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Strength analysis of reinforced concrete flexural members at not entirely use of reinforcement resistance

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The suggestions for the engineering calculation of the reinforced concrete flexural members strength during the elastic work of steel are given. The engineering technique is developed on the basis of a nonlinear deformation model application using fractionally rational function for describing the process of concrete compressed area deformation and other prerequisites that are recommended by current norms for the reinforced concrete structures design. The analytical dependences have been obtained for determining the neutral axis depth and the calculation of the internal bending moment value perceived by the beam in the normal section. The task of determining the load-bearing capacity of the bending element is reduced to searching by the direct analytical dependences the maximum value of the bending moment that can be perceived by the beam at given strains of the most compressed rib of a cross-section.

Keywords: reinforced concrete, beam, strength, analysis.

Розрахунок несучої здатності залізобетонних згинальних елементів при неповному використанні міцності арматури

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У будівельній практиці подекуди застосовуються переармовані залізобетонні елементи, а також такі, в яких при багаторядному розташуванні арматури напруження не в усіх арматурних стержнях досягають межі текучості. Розрахунок міцності таких конструкцій є складним та трудомістким процесом, оскільки інженерної методики для такого випадку розрахунку на основі нелінійної деформаційної моделі не розроблено. Розроблення інженерної методики розрахунку несучої здатності цих залізобетонних згинальних елементів здійснено з урахуванням вимог чинних норм, розподіл напружень в бетоні стиснутої зони здійснено відповідно до діаграми «напруження – деформації» у вигляді дробово-раціональної залежності. Задача з розрахунку міцності в нормальному перерізі розглядається для балки прямокутного поперечного перерізу, армованого одиночною арматурою в момент досягнення бетоном на рівні найбільш стиснутої фібри таких значень деформацій, коли несуча здатність елемента буде максимальною. При цьому арматура в поперечному перерізі працюватиме з неповним розрахунковим опором. Отримані аналітичні залежності для обчислення висоти стиснутої зони бетону та значення внутрішнього згинального моменту, що сприймається балкою в нормальному перерізі. Розрахунок несучої здатності згинального елемента зведено до обчислення максимального значення згинального моменту, який може сприйняти балка при граничних значеннях деформацій найбільш стиснутої грані перерізу. Розглянуто також випадок розрахунку міцності згинальних елементів з багаторядним розташуванням розтягнутої арматури, при котрому напруження, які відповідають межі текучості, досягаються не в усіх рядах арматури. Застосування дробово-раціональної залежності для описання процесу деформування бетону стиснутої зони згинальних елементів дало змогу суттєво спростити та наблизити до інженерного розв’язку задачу з визначення несучої здатності елементів з неповним використанням міцності розтягнутої арматури.

Ключові слова: залізобетон, балка, міцність, розрахунок.

Introduction

Design of reinforced concrete members and structures on the basis of nonlinear deformation of materials more accurately reflects the actual work of materials and is recommended by the current norms [1, 2]. Practical methods for solving problems of the reinforcement selection, and determining the reinforced concrete elements strength have been developed on the basis of this approach. In practice there are sometimes overreinforced elements or those where, when multi-row arrangement of tensile reinforcement, stresses reach the yield strength not in all reinforcing bars. Determining the strength of such structures is a much more complicated calculation process, since the engineering methodology for such a calculation case is not developed. Even the strength calculation of rectangular section reinforced concrete beam cannot be performed without special computer programs, where the physical essence of the process is closed from the designer, which deprives him of understanding the design theoretical foundations.

Review of research sources and publications

Proposals for engineering calculations of the bending reinforced concrete elements strength are given in works [3 – 8]. In particular, in the publications [3 – 5], these proposals are implemented on the basis of the fractional-rational "stress-strain" dependence in the concrete compressed area use. In [6] examples of calculating the strength and determining the area of reinforcement on the basis of the reinforced concrete resistance concept are considered. In the article [7] methods of the problem engineering solution of determining the longitudinal reinforcement area of the bending elements is given. All of the above proposals are obtained only for elements with entire use of the reinforcement strength. In the paper [8], the problem of determining the strength of both normal and overreinforced reinforced concrete elements is proposed to be solved on the basis of the application of a uniform stresses distribution in the concrete of the compressed area. Publications [9 – 14] are devoted to the development of methods for calculating the strength of any reinforced concrete elements based on the deformation model in the general case and are proposed for implementation only with the use of modern computer programs.

