Збірник праць Ін-ту математики НАН України 2017, том 14, № 1, 116–127

УДК 517.956.223

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On the description of quantum correlations by means of a one-particle density operator

Dedicated to Prof. Yu. B. Zelinskii on the occasion of his 70^{th} *birthday*

We develop an approach to the description of processes of the creation of correlations and the propagation of initial correlations in large particle quantum systems by means of a one-particle density operator that is a solution of the generalized quantum kinetic equation with initial correlations. Moreover, mean field asymptotic behavior of the constructed correlation operators of the system state is established.

Розвинуто підхід до опису процесів народження кореляцій та поширення початкових кореляцій у квантових системах багатьох частинок за допомогою одночастинкового оператора густини, який є розв'язком узагальненого квантового кінетичного рівняння з початковими кореляціями. Крім того, встановлено асимптотичну поведінку побудованих кореляційних операторів стану системи у наближенні самоузгодженого поля.

1. Introduction. As known, the marginal correlation operators give an equivalent approach to the description of the evolution of states of large particle quantum systems in comparison with marginal density operators. The physical interpretation of marginal correlation operators is that the macroscopic characteristics of fluctuations of mean values of observables are determined by them on the microscopic level [1, 2].

Traditionally marginal correlation operators are introduced by means of the cluster expansions of the marginal density operators [3]. In article [4] we developed an approach based on the definition of the marginal correlation operators within the framework of dynamics of correlations governed by the von Neumann hierarchy [5]. As a result of which it is established that the marginal correlation operators governed by the hierarchy of nonlinear evolution equations, known as the quantum nonlinear BBGKY (Bogolyubov–Born–Green–Kirkwood–Yvon) hierarchy, are represented in the form of series expansions over the number of particles of subsystems which generating operators are the corresponding-order cumulants of the groups of nonlinear operators of the von Neumann hierarchy for a sequence of correlation operators [5].

In this paper we consider the problem of the rigorous description of the evolution of states of large particle quantum systems within the framework of a one-particle (marginal) density operator that is a solution of the generalized quantum kinetic equation with initial correlations. We remark that initial states specified by correlations are typical for the condensed states of many-particle systems in contrast to their gaseous state [1, 6].

Moreover, in the paper mean field asymptotic behavior of processes of the creation of correlations and the propagation of initial correlations in large particle quantum systems is established.

We note that the conventional approach to the problem of the description of the propagation of initial chaos [7], i.e. in case of initial states specified by a one-particle density operator without correlation operators, is based on the consideration of an asymptotic behavior of a solution of the quantum BBGKY hierarchy for marginal density operators constructed within the framework of the perturbation theory [8–10].

2. Preliminaries: marginal correlation operators. Let the space $\mathcal{H}$ be a one-particle Hilbert space, then the n-particle space $\mathcal{H}_n = \mathcal{H}^{\otimes n}$ is a tensor product of n Hilbert spaces $\mathcal{H}$. We adopt the usual convention that $\mathcal{H}^{\otimes 0} = \mathbb{C}$. The Fock space over the Hilbert space $\mathcal{H}$ we denote by $\mathcal{F}_{\mathcal{H}} = \bigoplus_{n=0}^{\infty}\mathcal{H}_n$. A self adjoint operator f_n defined on the n-particle Hilbert space $\mathcal{H}_n = \mathcal{H}^{\otimes n}$ will be also denoted by the symbol $f_n(1,\ldots,n)$.

Let $\mathfrak{L}^1(\mathcal{H}_n)$ be the space of trace class operators

$$f_n \equiv f_n(1,\ldots,n) \in \mathfrak{L}^1(\mathcal{H}_n)$$

that satisfy the symmetry condition: $f_n(1,\ldots,n) = f_n(i_1,\ldots,i_n)$ for arbitrary $(i_1,\ldots,i_n) \in (1,\ldots,n)$, and equipped with the norm:

$$\|f_n\|_{\mathfrak{L}^1(\mathcal{H}_n)} = \mathrm{Tr}_{1,\ldots,n}|f_n(1,\ldots,n)|,$$

where $\mathrm{Tr}_{1,\ldots,n}$ are partial traces over $1,\ldots,n$ particles. We denote by $\mathfrak{L}_0^1(\mathcal{H}_n)$ the everywhere dense set of finite sequences of degenerate operators with infinitely differentiable kernels with compact supports.

