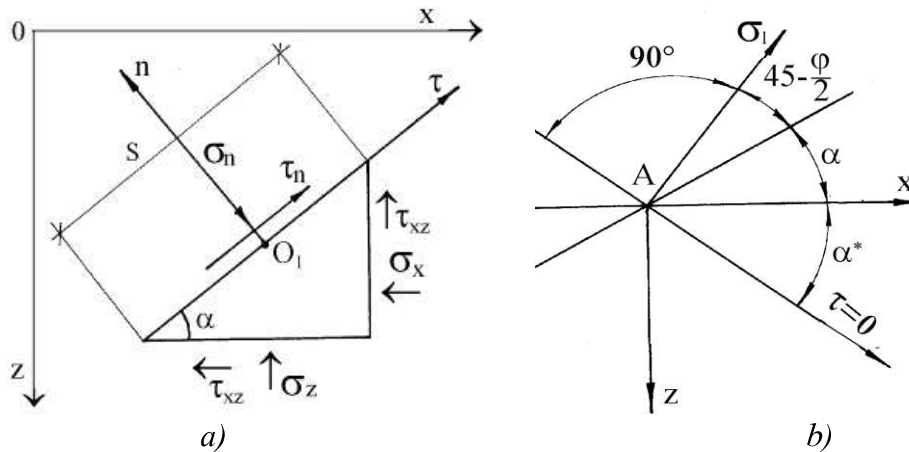


Fig. 2. Loading diagram for definition of normal σ_n and tangent τ_n component of tension operating on an inclined platform (a); loading diagram to definition of a corner α^* (b) (It is quoted in work [11])

It is easy to see that at $K=1$ expression (5) turns into a condition of durability of Coulomb in a look, offered by G. Kako [8]. Substituting values (4) into formula (5), we will receive expression for function K which as it is accepted, we will call coefficient of a stock of stability in a point of the soil body, and numerator and a denominator of the received fraction - according to holding F_{hol} and shifting F_{sh} forces operating along the most probable platform of shift

$$K = \frac{[\sigma_z \cos^2 \alpha + \sigma_x \sin^2 \alpha + 2\tau_{xz} \sin \alpha \cos \alpha + \sigma_{\text{CB}}] \operatorname{tg} \varphi}{(\sigma_x - \sigma_z) \sin \alpha \cos \alpha + \tau_{xz} (\cos^2 \alpha - \sin^2 \alpha)} . \quad (6)$$

Let's note that the expressions similar to (6), are given in works [9, 10].

Let's assume that some point A (see fig. 2b) after increase in external loading "passed" into the area of plastic deformations.

It is known [5] that the shift platform in plastic area is focused at an angle $45^\circ - \varphi/2$ to the direction of the maximum main tension of σ_1 , and the tangent tension τ_n on a platform, perpendicular to σ_1 , is equal to zero. Equating a denominator of expression (6) to zero and, solving the received equation relatively to α , we have

$$\operatorname{tg} 2\alpha^* = \frac{2\tau'_{xz}}{\sigma'_z - \sigma'_x}, \quad (7)$$

where σ'_z ; σ'_x и τ'_{xz} – tension components in plastic area.

From figure (2б) can be seen that

$$|\alpha^*| = 45^\circ + \varphi - \alpha, \quad (8)$$

where α – the tilt angle of the most probable surface of destruction provided that a considered point A is in a up to limit condition.

However the corner α^* is counted as it follows from fig. 2b, from the positive direction of an axis X clockwise and, therefore, according to the standard rule of signs, finally we have

$$\alpha^* = \alpha - (45^\circ + \frac{\varphi}{2}). \quad (9)$$

Above experimentally reasonable assumption is accepted that the vertical component of tension of σ_z in a considered point upon "transition" of the last from elastic area in plastic can be defined on the basis of the relevant decision of a task of the linear theory of elasticity. This assumption, we will emphasize once again, means for us that it is possible to define a vertical component of full tension of σ'_z in any point of area of plastic deformations on the basis of the analytical solution of the corresponding task of the theory of elasticity [11] or any other including a numerical, known method, i.e.

$$\sigma'_z = \sigma_z. \quad (10)$$

Thus, a condition of plasticity (3), expressions (7), (9) and equality (10) represent, at the assumptions accepted by us, closed system of equations of a tension in plastic area

$$\left. \begin{aligned} \sigma'_z &= \sigma_z; \\ (\sigma'_x + \sigma'_z)^2 + 4\tau'^2_{xz} &= (\sigma'_x + \sigma'_z + 2\sigma_{\text{coh}})^2 \sin^2 \varphi; \\ \operatorname{tg} 2\alpha^* &= \frac{2\tau'_{xz}}{\sigma'_z - \sigma'_x}; \\ \alpha^* &= \alpha - (45^\circ + \frac{\varphi}{2}). \end{aligned} \right\} \quad (11)$$

The sign (\square) says that tension operating in the field of plastic deformations are meant.