Definition of non-solved aspects of the problem

Thus, in the theory of structural analysis of reinforced concrete elements a gap was created, which concerns the lack of engineering calculations of elements with reinforcement in an elastic stage.

Problem statement

Consequently, the development of an engineering method for calculating the carrying capacity of reinforced concrete bending elements with partial use of the reinforcement strength is an urgent task.

Basic material and results

The solution of the problem is carried out according to the design scheme depicted in Figure 1. In this case, the preconditions for calculating according to the norms [1, 2] are considered. In the scheme, the function-approximation of the "strain-strain" diagram in the form of fractional-rational dependence (3.4) from [1] was used in the image of the curvilinear diagram of the stresses distribution in the concrete compressed area.

The problem of calculating the strength is considered for a single reinforced beam of rectangular cross-section (Fig. 1) at the time when the most compressed concrete fibers reaching the level of such strain values, when the carrying capacity of the element is maximal. In this case, the reinforcement in the cross section works with partial design strength, i.e. $\sigma_s < f_{yd}$.

In the task, it is taken as known values: the area of the longitudinal reinforcement A_s in the cross-section, the dimensions of the cross-section of the beam $b \times h$, the physical and mechanical characteristics of the concrete f_{cd}, E_{cd}, ε_{c1} and reinforcement f_{yd}, E_s. For the considered beam it is determined by the engineering method [5], that $\xi > \xi_R$.

Unknown values are the maximum value of the resisting moment M_{Rd}, which can be perceived by the beam, and the corresponding value of strains in the most compressed concrete fibers $\varepsilon_{c(1)} = \varepsilon_{cu}$ (its level $\eta = \eta_u$), at which the beam maximizes resistance to the external load.

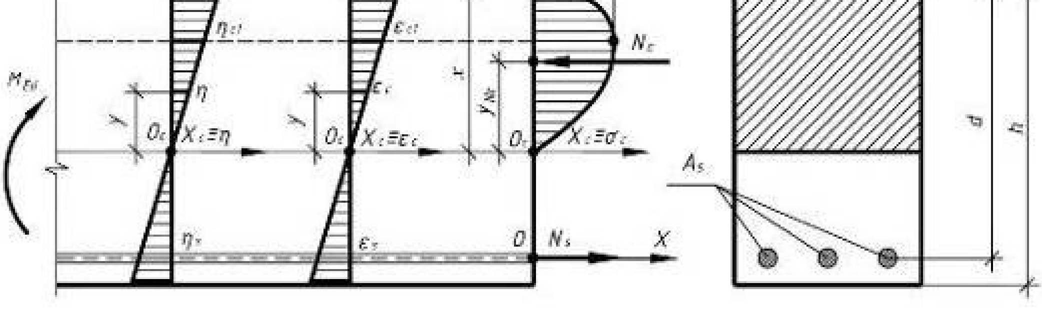

Figure 1 – Design scheme of the reinforced concrete beam

To solve the problem according to the accepted design scheme (Fig. 1) it is used:

– equation of equilibrium:

$$\sum X=0;\ N_s-N_c=0, \quad (1)$$

$$\sum M_O=0;\ M_{Ed}-N_c(d-x+y_{Nc})=0, \quad (2)$$

where N_s, N_c – resultant forces respectively in reinforcement and concrete;
d, x, y_{Nc} – the effective depth of the cross-section, the neutral axis depth, the distance from the neutral axis to the point of force N_c application;

– the "stress-strain" diagram for concrete at the axial compression according to [1]

$$\sigma_c=\frac{f_{cd}(k\eta-\eta^2)}{(1+(k-2)\eta)}, \quad (3)$$

where $k=1{,}05(E_{cd}\,\varepsilon_{c1,cd}/f_{cd})$, $\eta=(\varepsilon_c/\varepsilon_{c1,cd})<k$;
f_{cd}, E_{cd} – accordingly, the design values of concrete strength at axial compression and its elastic modulus;

– condition of strains compatibility of concrete and reinforcement

$$\varepsilon_c=\varepsilon_s; \quad (4)$$

– the "stress-strain" diagram for reinforcing steel in tension (compression)

$$\sigma_s=E_s\varepsilon_s \text{ at } 0<\varepsilon_s\le f_{yd}/E_s; \quad (5)$$

$$\sigma_s=f_{yd} \text{ at } f_{yd}/E_s<\varepsilon_s\le\varepsilon_{ud}; \quad (6)$$

– criterion for the maximum strength of the cross section of the beam element

$$M_u(\varepsilon_{cu})=M_u=\max M(\varepsilon_{c(1)})$$

or

$$M_u(\eta_u)=M_u=\max M(\eta), \quad (7)$$

in which the ultimate (characteristic) value of the compressive strain in the concrete (or its level) satisfies the condition of the extreme strength criterion of this section in the beam [2, p. 4.1.1], thereby ensuring, in the general case, the duality of the beam element strength problem solution in cross section.