On the space $\mathfrak{L}^1(\mathcal{F}_{\mathcal{H}}) = \oplus_{n=0}^{\infty}\mathfrak{L}^1(\mathcal{H}_n)$ of sequences $f = (f_0, f_1, \ldots, f_n, \ldots)$ of trace class operators $f_n \in \mathfrak{L}^1(\mathcal{H}_n)$ and $f_0 \in \mathbb{C}$ it is defined the following nonlinear one-parameter mapping

$$\mathcal{G}(t;1,\ldots,s \mid f) \doteq \sum_{\mathrm{P}:\,(1,\ldots,s)=\bigcup_j X_j} \mathfrak{A}_{|\mathrm{P}|}(t,\{X_1\},\ldots,\{X_{|\mathrm{P}|}\}) \prod_{X_j\subset \mathrm{P}} f_{|X_j|}(X_j), s\geq 1, \tag{1}$$

where the symbol $\sum_{\mathrm{P}:\,(1,\ldots,s)=\bigcup_j X_j}$ means the sum over all possible partitions P of the set $(1,\ldots,s)$ into $|\mathrm{P}|$ nonempty mutually disjoint subsets X_j, the set $(\{X_1\},\ldots,\{X_{|\mathrm{P}|}\})$ consists from elements of which are subsets $X_j \subset (1,\ldots,s)$, i.e., $|(\{X_1\},\ldots,\{X_{|\mathrm{P}|}\})| = |\mathrm{P}|$. The generating operator $\mathfrak{A}_{|\mathrm{P}|}(t)$ of expansion (1) is the $|\mathrm{P}|th$-order cumulant of the groups of operators defined by the following expansion

$$\mathfrak{A}_{|\mathrm{P}|}(t,\{X_1\},\ldots,\{X_{|\mathrm{P}|}\}) \doteq \sum_{\mathrm{P}':\,(\{X_1\},\ldots,\{X_{|\mathrm{P}|}\})=\bigcup_k Z_k} (-1)^{|\mathrm{P}'|-1}(|\mathrm{P}'|-1)! \prod_{Z_k\subset \mathrm{P}'} \mathcal{G}^*_{|\theta(Z_k)|}(t,\theta(Z_k)), \tag{2}$$

where θ is the declusterization mapping: $\theta(\{X_1\},\ldots,\{X_{|\mathrm{P}|}\}) \doteq (1,\ldots,s)$, and on the space $\mathfrak{L}^1(\mathcal{H}_n)$ the one-parameter mapping $\mathcal{G}_n^*(t)$ is defined by the formula

$$\mathbb{R}^1 \ni t \mapsto \mathcal{G}_n^*(t)f_n \doteq e^{-itH_n}f_n e^{itH_n}. \tag{3}$$

In (3) the operator H_n is the Hamiltonian of a system of n particles, obeying Maxwell–Boltzmann statistics, and we use units, where $h = 2\pi\hbar = 1$ is a Planck constant and $m = 1$ is the mass of particles. The inverse group to the group $\mathcal{G}_n^*(t)$ we denote by $(\mathcal{G}_n^*)^{-1}(t) = \mathcal{G}_n^*(-t)$. On its domain of the definition the infinitesimal generator $\mathcal{N}_n^*$ of the group of operators (3) is determined in the sense of the strong convergence of the space $\mathfrak{L}^1(\mathcal{H}_n)$ by the operator

$$\lim_{t\to 0}\frac{1}{t}\big(\mathcal{G}_n^*(t)f_n - f_n\big) = -i\,(H_n f_n - f_n H_n) \doteq \mathcal{N}_n^* f_n, \tag{4}$$

that has the structure: $\mathcal{N}_n^* = \sum_{j=1}^n \mathcal{N}^*(j) + \epsilon\sum_{j_1<j_2=1}^n \mathcal{N}^*_{\mathrm{int}}(j_1,j_2)$, where the operator $\mathcal{N}^*(j)$ is a free motion generator of the von Neumann equation

[3], the operator $\mathcal{N}^{*}_{\mathrm{int}}$ is defined by means of the operator of a two-body interaction potential Φ by the formula:

$$\mathcal{N}^{*}_{\mathrm{int}}(j_1,j_2)f_n \doteq -i\left(\Phi(j_1,j_2)f_n - f_n\Phi(j_1,j_2)\right),$$

and we denote a scaling parameter by $\epsilon > 0$.