Solving system (11) according to σ'_z , σ'_x and τ'_{xz} , we will receive:

$$\left. \begin{aligned} \sigma'_z &= \sigma_z; \\ \sigma'_x &= \frac{\sigma_z(l - \sin \varphi) - 2\sigma_{\text{coh}} \sin \varphi}{l + \sin \varphi}; \\ \tau'_{xz} &= \frac{(\sigma_z + \sigma_{\text{coh}})b \sin \varphi}{l + \sin \varphi}, \end{aligned} \right\} \quad (12)$$

where $b = \operatorname{tg} 2\alpha^*$; $l = (1 + b^2)^{\frac{1}{2}}$.

In case of ideally coherent soil environment ($\varphi=0$) of a formula (12) give the hydrostatic law of distribution of tension in the plastic area $\sigma_z=\sigma_x$; $\tau_{xz}=0$, that corresponds to this limit case [3].

Numerical value of a corner α can be found on the basis of the proposal of prof. V.K. Tsvetkov [10] which is consolidated to performance of a condition of $K=K_{\min}$.

Let's consider a way of definition of a corner α offered by us from a condition of a minimality of coefficient of residual resistance to shift.

The condition of accessory of some point of the soil body of area of elastic deformations at this tension can be described by two expressions

$$\left. \begin{aligned} K\tau_n &= (\sigma_n + \sigma_{\text{coh}})tg\varphi; \\ \tau_n + \tau_{\text{bal}} &= (\sigma_n + \sigma_{\text{coh}})tg\varphi, \end{aligned} \right\} \quad (13)$$

(from (13) it is visible that at $K=1$ and $\tau_{\text{bal}}=0$ both equalities degenerate in a Coulomb's condition of durability)

Equating the left parts of expressions (13), we will receive

$$\tau_n(K-1) = \tau_{\text{bal}}. \quad (14)$$

Substituting τ_{bal} value from (14) in the second from equalities (13), we will receive

$$\tau_n + \tau_n(K-1) = (\sigma_n + \sigma_{\text{coh}})tg\varphi \quad (15)$$

or, that is the same

$$f = \frac{(\sigma_n + \sigma_{\text{coh}})tg\varphi - \tau_n}{\tau_n}, \quad (16)$$

where $f=(K-1)$ - coefficient of residual resistance to shift.

Having taken a derivative $\frac{\partial f}{\partial \alpha}$, and comparing it to a derivative $\frac{\partial K}{\partial \alpha}$, we can see that $\frac{\partial f}{\partial \alpha} = \frac{\partial K}{\partial \alpha}$, i.e. the tilt angle of the platform having the minimum value of coefficient of residual resistance to shift, is equal to a platform tilt angle with the minimum coefficient of a stock of stability.

For minimization of size f we use known conditions $\frac{\partial f}{\partial \alpha} = 0$ and $\frac{\partial^2 f}{\partial \alpha^2} > 0$ that leads to the solution of the following trigonometrical equation

$$2(\sigma_x + \sigma_z)\tau_{xz} \sin 2\alpha - (\sigma_x^2 + \sigma_z^2) \cos 2\alpha + (\sigma_x + \sigma_z)^2 + 4\tau_{xz}^2 = 0 \quad (17)$$

Expressing $\cos 2\alpha$ through $\sin 2\alpha$, entering the designations given below and carrying out the corresponding transformations, we reduce the equation (17) to square relatively $\sin 2\alpha$, which roots are calculated as follows

$$\sin 2\alpha_{1,2} = \frac{-A_1 D_1 \pm \sqrt{A_1^2 D_1^2 + (A_1^2 + B_1^2) B_1^2}}{A_1^2 + B_1^2}, \quad (18)$$

where $A_1 = 2(\sigma_x + \sigma_z)\tau_{xz}$; $B_1 = (\sigma_x^2 - \sigma_z^2)$; $D_1 = (\sigma_x - \sigma_z)^2 + 4\tau_{xz}^2$.

Thus, all parameters entering into formulas (12) are determined and they can be used for calculation of numerical values of tension in the points contained by plastic areas.

Definition of position and border form between areas of elastic and plastic deformations. For delimitation between areas of elastic and plastic deformations we will use the equations of balance and a condition of a continuity of a field of tension in each point of border: normal and tangents to tension border in each its point have to be identical on both of its parties.

Let curve AA' – be border between elastic and plastic areas and the point B lies on this curve (fig. 2a). Let's cut out mentally near point B an infinitesimal rectangular prism of single height $DEKM$ located in such a way that point B lies on its diagonal, and the diagonal DK is a tangent to curve AA' in B . Let's divide rectangular prism of $DEKM$ into two triangular prisms of DEK and DMK in such a way that the first of them will be in area of elastic deformations, and the second - in plastic area. Considering that the equations of balance are carried out in all volume of the soil body, we will make them for both prisms and will express from there normal and tangent components of the full tension operating on inclined sides of prisms of DEK and DMK through components of full tension of $\sigma_z; \sigma_x; \tau_{xz}$ и $\sigma'_z; \sigma'_x; \tau'_{xz}$.