To solve the problem, it is necessary first to express the components N_s, N_c, x and y_{Nc} in equations (1) and (2) functionally through σ_s, x, $\varepsilon_{c(1)}$ or through σ_s, x, $\eta_{c(1)}$.

To achieve this goal there is a need to have the law of stress distribution in the concrete of the compressed area in the form $\sigma_c=f(y,\eta_{c(1)},\ldots)$. Such a law describes the distribution of stresses in the concrete of the compressed area in the coordinate system $Y_cO_cX_c$ with its start on the neutral axis (Fig. 1), is obtained as follows

$$\sigma_c(y,\eta_{c(1),\ldots})=\frac{f_{cd}\eta_{c(1)}y(kx-\eta_{c(1)}y)}{x(x+(k-2)\eta_{c(1)}y)}, \quad (8)$$

where y – the current coordinate;
x – the neutral axis depth.

Applying the dependence (8), the components of equations (1) and (2), after performing the necessary mathematical actions, are given to the expressions:

$$N_c=b\int_0^x\frac{f_{cd}\eta_{c(1)}y(kx-\eta_{c(1)}y)}{x(x+(k-2)\eta_{c(1)}y)}dy=f_{cd}bx\omega(\eta_{c(1)}); \quad (9)$$

$$y_{Nc}=S_c/N_c=x\frac{\varphi(\eta_{c(1)})}{\omega(\eta_{c(1)})}; \quad (10)$$

$$S_c=b\int_0^x\frac{f_{cd}\eta_{c(1)}y(kx-\eta_{c(1)}y)y}{x(x+(k-2)\eta_{c(1)}y)}dy=f_{cd}bx^2\varphi(\eta_{c(1)}); \quad (11)$$

$$\left.\begin{aligned}&\omega(\eta_{c(1)})=\frac{(k-1)^2(c-\ln c-1)}{(k-2)^3\eta_{c(1)}}-\frac{\eta_{c(1)}}{2(k-2)} \text{ при } k\neq2,\\&\omega(\eta_{c(1)})=\eta_{c(1)}(1-\eta_{c(1)}/3) \text{ при } k=2,\end{aligned}\right\}; \quad (12)$$

$$\left.\begin{aligned}&\varphi(\eta_{c(1)})=\frac{(k-1)^2\left[(c-2)^2+2\ln c-1\right]}{2(k-2)^4\eta_{c(1)}^2}-\frac{\eta_{c(1)}}{3(k-2)} \text{ при } k\neq2,\\&\varphi(\eta_{c(1)})=\eta_{c(1)}\left(\frac{2}{3}-\frac{\eta_{c(1)}}{4}\right) \text{ при } k=2,\end{aligned}\right\}, \quad (13)$$

where $\omega(\eta_{c(1)})$ – the coefficient of the stress diagram fullness in the concrete compressed area, as it can be seen from formula (9).

Formulas (12) and (13) are derived when developing the expressions (9) – (11), while the notation was made $c=1+(K-2)\eta_{c(1)}$.

After the substitution of the values of N_s, N_c, x and y_{Nc} expressed in terms of $\eta_{c(1)}$ into (1) and (2), with $\sigma_s<f_{yd}$, the latter acquire the form:

$$E_s\varepsilon_sA_s-f_{cd}bx\omega=0; \quad (14)$$

$$M_{Ed}-f_{cd}bx\omega(d-\chi\omega x)=0, \quad (15)$$

where coefficient

$$\chi=\frac{\omega-\varphi}{\omega^2}, \quad (16)$$

shows which part of the depth x the distance from the most compressed fibers to the point of application of the resultant N_c constitutes.