The evolution of all possible states of large particle quantum systems, obeying the Maxwell–Boltzmann statistics, can be described by means of the sequence $G(t) = (I, G_1(t), G_2(t), \ldots, G_s(t), \ldots) \in \mathfrak{L}^1(\mathcal{F}_{\mathcal{H}})$ of marginal correlation operators governed by the hierarchy of nonlinear evolution equations known as the quantum nonlinear BBGKY hierarchy [1]. If $G(0) = (I, G_1^{0,\epsilon}(1), \ldots, G_s^{0,\epsilon}(1,\ldots,s), \ldots)$ is a sequence of initial marginal correlation operators, then a nonperturbative solution of the Cauchy problem of the quantum nonlinear BBGKY hierarchy is represented by a sequence of the following operators [4]:

$$\begin{aligned} &G_s(t,1,\ldots,s) = \\ &\quad = \sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{Tr}_{s+1,\ldots,s+n}\, \mathfrak{A}_{1+n}(t; \{1,\ldots,s\}, s+1, \ldots, s+n \mid G(0)), \\ &s \geq 1, \end{aligned} \tag{5}$$

where the generating operator $\mathfrak{A}_{1+n}(t;\{1,\ldots,s\}, s+1,\ldots,s+n \mid G(0))$ of series expansion (5) is the $(1+n)th$-order cumulant of groups of nonlinear operators (1) of the von Neumann hierarchy for correlation operators:

$$\begin{aligned} &\mathfrak{A}_{1+n}(t;\{1,\ldots,s\}, s+1,\ldots,s+n \mid G(0)) \doteq \\ &\qquad \sum_{\mathrm{P}:\,(\{1,\ldots,s\},s+1,\ldots,s+n)=\bigcup_k X_k} (-1)^{|\mathrm{P}|-1}\,(\,|\mathrm{P}|-1)!\,\mathcal{G}(t;\theta(X_1)\mid \ldots \\ &\qquad\quad \mathcal{G}(t;\theta(X_{|\mathrm{P}|})\mid G(0))\ldots)\,, \quad n \geq 0, \end{aligned} \tag{6}$$

and the composition of mappings (1) of the corresponding noninteracting groups of particles we denote by the symbol

$$\mathcal{G}(t;\theta(X_1)\mid \ldots \mathcal{G}(t;\theta(X_{|\mathrm{P}|})\mid G(0))\ldots).$$

We remark that nonperturbative solution (5) of the quantum nonlinear BBGKY hierarchy is transformed to the solution represented be perturbation (iteration) series as a result of the application of analogs of the Duhamel equation to cumulants (2) of the groups of operators (3).

In case of initial states specified in terms of a one-particle (marginal) density operator and correlation operators the evolution of all possible states of large particle quantum systems can be described in an equivalent way within the framework of a one-particle density operator governed by the kinetic equation, i.e. without any approximations.

3. A main result: marginal correlation functionals of the state. We shall consider the case of initial states specified by a one-particle marginal density operator with correlations, namely, initial states specified by the following sequence of marginal correlation operators:

$$G^{(c)} = \big(I, G_1^{0,\epsilon}(1), g_2^{\epsilon}(1,2)\prod_{i=1}^{2} G_1^{0,\epsilon}(i), \ldots, g_n^{\epsilon}(1,\ldots,n)\prod_{i=1}^{n} G_1^{0,\epsilon}(i), \ldots\big), \quad (7)$$

where the operators $g_n^{\epsilon}(1,\ldots,n) \equiv g_n^{\epsilon} \in \mathfrak{L}_0^1(\mathcal{H}_n)$, $n \geq 2$, are specified the initial correlations. We remark that such assumption about initial states is intrinsic for the kinetic description of many-particle systems. On the other hand, initial data (7) is typical for the condensed states of large particle quantum systems, for example, the equilibrium state of the Bose condensate satisfies the weakening of correlation condition with the correlations which characterize the condensed state [1, 6].