We have

$$\left. \begin{aligned} \sigma_n &= \frac{1}{2}(\sigma_x - \sigma_z) \cos 2\theta + \frac{1}{2}(\sigma_x + \sigma_z) + \tau_{xz} \sin 2\theta ; \\ \tau_n &= \frac{1}{2}(\sigma_x - \sigma_z) \sin 2\theta + \tau_{xz} \cos 2\theta . \end{aligned} \right\} \quad (19)$$

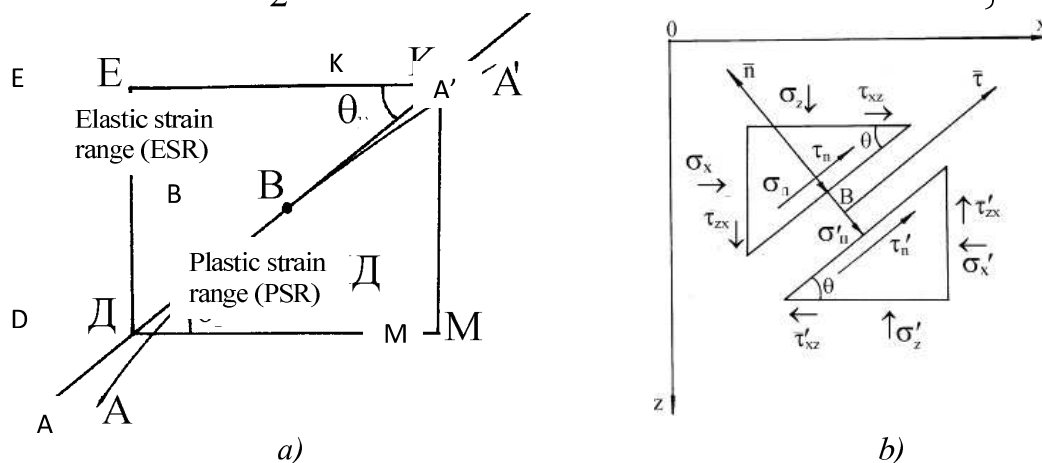


Fig. 3. Elementary prism of soil on border of elastic and plastic areas (a); scheme for drawing up the equations of balance (b)

$$\left. \begin{aligned} \sigma'_n &= \frac{1}{2}(\sigma'_x - \sigma'_z) \cos 2\theta + \frac{1}{2}(\sigma'_x + \sigma'_z) + \tau'_{xz} \sin 2\theta ; \\ \tau'_n &= \frac{1}{2}(\sigma'_x - \sigma'_z) \sin 2\theta - \tau'_{xz} \cos 2\theta . \end{aligned} \right\} \quad (20)$$

Taking into account that

$$\tau_n = \tau'_n ; \quad \sigma_n = \sigma'_n. \quad (21)$$

we substitute in (21) values from (19) and (20) and meaning that $\sigma_z = \sigma'_z$ and σ'_x ; τ'_{xz} are defined by formulas (12), we will receive the expressions describing border of elastic and plastic areas

$$\operatorname{tg} 2\theta = \frac{2 \cdot (\tau_{zx} + \tau'_{zx})}{2 \cdot \sigma'_x - \sigma_x - \sigma_z}, \quad (22)$$

$$\sin \varphi_\theta = \frac{l \cdot (\sigma_z \cdot (3 + \cos 2\theta) + \sigma_x \cdot (1 - \cos 2\theta) + 2 \cdot \tau_{zx} \cdot \sin 2\theta)}{- (\sigma_z + \sigma_x) + 2 \cdot \sigma_{coh} + \cos 2\theta \cdot (\sigma_x - 3 \cdot \sigma_z - 2 \cdot \sigma_{coh}) - 2 \cdot \sin 2\theta \cdot (\tau_{zx} + b \cdot (\sigma_z + \sigma_{coh}))}, \quad (23)$$

where θ – corner between a tangent to border AA' in a point B and the positive direction of the OX axis; φ_θ – "limit" value of internal friction of soil corner at which this point of the massif "passes" at this stage of loading to area of plastic deformations; b and l – the same, as in formulas (12); σ_z ; σ_x ; τ_{xz} – components of the full tension found from the analytical solution of a task of the theory of elasticity for this stage of loading.