Based on the hypothesis application of strains linear distribution, an expression is used to determine the deformations of the tensile reinforcement

$$\varepsilon_s=\frac{\varepsilon_{c(1)}(d-x)}{x}. \quad (17)$$

After substitution (17) into the first equation of equilibrium (14) a formula for the direct determination of the neutral axis depth is obtained

$$x=-B+\sqrt{B^2+2Bd}, \quad (18)$$

where

$$B=\frac{\varepsilon_{c(1)}E_sA_s}{2f_{cd}b\omega}. \quad (19)$$

Thus, the problem of determining the strength is reduced to finding the maximum value of the bending moment that can be perceived by the beam at the predefined deformations $\varepsilon_{c(1)}$ of the most compressed rib from the second equation of equilibrium

$$M_{Rd}=f_{cd}bx\omega(d-\chi\omega x). \quad (20)$$

In practice, bending elements with multi-row arrangement of tensile reinforcement are often used. In this case, the stresses in the reinforcement of different rows may reach the yield strength or not. For this case, the design scheme acquires the form shown in Fig. 2. Equation for equilibrium for the considered design scheme (Fig. 2) is written as follows:

$$\sum X = 0;\ N_{si} + N_{sj} - N_c = 0, \qquad (21)$$

$$\sum M_A = 0;\ M_{Ed} - N_{si}(d_i - x + y_{Nc}) - N_{sj}(d_j - x + y_{Nc}) = 0, \qquad (22)$$

in which N_{si}, N_{sj} – the resultant forces in the tensile reinforcement, the stress at which $\sigma_s = f_{yd}$ and $\sigma_s < f_{yd}$, respectively;
d_i – the distance from the most compressed border of the cross-section to the i-th row of reinforcement, the stress of which reaches the yield strength ($\sigma_s = f_{yd}$);
d_j – the distance from the most compressed border of the cross-section to the j-th row of reinforcement, where the stress does not reach the yield strength ($\sigma_s < f_{yd}$).

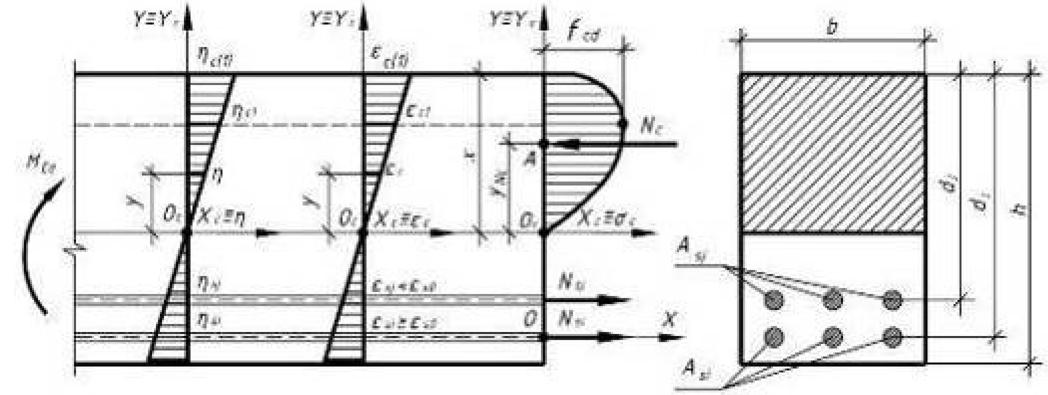

Figure 2 – Design scheme of the reinforced concrete beam with multi-row arrangement of tensile reinforcement

After substituting the values of N_{si}, N_c, x and y_{Nc} into equations (1) and (2) with $\sigma_s < f_{yd}$, the latter acquire the form:

$$E_s \sum_{i=1}^{n} \varepsilon_{si} A_{si} + f_{yd} \sum_{j=1}^{k} A_{sj} - f_{cd} b x \omega = 0\,, \qquad (23)$$

$$M_{Ed} - \varepsilon_{c(1)} E_s \sum_{i=1}^{n} (d_i / x - 1) A_{si} (d_i - \chi \omega x) - f_{yd} \sum_{j=1}^{k} A_{sj} (d_j - \chi \omega x) = 0, \qquad (24)$$

where n – the number of reinforcement bars rows, where the stresses do not reach the yield strength;
k – the number of reinforcement bars rows, where the stresses reach the yield strength.