For initial states specified in terms of a one-particle density operator and correlation operators (7) the evolution of states given in the framework of the sequence $G(t) = (I, G_1(t), \ldots, G_s(t), \ldots)$ of marginal correlation operators (5) can be described by means of the sequence

$$G(t \mid G_1(t)) = (I, G_1(t), G_2(t \mid G_1(t)), \ldots, G_s(t \mid G_1(t)), \ldots)$$

of marginal correlation functionals: $G_s(t, 1, \ldots, s \mid G_1(t))$, $s \geq 2$, with respect to the one-particle correlation operator $G_1(t)$ governed by the kinetic equation.

In this case the marginal correlation functionals $G_s(t \mid G_1(t))$, $s \geq 2$, are defined with respect to the one-particle (marginal) density operator

$$G_1(t,1) = \sum_{n=0}^{\infty} \frac{1}{n!} \mathrm{Tr}_{2,\ldots,1+n}\, \mathfrak{A}_{1+n}(t, 1, \ldots, n+1) \times \quad (8)$$

$$\times \sum_{\mathrm{P}\,:\,(1,\ldots,n+1)=\bigcup_i X_i} \;\prod_{X_i \subset \mathrm{P}} g^{\epsilon}_{|X_i|}(X_i) \prod_{i=1}^{n+1} G_1^{0,\epsilon}(i),$$

where the generating operator $\mathfrak{A}_{1+n}(t)$ is the $(1+n)-th$ order cumulant (2) of the groups of operators (3), and these functionals are represented by the series expansions:

$$G_s\big(t,1,\ldots,s \mid G_1(t)\big)=\sum_{n=0}^{\infty}\frac{1}{n!}\,\mathrm{Tr}_{s+1,\ldots,s+n}\,\mathfrak{G}_{s+n}\big(t,\theta(\{1,\ldots,s\}),\qquad (9)$$

$$s+1,\ldots,s+n\big)\prod_{i=1}^{s+n}G_1(t,i),\quad s\geq 2,$$

where the $(s+n)th$-order generating operator $\mathfrak{G}_{s+n}(t)$, $n\geq 0$, of this series is determined by the following expansion

$$\mathfrak{G}_{s+n}\big(t,\theta(\{1,\ldots,s\}),s+1,\ldots,s+n\big)= \qquad (10)$$

$$=n!\sum_{k=0}^{n}(-1)^k\sum_{n_1=1}^{n}\ldots\sum_{n_k=1}^{n-n_1-\ldots-n_{k-1}}\frac{1}{(n-n_1-\ldots-n_k)!}\times$$

$$\times\breve{\mathfrak{A}}_{s+n-n_1-\ldots-n_k}(t,\theta(\{1,\ldots,s\}),s+1,\ldots,s+n-n_1-\ldots-n_k)\times$$

$$\times\prod_{j=1}^{k}\sum_{\substack{\mathrm{D}_j\,:\,Z_j=\bigcup_{l_j}X_{l_j},\\ |\mathrm{D}_j|\leq s+n-n_1-\cdots-n_j}}\frac{1}{|\mathrm{D}_j|!}\sum_{i_1\neq\ldots\neq i_{|\mathrm{D}_j|}=1}^{s+n-n_1-\ldots-n_j}\prod_{X_{l_j}\subset\mathrm{D}_j}\frac{1}{|X_{l_j}|!}\times$$

$$\times\breve{\mathfrak{A}}_{1+|X_{l_j}|}(t,i_{l_j},X_{l_j}).$$

In formula (10) the sum over all possible dissections [15] of the linearly ordered set $Z_j\equiv(s+n-n_1-\ldots-n_j+1,\ldots,s+n-n_1-\ldots-n_{j-1})$ on no more than $s+n-n_1-\ldots-n_j$ linearly ordered subsets we denote by $\sum_{\mathrm{D}_j:Z_j=\bigcup_{l_j}X_{l_j}}$ and the $(s+n)$ $^{\mathrm{th}}$-order scattering cumulant is defined by the formula