Let's stop on a question where and in what direction will pass the origin of plastic areas. The direction of a beam along which zones of plastic deformations start developing, has to be defined by the direction of the maximum tangent tension on condition of their equality in elastic and plastic areas. Taking derivatives $\frac{d\tau_n}{d\theta}$ and $\frac{d\tau'_n}{d\theta}$, equating them and meaning formulas (12), we will receive the expression defining the direction of origin of zones of plastic deformations

$$\operatorname{tg} 2\theta_3 = \frac{l(\sigma_x - \sigma_z) + (\sigma_x - 2\sigma_{coh} - 3\sigma_z) \sin \varphi}{2\{\sin \varphi[(\sigma_z + \sigma_{coh})b + \tau_{xz}] + \tau_{xz}l\}}, \quad (24)$$

where θ_3 – a tilt angle of a beam along which there is an origin of area of plastic deformations.

It is known [5, 6, 12] that in case of ideally coherent environment, tangents to both families of surfaces of destruction coincide with the direction of the maximum tangent tension in a considered point of the soil body. If in a formula (24) put $\varphi=0$, it will assume well-known in the elasticity theory look

$$\operatorname{tg} 2\theta_3 = \frac{\sigma_x - \sigma_z}{2\tau_{xz}}. \quad (25)$$

Besides, the formula (25) is completely identical to the expression given in work [10], for definition of a tilt angle of a tangent to the most probable surface of destruction at the point lying on a contour of a slope and possessing the minimum value of coefficient of stability.

In fig. 4 as an example areas of plastic deformations are given in the uniform basis of the buried base provided that tension in an active zone of the base is counted on the basis of the analytical solution of the first main objective of the theory of elasticity for one-coherent area with trapezoid cut on border [11] methods of the theory of functions complex variable [13].

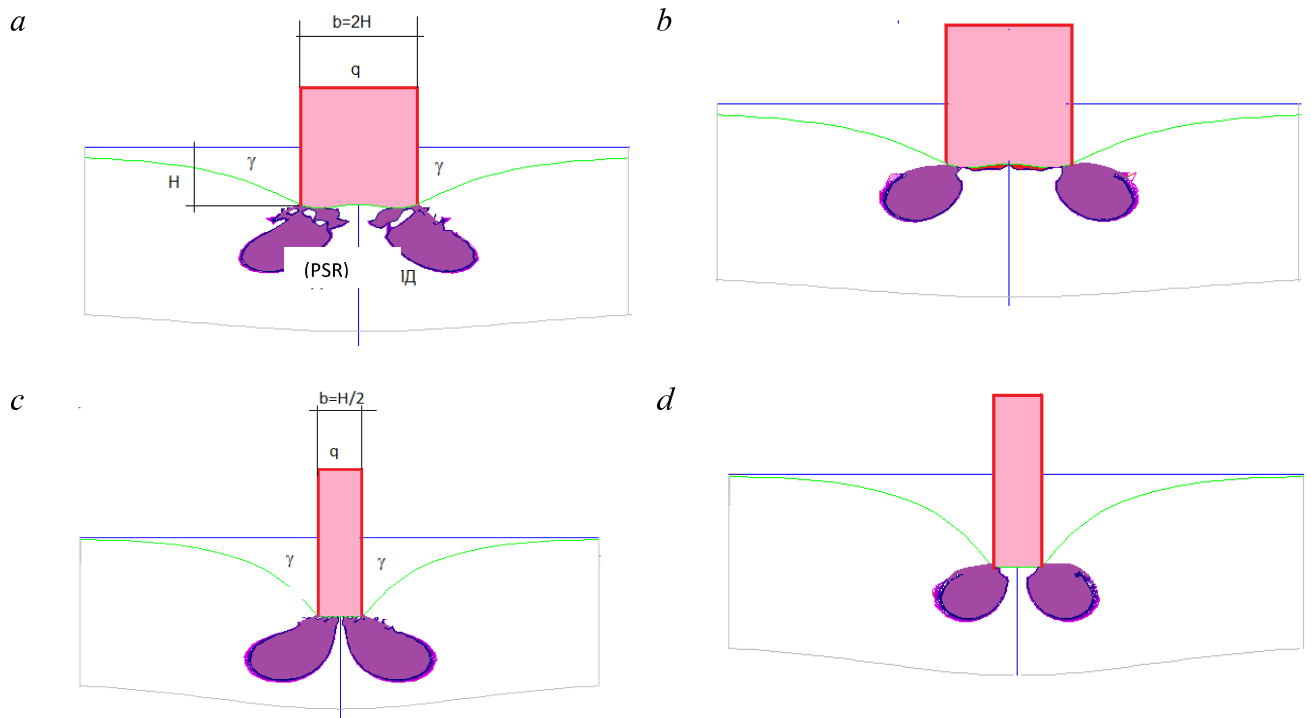


Fig. 4. Areas of plastic deformations in the uniform basis of the buried base at $\xi_o=0,75$ (a, c) and at $\xi_o=0,30$ (b, d)

The size of the relation of width of the base to depth of its run makes $b/H=2$ (fig. 4 a, b) and $b/H=0,5$ (fig. 4 c, d). The corner of internal friction of soil is constant and equal to $\varphi=16^\circ$, and specific coupling of C, the volume weight of soil γ and depth of a run of the base are that that the size of reduced pressure of connectivity $\sigma_{coh} = C(\gamma H \tan \varphi)^{-1} = 2,58$. Thus the size of intensity of evenly distributed loading is constant and equal $q = 2\gamma H$. Difference between figures 4a and 4b, 4c and 4d consists that in fig. 4a and 4c of area of plastic deformations are constructed at the size of coefficient of lateral pressure of soil $\xi_o=0,75$, and at creation of fig. 4b and 4d the size of coefficient of lateral pressure is accepted by equal $\xi_o=0,30$.