After substitution (17) into the first equation of equilibrium (23) a formula for the direct determination of the neutral axis depth is obtained

$$x = -B + \sqrt{B^2 + \frac{\varepsilon_{c(1)} E_s \sum_{j=1}^{k} A_{sj} d_{sj}}{f_{cd} b \omega}}\,, \qquad (25)$$

where

$$B = \frac{\varepsilon_{c(1)} E_s \sum_{i=1}^{n} A_{si} - f_{yd} \sum_{j=1}^{k} A_{sj}}{2 f_{cd} b \omega}\,. \qquad (26)$$

Thus, in the same way as in the previous case, the problem of determining the strength is reduced to finding the maximum value of the bending moment that can be perceived by the beam at the predefined strains $\varepsilon_{c(1)}$ of the most compressed border of cross-section from the second equation of equilibrium

$$M_{Rd} = \varepsilon_{c(1)} E_s \sum_{i=1}^{n} (d_i / x - 1) A_{si} (d_i - \chi \omega x) + f_{yd} \sum_{j=1}^{k} A_{sj} (d_j - \chi \omega x). \qquad (27)$$

In this case, the iterative search should begin with the value of strains $\varepsilon_{c(1)} = \eta_u \varepsilon_{c1}$, where η_u is the limiting deformation level of the most compressed fibre of the concrete in [5], which corresponds to the destruction of the element with the full use of the reinforcement strength, that is, when reinforcement reaches the stresses f_{yd}. Increment deformation at each step is $\Delta\varepsilon_{c(1)} = 0{,}1\varepsilon_{cu1}$.

Typically, with $\Delta\varepsilon_{c(1)} = 0{,}1\varepsilon_{cu1}$, results are obtained with sufficient accuracy, but if it is necessary to improve the accuracy of the calculation, smaller values for deformation growth at each step can be used.

The search should be carried out to the value of the strains $\varepsilon_{c(1)}$, where the value of the bending moment M_{Rd} determined from (20) or (27), depending on the calculation case, begins to decrease, thus, it is smaller than in the previous step. The largest of the obtained M_{Rd} values determines the bearing capacity in the normal section of the bending element with incomplete use of the reinforcement strength.

Conclusions

Thus, the application of fractional-rational dependence for describing the process of deforming the concrete of the bending elements compressed area enabled to simplify and approximate significantly to the engineering solution the problem of the elements bearing capacity determining with incomplete use of the tensile reinforcement strength.

References

1. ДБН В.2.6-98:2009. *Конструкції будинків та споруд. Бетонні та залізобетонні конструкції. Основні положення*. (2011). Київ: Мінрегіонбуд України.

2. ДСТУ Б В.2.6-156:2010. (2011). *Конструкції будинків та споруд. Бетонні та залізобетонні конструкції з важкого бетону. Правила проектування*. Київ: Мінрегіонбуд України.

3. Павліков, А.М., Бойко, О.В. (2007). Застосування нелінійної деформаційної моделі в інженерних розрахунках міцності залізобетонних елементів. *Ресурсоекономні матеріали, конструкції, будівлі та споруди: зб. наук. праць,* 7, 355-363.

4. Павліков, А.М. (2007). *Нелінійна модель напружено-деформованого стану косозавантажених залізобетонних елементів у закритичній стадії: монографія.* Полтава: ПолтНТУ.

5. Павліков, А.М., Бойко, О.В. (2012). *Розрахунок міцності нормальних перерізів балкових елементів за нелінійною деформаційною моделлю (на основі ДБН В.2.6-98:2009)*. Полтава: ПолтНТУ.

6. Pavlikov, A.M., Kochkarev, D.V. & Garkava, O.V. (2017). Strength of reinforced of concrete in bending elements calculations. *Збірник наукових праць. Серія: Галузеве машинобудування, будівництво*, 1(48), 62-71.
http://znp.pntu.edu.ua/files/archive/ua/48_2017/11.pdf

7. Бамбура, А.М., Дорогова, О.В., Сазонова, І.Р. (2017). Попереднє призначення площі розтягнутої арматури для конструкцій прямокутного перерізу при згині. *Наука та будівництво*, 3(13), 32-39.