$$\breve{\mathfrak{A}}_{s+n}(t,\theta(\{1,\ldots,s\}),s+1,\ldots,s+n)=$$
$$=\mathfrak{A}_{s+n}(t,1,\ldots,s+n)g^{\epsilon}_{s+n}(1,\ldots,s+n)\prod_{i=1}^{s+n}\mathfrak{A}_1^{-1}(t,i),$$

where the operator $g^{\epsilon}_{s+n}(1,\ldots,s+n)$ is specified initial correlations (7), and notations accepted above were used. We adduce simplest examples of generating operators (10):

$$\begin{aligned}
&\mathfrak{G}_s(t,\theta(\{1,\ldots,s\})) = \breve{\mathfrak{A}}_s(t,\theta(\{1,\ldots,s\})) = \\
&\quad = \mathfrak{A}_s(t,1,\ldots,s))g_s^\epsilon(1,\ldots,s)\prod_{i=1}^{s}\mathfrak{A}_1^{-1}(t,i), \\
&\mathfrak{G}_{s+1}(t,\theta(\{1,\ldots,s\}),s+1) = \\
&\quad = \mathfrak{A}_{s+1}(t,1,\ldots,s+1)g_{s+1}^\epsilon(1,\ldots,s+1)\prod_{i=1}^{s+1}\mathfrak{A}_1^{-1}(t,i)- \\
&\quad -\mathfrak{A}_s(t,1,\ldots,s)g_s^\epsilon(1,\ldots,s)\prod_{i=1}^{s}\mathfrak{A}_1^{-1}(t,i)\sum_{j=1}^{s}\mathfrak{A}_2(t,j,s+1)\times \\
&\quad \times g_2^\epsilon(j,s+1)\mathfrak{A}_1^{-1}(t,j)\mathfrak{A}_1^{-1}(t,s+1).
\end{aligned}$$

A method of the construction of marginal correlation functionals (9) is based on the application of kinetic cluster expansions [3] to the generating operators of series (5). If $\|G_1(t)\|_{\mathfrak{L}^1(\mathcal{H})} < e^{-(3s+2)}$, then for arbitrary $t \in \mathbb{R}$ series expansion (9) converges in the norm of the space $\mathfrak{L}^1(\mathcal{H}_s)$.

We emphasize that marginal correlation functionals (9) describe the all possible correlations generated by dynamics of large particle quantum systems with initial correlations by means of a one-particle density operator.

Now we establish the evolution equation for one-particle (marginal) density operator (8). As a result of the differentiation over time variable of the operator represented by series (8) in the sense of the norm convergence of the space $\mathfrak{L}^1(\mathcal{H})$, then due to the application of the kinetic cluster expansions [13] to the generating operators of obtained series expansion, for one-particle density operator (8) we derive the following identity:

$$\begin{aligned}
\frac{\partial}{\partial t}G_1(t,1) &= \mathcal{N}^*(1)G_1(t,1) + \epsilon\,\mathrm{Tr}_2\,\mathcal{N}^*_{\mathrm{int}}(1,2)G_1(t,1)G_1(t,2) + \\
&+\epsilon\,\mathrm{Tr}_2\,\mathcal{N}^*_{\mathrm{int}}(1,2)G_2\big(t,1,2\mid G_1(t)\big),
\end{aligned} \tag{11}$$

where the second part of the collision integral in equality (11) is determined in terms of the marginal correlation functional represented by series expansions (9) in case of $s = 2$. This identity we treat as the quantum kinetic equation and we refer to this evolution equation as the generalized quantum kinetic equation with initial correlations.

We emphasize that the coefficients in an expansion of the collision integral of the non-Markovian kinetic equation (11) are determined by the operators specified initial correlations (7).

On the space $\mathfrak{L}^1(\mathcal{H})$ for the Cauchy problem of the established generalized quantum kinetic equation with initial correlations the following statement is true.