On fig. 5–10 examples of an evolving of areas of plastic deformations in the uniform basis of the flexible base (in view of symmetry half of loading diagrams are shown) are given at action of evenly distributed loading increasing in time which are received using a Coulomb's condition of plasticity in the form of (5) provided that $K=1$ (fig. 5, 7, 9) and by means of the expressions (22) and (23) received at the solution of the mixed task of the theory of elasticity and the theory of plasticity of soil [11] (fig. 6, 8, 10).

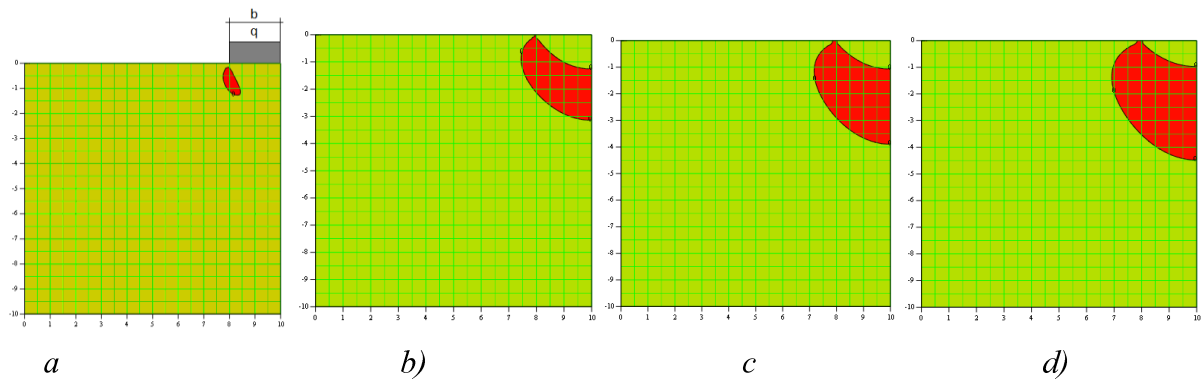


Fig. 5. Loading diagram of the loaded uniform basis and stages of evolution of areas of plastic deformations in the basis of not buried base at $2b=2$ m and the size of intensity of evenly distributed loading of $q/\gamma b=3,0; 4,0; 5,0; 6,0$ constructed on the basis of a plasticity condition (a – d)

Thus soil has the following characteristics: specific cohesion $C=16\text{kPa}$, corner of internal friction of $\varphi=16^\circ$, specific gravity $1,96 \text{ t/m}^3$. The size of coefficient of lateral pressure of soil is accepted to be equal $\xi_0=0,75$. Base width when carrying out calculations consistently accepts values $2b=2,0; 1,0; 0,5$ m.

Let's note that all our calculations and graphic constructions are carried out by means of the computer program [15] developed at the Volgograd State University of Architecture and Civil Engineering.