8. Mosley, W.H., Bungey, J.H. & Hulse, R. (2007). *Reinforced Concrete Design to Eurocode 2.* Palgrave Macmillan.

9. Naaman, A.E. (1995). Unified Bending Strength Design of Concrete Members: AASHTO LRFD Code. *Journal of Structural Engineering*, 121(6), 964-982
https://doi.org/10.1061/(ASCE)0733-9445(1995)121:6(964)

10. Ho, J.C.M., Kwan, A.K.H. & Pam, H.J. (2003). Theoretical analysis of post-peak flexural behaviour of normal- and high-strength concrete beams. *The structural design of tall and special buildings*, 12(2), 109-125.
https://doi.org/10.1002/tal.216

11. Kwan, A.K.H., Ho, J.C.M. & Pam, H.J. (2002). Flexural strength and ductility of reinforced concrete beams. *Proceedings of the Institution of Civil Engineers: Structures and Buildings*, 152(4), 361-369.
http://dx.doi.org/10.1680/stbu.152.4.361.40817

12. Wu, Yu-Fei (2016). Theorems for Flexural Design of RC Members. *Journal of Structural Engineering*, 142(5), 174-193.
https://doi.org/10.1061/(ASCE)ST.1943-541X.0001454

13. Buyukozturk, O. (1977). Nonlinear analysis of reinforced concrete structures. *Computers & Structures*, 7(1), 149-156.
https://doi.org/10.1016/0045-7949(77)90069-4

14. Wang, T. & Hsu, T. (2001). Nonlinear finite element analysis of concrete structures using new constitutive models. *Computers & Structures*, 79(32), 2781-2791.
https://doi.org/10.1016/S0045-7949(01)00157-2

1. SBN B.2.6-98:2009. *Constructions of buildings and structures. Concrete and reinforced concrete structures. Basic provisions.* (2011). Kyiv: Minregionbuild of Ukraine.

2. SSTU B V.2.6-156:2010. (2011). *Constructions of buildings and structures. Concrete and reinforced concrete structures of heavy-weight concrete. Design rules*. Kyiv: Minregionbuild of Ukraine.

3. Pavlikov, A.M. & Boiko, O.V. (2007). Application of nonlinear deformation model in engineering calculations of reinforced concrete elements strength. *Economical materials, structures, buildings and structures: coll. of scient. papers,* 7, 355-363.

4. Pavlikov, A.M. (2007). *Nonlinear model of deformation mode of biaxially loaded reinforced concrete elements in the overcritical stage: monograph.* Poltava: PoltNTU.

5. Pavlikov, A.M. & Boiko, O.V. (2012). S*trength calculation of the normal cross sections of the beam elements by a nonlinear deformation model (based on SBN B.2.6-98:2009)*. Poltava: PoltNTU.

6. Pavlikov, A.M, Kochkarev, D.V. & Garkava, O.V. (2017). Strength of reinforced of concrete in bending elements calculations. *Academic journal. Industrial Machine Building, Civil Engineering*, 1(48), 62-71.
http://znp.pntu.edu.ua/files/archive/ua/48_2017/11.pdf

7. Bambura, A.M., Dorohova, O.V., Sazonova, I.R. (2017). Preliminary designation of the area of the tensile reinforcement for structures of rectangular section at bending. *Science and Construction*, 3(13), 32-39.

8. Mosley, W.H., Bungey, J.H. & Hulse, R. (2007). *Reinforced Concrete Design to Eurocode 2.* Palgrave Macmillan.

9. Naaman, A.E. (1995). Unified Bending Strength Design of Concrete Members: AASHTO LRFD Code. *Journal of Structural Engineering*, 121(6), 964-982
https://doi.org/10.1061/(ASCE)0733-9445(1995)121:6(964)

10. Ho, J.C.M., Kwan, A.K.H. & Pam, H.J. (2003). Theoretical analysis of post-peak flexural behaviour of normal- and high-strength concrete beams. *The structural design of tall and special buildings*, 12(2), 109-125.
https://doi.org/10.1002/tal.216

11. Kwan, A.K.H., Ho, J.C.M. & Pam, H.J. (2002). Flexural strength and ductility of reinforced concrete beams. *Proceedings of the Institution of Civil Engineers: Structures and Buildings*, 152(4), 361-369.
http://dx.doi.org/10.1680/stbu.152.4.361.40817

12. Wu, Yu-Fei (2016). Theorems for Flexural Design of RC Members. *Journal of Structural Engineering*, 142(5), 174-193.
https://doi.org/10.1061/(ASCE)ST.1943-541X.0001454

13. Buyukozturk, O. (1977). Nonlinear analysis of reinforced concrete structures. *Computers & Structures*, 7(1), 149-156.
https://doi.org/10.1016/0045-7949(77)90069-4

14. Wang, T. & Hsu, T. (2001). Nonlinear finite element analysis of concrete structures using new constitutive models. *Computers & Structures*, 79(32), 2781-2791.
https://doi.org/10.1016/S0045-7949(01)00157-2