Theorem 1. *If* $\|G_1^{0,\epsilon}\|_{\mathfrak{L}^1(\mathcal{H})} < (e(1+e^9))^{-1}$, *a global in time solution of the Cauchy problem of kinetic equation* (11) *is determined by series expansion* (8). *For initial data* $G_1^{0,\epsilon} \in \mathfrak{L}^1_0(\mathcal{H})$ *it is a strong solution and for an arbitrary initial data it is a weak solution.*

The proof of this existence theorem is similar to the proof in the case of the generalized quantum kinetic equation given in [15].

4. On a propagation of initial correlations in a mean field limit. Further we establish the mean field asymptotic behavior of constructed marginal correlation functionals (9) in case of initial states specified by the one-particle density operator with correlations (7).

We assume the existence of a mean field limit of initial one-particle density operator in the following sense

$$\lim_{\epsilon\to 0}\big\|\epsilon G_1^{0,\epsilon} - g_1^0\big\|_{\mathfrak{L}^1(\mathcal{H})} = 0, \tag{12}$$

and initial correlations as follows:

$$\lim_{\epsilon\to 0}\big\|g_n^\epsilon - g_n\big\|_{\mathfrak{L}^1(\mathcal{H}_n)} = 0, \quad n \geq 2. \tag{13}$$

Let us observe that for arbitrary finite time interval for an asymptotically perturbed first-order cumulant of groups of operators (3), i.e. for strongly continuous groups (3), the following equality is valid

$$\lim_{\epsilon\to 0}\Big\|\mathcal{G}_s^*(t,1,\ldots,s)f_s - \prod_{j=1}^{s}\mathcal{G}_1^*(t,j)f_s\Big\|_{\mathfrak{L}^1(\mathcal{H}_s)} = 0.$$

As a result of this fact for the $(s+n)-th$ order cumulants of asymptotically perturbed groups of operators (3) the following equalities are true:

$$\lim_{\epsilon\to 0}\Big\|\frac{1}{\epsilon^n}\,\mathfrak{A}_{s+n}(t,1,\ldots,s+n)f_{s+n}\Big\|_{\mathfrak{L}^1(\mathcal{H}_{s+n})} = 0, \quad s \geq 2. \tag{14}$$

In consequence of the validity of equalities (14) for one-particle density operator (8) the following mean field limit theorem holds.

Theorem 2. *If conditions* (12), (13) *holds, then for series expansion (*8*) the equality is true*

$$\lim_{\epsilon\to 0}\big\|\epsilon G_1(t)-g_1(t)\big\|_{\mathfrak{L}^1(\mathcal{H})}=0,$$

where for finite time interval the limit one-particle density operator $g_1(t)$ *is given by the following norm convergent series on the space* $\mathfrak{L}^1(\mathcal{H})$

$$\begin{aligned} &g_1(t,1)= \qquad (15)\\ &\sum_{n=0}^{\infty}\int\limits_0^t dt_1\ldots\int\limits_0^{t_{n-1}}dt_n\,\mathrm{Tr}_{2,\ldots,n+1}\mathcal{G}_1^*(t-t_1,1)\mathcal{N}^*_{\mathrm{int}}(1,2)\prod_{j_1=1}^{2}\mathcal{G}_1^*(t_1-\\ &-t_2,j_1)\ldots\prod_{i_n=1}^{n}\mathcal{G}_1^*(t_n-t_n,i_n)\sum_{k_n=1}^{n}\mathcal{N}^*_{\mathrm{int}}(k_n,n+1)\prod_{j_n=1}^{n+1}\mathcal{G}_1^*(t_n,j_n)\times\\ &\times\sum_{\mathrm{P}\,:\,(1,\ldots,n+1)=\bigcup_i X_i}\;\prod_{X_i\subset\mathrm{P}}g_{|X_i|}(X_i)\prod_{i=1}^{n+1}g_1^0(i). \end{aligned}$$

In series expansion (15) the operator $\mathcal{N}^*_{\mathrm{int}}(j_1,j_2)$ is defined according to formula (4) and the group of operators $\mathcal{G}_1^*(t)$ is defined by (3). For bounded interaction potentials series (15) is norm convergent on the space $\mathfrak{L}^1(\mathcal{H})$ under the condition that: $t<t_0\equiv\big(2\,\|\Phi\|_{\mathfrak{L}(\mathcal{H}_2)}\|g_1^0\|_{\mathfrak{L}^1(\mathcal{H})}\big)^{-1}$.