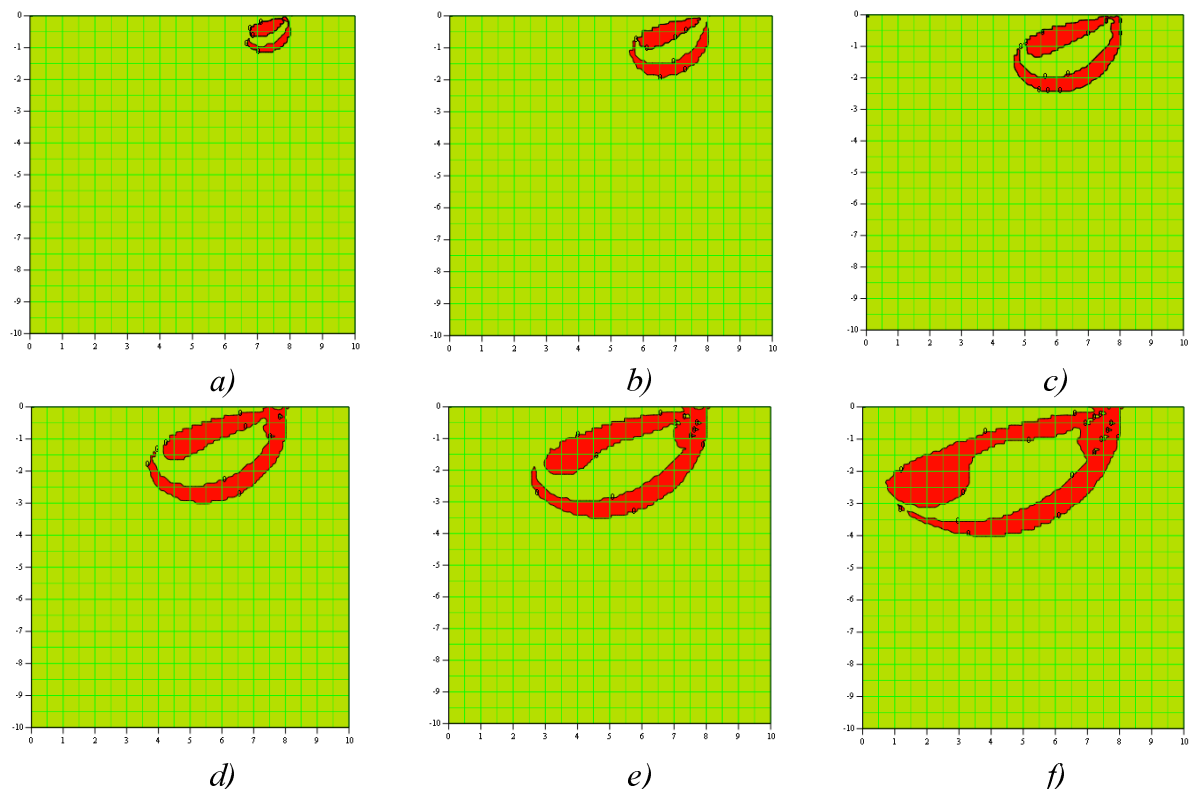


Fig. 6. Stages of evolution of areas of plastic deformations in the basis of not buried base at $2b=2$ m and the size of intensity of evenly distributed loading of $q/\gamma b=1,0; 2,0; 3,0; 4,0; 5,0; 6,0$ constructed on the basis of the solution of the mixed task of the theory of elasticity and the theory of plasticity of soil [11] (a – f)

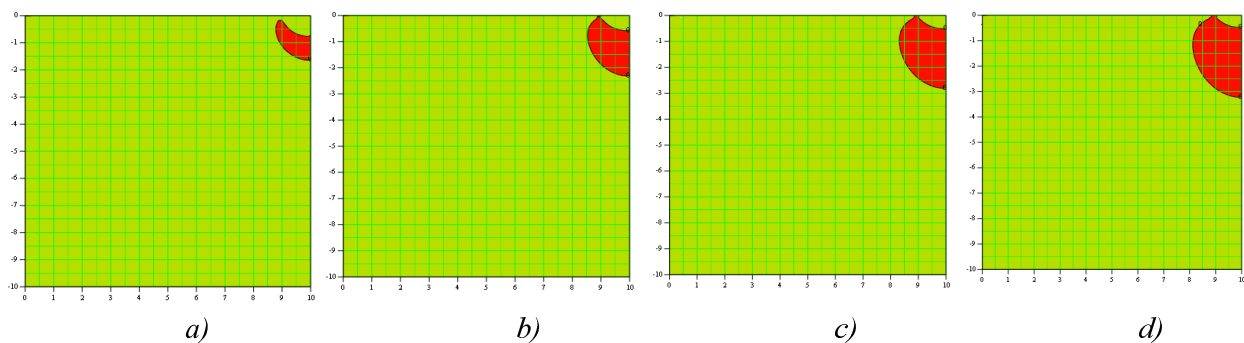


Fig. 7. Stages of evolution of areas of plastic deformations in the basis of not buried base at $2b=1$ m and the size of intensity of evenly distributed loading of $q/\gamma b=3,0; 4,0; 5,0; 6,0$ constructed on the basis of a plasticity condition (a – d)

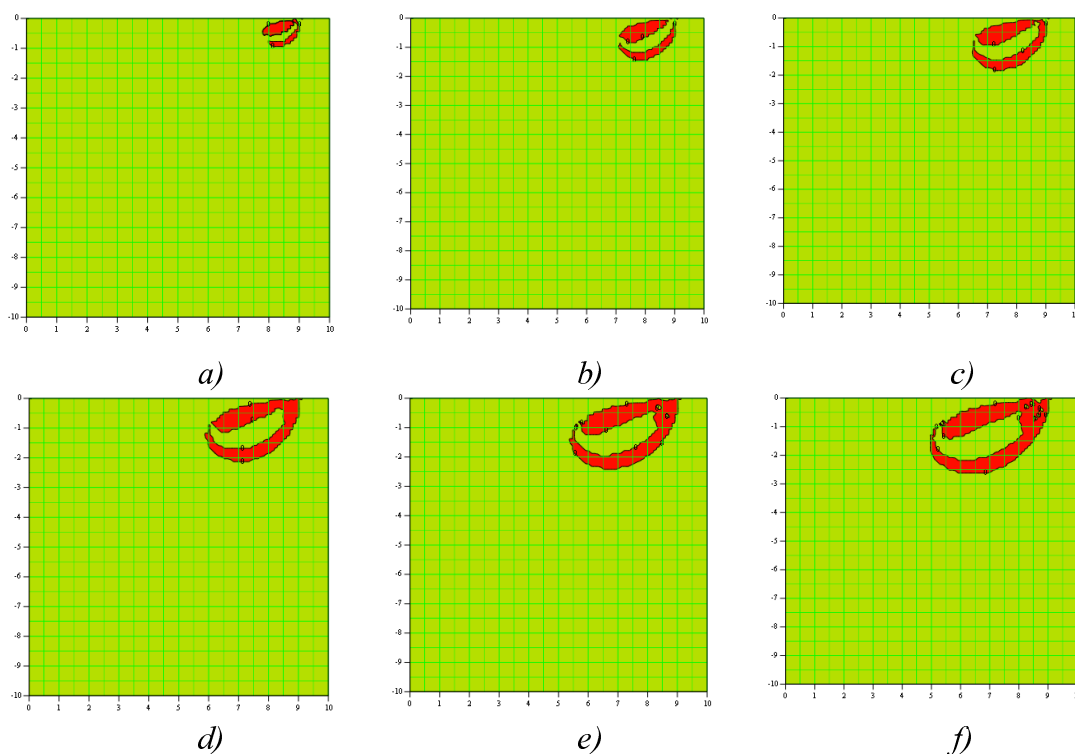


Fig. 8. Stages of evolution of areas of plastic deformations in the basis of not buried base at $2b=1$ m and the size of intensity of evenly distributed loading of $q/\gamma b=1,0; 2,0; 3,0; 4,0; 5,0; 6,0$ constructed on the basis of the solution of the mixed task of the theory of elasticity and the theory of plasticity of soil [11] (a – f)

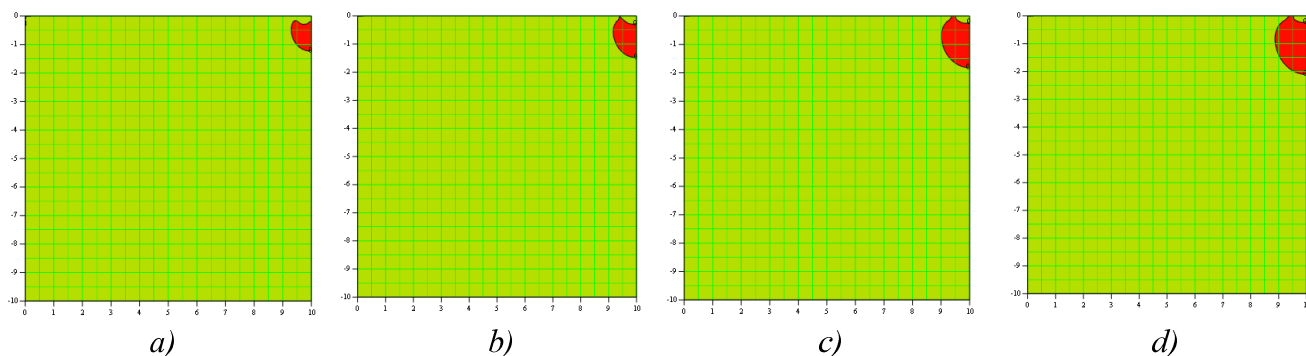


Fig. 9. Stages of evolution of areas of plastic deformations in the basis of not buried base at $2b=0,5$ m and the size of intensity of evenly distributed loading of $q/\gamma b=3,0; 4,0; 5,0; 6,0$ constructed on the basis of a plasticity condition (a – d)