According to Theorem 2, for marginal correlation functionals (9) the following limit theorem holds.

Theorem 3. *Under conditions* (12), (13) *on initial state* (7) *there exists a mean field limit of marginal correlation functionals* (9) *in the following sense:*

$$\lim_{\epsilon\to 0}\Big\|\epsilon^s G_s\big(t,1,\ldots,s\mid G_1(t)\big)-g_s\big(t,1,\ldots,s\mid g_1(t)\big)\Big\|_{\mathfrak{L}^1(\mathcal{H}_s)}=0,\ s\geq 2,$$

where the limit marginal correlation functionals $g_s\big(t\mid g_1(t)\big)$, $s\geq 2$, *are represented by the expansions*:

$$g_s\big(t,1,\ldots,s\mid g_1(t)\big)=$$

$$= \prod_{i_1=1}^{s} \mathcal{G}_1^*(t,i_1) g_s(1,\ldots,s) \prod_{i_2=1}^{s} (\mathcal{G}_1^*)^{-1}(t,i_2) \prod_{j=1}^{s} g_1(t,j), \tag{16}$$

and, respectively, the limit one-particle density operator $g_1(t)$ is represented by series expansion (15).

The proof of these statements is based on the validity of equality (14) for cumulants of asymptotically perturbed groups of operators (3) and the explicit structure of the generating operators of series expansions (9) of marginal correlation functionals and series expansion (8).

We remark that limit marginal correlation functionals (15), (16) are a solution of the Cauchy problem of the quantum Vlasov hierarchy of nonlinear evolution equations [2], which describes a mean field asymptotic behavior of marginal correlation operators in case of arbitrary initial states, namely,

$$\begin{aligned}
&\frac{\partial}{\partial t} g_s(t,1,\ldots,s) = \sum_{i=1}^{s} \mathcal{N}^*(i) g_s(t,1,\ldots,s) + \\
&\quad + \mathrm{Tr}_{s+1} \sum_{i=1}^{s} \mathcal{N}^*_{\mathrm{int}}(i,s+1) \big(g_{s+1}(t,1,\ldots,s+1) + \\
&\quad + \sum_{\substack{\mathrm{P}\,:\,(1,\ldots,s+1) = X_1 \bigcup X_2,\\ i \in X_1;\, s+1 \in X_2}} g_{|X_1|}(t,X_1) g_{|X_2|}(t,X_2) \big), \\
&g_s(t)\big|_{t=0} = g_s^0, \quad s \geq 1,
\end{aligned}$$

where we used notations similar to accepted above.

It should be noted that limit marginal correlation functionals (16) describe the process of the evolution of correlations of large particle quantum systems by means of a one-particle density operator in a mean field approximation.

Similar to the derivation of kinetic equation (11) we establish that the one-particle density operator represented by series expansion (15) is a solution of the Cauchy problem of the Vlasov-type quantum kinetic equation with initial correlations:

$$\frac{\partial}{\partial t} g_1(t,1) = \mathcal{N}^*(1) g_1(t,1) + \tag{17}$$

$$+\mathrm{Tr}_2\, \mathcal{N}^*_{\mathrm{int}}(1,2) \prod_{i_1=1}^{2} \mathcal{G}_1^*(t,i_1)(g_2(1,2)+I) \prod_{i_2=1}^{2} (\mathcal{G}_1^*)^{-1}(t,i_2) g_1(t,1) g_1(t,2),$$

$$g_1(t)|_{t=0} = g_1^0, \tag{18}$$

and consequently, for pure states we derive the Hartree-type equation with initial correlations. We point out that equation (17) is the non-Markovian quantum kinetic equation.

Thus, we established that a mean field behavior of processes of the creation of correlations and the propagation of initial correlations in large particle quantum systems are governed by kinetic equation (17).

5. Conclusion. The concept of quantum kinetic equations in case of initial states specified in terms of a one-particle density operator and correlation operators (11), for instance, the initial correlation operators, characterizing the condensed states [1, 6] or their influence on ultrafast relaxation processes in plasmas [11], was considered.