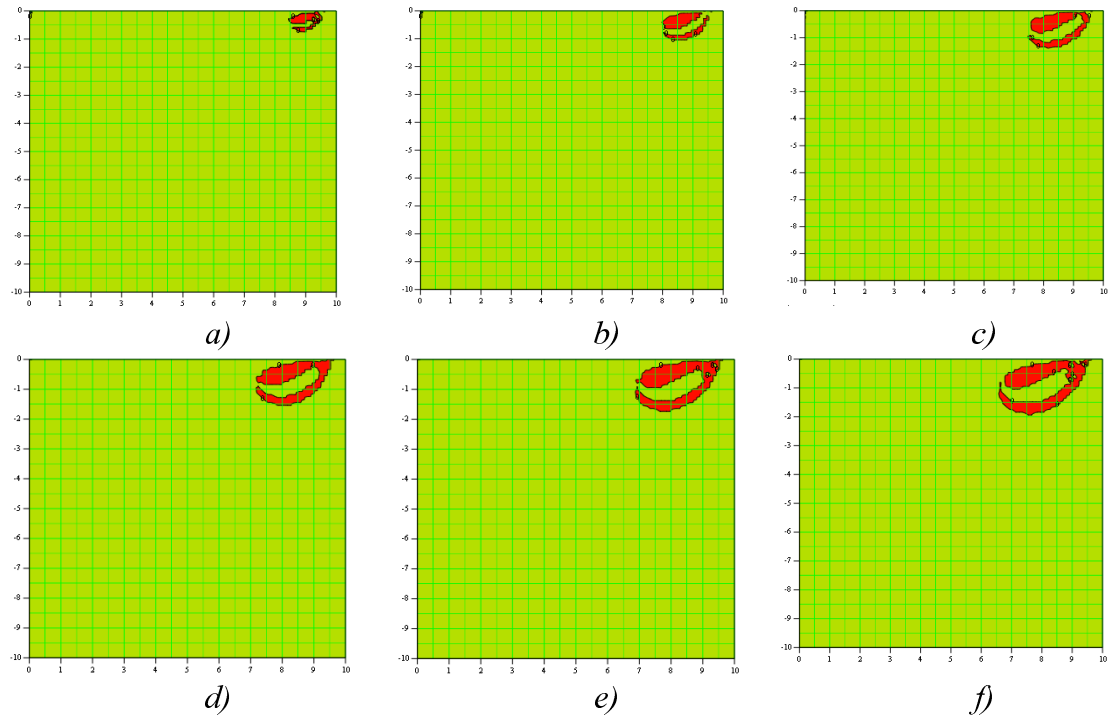


Fig. 10. Stages of evolution of areas of plastic deformations in the basis of not buried base at $2b=0,5$ m and the size of intensity of evenly distributed loading of $q/\gamma b=1,0$; $2,0$; $3,0$; $4,0$; $5,0$; $6,0$ constructed on the basis of the solution of the mixed task of the theory of elasticity and theory of soil plasticity [11] (a – f)

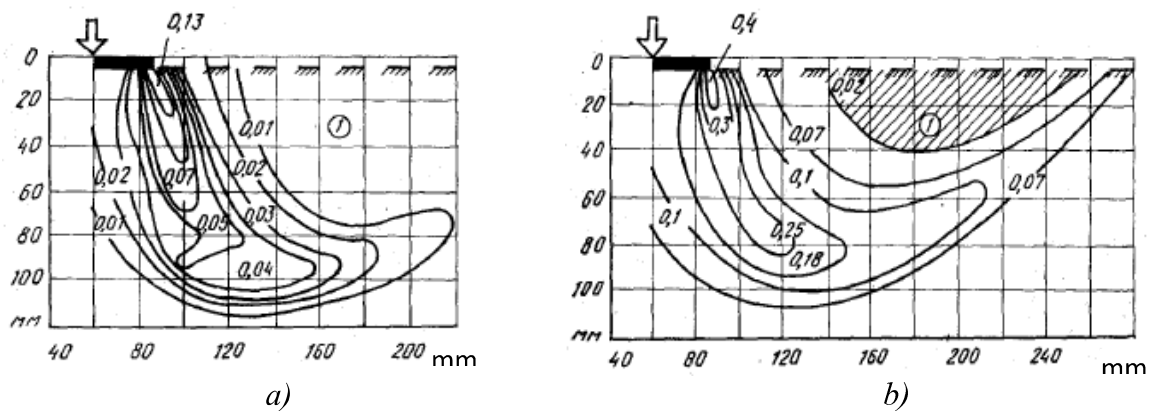


Fig. 11. Isolines of deformations of shift at a deposit of the stamp, corresponding to the moment before a maximum load (a); to corresponding ultraboundary loading on stability (b) (It is quoted on work [14])

Conclusions. 1. Areas of the plastic deformations which construction is carried out on the basis of use of a trivial condition of plasticity, in process of increase in intensity of loading develop deep into the bases under the base. At their joining under a sole of the base the elastic semielliptical kernel is formed. To start formation of these areas it is necessary that intensity of external influence reaches a certain value.

2. Areas of plastic deformations constructed on the basis of the decision provided in work, develop in different directions from the base, without being crossed and without being closed. Process of emergence of these plastic areas begins right after application of load: points of growth are observed on the

monitor screen at once, no matter how small intensity of external influence would be. In PSR there are zones being in a breaking point condition.

3. Lower bounds of areas of plastic deformations are very similar to an outline of lines of the sliding received by V.V. Sokolovsky [5, 6], at the analytical solution of the corresponding task of the theory of limit balance.

4. Position, form, direction of development of areas of the plastic deformations given in the real work, qualitatively coincide with corresponding parameters of the PSR, received irrespective from us by other authors while carrying out pilot studies (see fig. 5 – 11).

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