This paper dealt with a quantum system of a non-fixed, i.e. arbitrary but finite, number of identical (spinless) particles obeying Maxwell–Boltzmann statistics. The obtained results can be extended to large particle quantum systems of bosons and fermions like in paper [5].

In case of pure states the quantum Vlasov-type kinetic equation with initial correlations (17) can be reduced to the Gross–Pitaevskii-type kinetic equation. It was also established that in this case mean field dynamics does not create new correlations except of those that generating by initial correlations (16).

We note that in papers [12, 13] two other approaches to the description of the propagation of initial correlations of large particle quantum systems in a mean field scaling limit where developed. In paper [12] the process of the propagation of initial correlations was proved within the framework of the description of the evolution by means of marginal observables [14] and in paper [13] it was established by another method in terms of marginal density functionals with respect to a one-particle density operator governed by the generalized quantum kinetic equation [15].

The developed approach to the derivation of the quantum Vlasov-type kinetic equation with initial correlations (17) from underlying dynamics governed by the generalized quantum kinetic equation with initial correlations (11) enables to construct the higher-order corrections to the mean field evolution of large particle quantum systems.

References

[1] *Боголюбов М. М.* Лекції з квантової статистики. Питання статистичної механіки квантових систем. — К.: Рад. Школа, 1949. — 228 с.

[2] *Gerasimenko V. I., Polishchuk D. O.* A nonperturbative solution of the nonlinear BBGKY hierarchy for marginal correlation operators // Math. Meth. Appl. Sci. — 2013. — **36**, No. 17. — P. 2311 – 2328.

[3] *Gerasimenko V. I.* Hierarchies of quantum evolution equations and dynamics of many-particle correlations // Statistical Mechanics and Random Walks: Principles, Processes and Applications. — New York: Nova Science Publ. Inc., 2013. — P. 233 – 288.

[4] *Gerasimenko V. I.* Processes of creation and propagation of correlations in quantum many-particle systems // Reports of NAS of Ukraine. — 2016. — **5**. — P. 58 – 66.

[5] *Gerasimenko V. I., Polishchuk D. O.* Dynamics of correlations of Bose and Fermi particles // Math. Meth. Appl. Sci. — 2011. — **34**, No. 1. — P. 76 – 93.

[6] *Saint-Raymond L.* Kinetic models for superfluids: a review of mathematical results / C.R. Physique /. — 2004. — **5**. — P. 65 — 75.

[7] *Cercignani C., Gerasimenko V., Petrina D.* Many-Particle Dynamics and Kinetic Equations, the Netherlands: Springer, 2012. — 248 p.

[8] *Golse F.* On the dynamics of large particle systems in the mean field limit. — 2013. — (ArXiv preprint / arXiv:1301.5494 [math.AP]).

[9] *Erdös L., Schlein B., Yau H-T.* Derivation of the cubic nonlinear Schrödinger equation from quantum dynamics of many-body systems // Invent. Math. — 2007. — **167**. — P. 515 – 614.

[10] *Chen X., Guo Y.* On the weak coupling limit of quantum many-body dynamics and the quantum Boltzmann equation // Kinetic and Related Models. — 2015. — **8**, No. 3. — P. 443 – 465.

[11] *Semkat D., Kremp D., Bonitz M.* Kadanoff–Baym equations with initial correlations // Phys. Rev. E. — 1999. — **59**. — P. 1557 – 1562.

[12] *Gerasimenko V. I.* New approach to derivation of quantum kinetic equations with initial correlations // Carpathian Math. Publ. — 2015. — **7**, No. 1. — P. 38 – 48.

[13] *Gerasimenko V. I., Tsvir Zh. A.* On quantum kinetic equations of many-particle systems in condensed states // Physica A: Stat. Mech. Appl. — 2012. — **391**, No. 24. — P. 6362 – 6366.

[14] *Gerasimenko V. I.* Heisenberg picture of quantum kinetic evolution in mean field limit // Kinet. Relat. Models. — 2011. — **4**. — P. 385 – 399.

[15] *Gerasimenko V. I. and Tsvir Zh. A.* A description of the evolution of quantum states by means of the kinetic equation // J. Phys. A: Math. Theor. — 2010. — **43**. — 485203.