Adaptive control methods

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STABILITY EVALUATION BASED ON THE SUSTAINABILITY TRIANGLE APPLICATION FOR TRANSFER FUNCTIONS ABOVE 2ND ORDER

This research subject represents the transfer function of a system above the second order. **Research goal:** to simplify the system stability assessment process for transfer functions higher than second order based on the stability triangle and inequations of the transfer functions coefficients' acceptable values. **Task:** to simplify the system stability assessment; to investigate the characteristic equation and determine limiting inequations. **Methods used**: Jury stability criterion, the analytical method for stability determining the when graphical representation used on the stability triangle basis, the characteristic equation analytical solution, and mathematic modeling. The following **results have been obtained**: the maximum permissible coefficient values are found, as well as limiting inequations for the transfer function coefficients. Found are the correlation coefficients between the easily determined maximum theoretical values of these coefficients and their practical values, the correction coefficients' functional dependences being obtained. **Conclusions:** based on the research done to estimate the coefficients' boundaries for frequency-dependent components above the second order, necessary is to determine the characteristic equation coefficients maximum values multiplying them by the correction coefficients obtained at this research issue. The obtained values represent the components' stability boundary at denominator coefficients' change. This greatly facilitates the stability evaluation when rebuilding the component transfer function's coefficients.

Keywords: Jury criterion; characteristic equation; stability boundaries; limiting inequations; frequency-dependent component above second order.

Introduction

The specialized computer, programmable mobile and robotic systems are functioning under difficult and unpredictable conditions that imply the need to restructure the system components transfer functions' coefficients [1-5]. Therefore, we face a problem when, prior to start the adjustment, it is necessary to evaluate the system stability while ensuring its new condition, by changing the system components transfer functions' coefficients [6–9]. For first and second orders transfer functions, this problem is solved easily, but when transfer functions above the second order, it is not a simple one [8–10].

The research aim is to simplify the process of system stability assessment for transfer functions higher that second order on the basis of stability triangle and inequations of transfer function coefficients' allowable values.

Research results

In most stability problems, considered is the system characteristic equation:

$$D(z)=\sum_{i=0}^{n} b_i z^{n-i},$$

where b_i – denominator coefficients at the system transfer function (in most cases, $b_0=1$, n – system order.

To assess the n-th order system stability is possible using the Jury criterion, according to which satisfied shall be the characteristic equation' inequations of the form [10]:

$$D(1)>0;\ (-1)^n D(-1)>0. \qquad (1)$$

This criterion works well with the system transfer function constant coefficients, and the restructuring of one or several coefficients involves problems arising. So we have to find these coefficients limits to ensure stability. For a second-order system, whose transfer function has the form [10, 11]:

$$H_2(z)=\frac{a_0+a_1z^{-1}+a_2z^{-2}}{1+b_1z^{-1}+b_2z^{-2}}.$$

The characteristic equation is equal to

$$D(z)=z^2+b_1z+b_2.$$

In this case, from (1) we can write:

$$1+b_1+b_2>0;$$

$$1-b_1+b_2>0.$$

Then in coordinates (b_1, b_2) we can find the stability triangle forming lines

$$\begin{cases} b_2>-1-b_1; \\ b_2>-1+b_1. \end{cases}$$

However, this triangle has no upper boundary $b_{2\max}$.

So next, we consider the variance with characteristic equation complex - conjugate roots

$$z_{1,2}=\alpha e^{\pm j\phi}, \qquad (2)$$

where α - modulus ($|\alpha|\le 1$), ϕ - argument.

It should be noted that the frequency-dependent components at computer systems above the first order contain complex-conjugate roots, for example, filters.

Then, based on the Viet theorem, we can write down [12]:

$$\begin{cases} z_1 + z_2 = -b_1; \\ z_1 \cdot z_2 = b_2. \end{cases} \quad (3)$$

Substituting (2) into (3) we shall get

$$\begin{cases} z_1 + z_2 = \alpha\left(e^{j\phi} + e^{-j\phi}\right); \\ z_1 \cdot z_2 = \alpha e^{j\phi} \cdot \left(\alpha e^{-j\phi}\right). \end{cases}$$

Based on the Euler formula and the rules of complex numbers operation, we obtain

$$\begin{cases} z_1 + z_2 = 2\alpha \cos\phi = -b_1; \\ z_1 \cdot z_2 = \alpha^2 = b_2. \end{cases}$$

then $b_{2\max} \le 1$, with respect to $|\alpha| \le 1$.

Based on the obtained relations, we can therefore build a stability triangle, Fig 1.

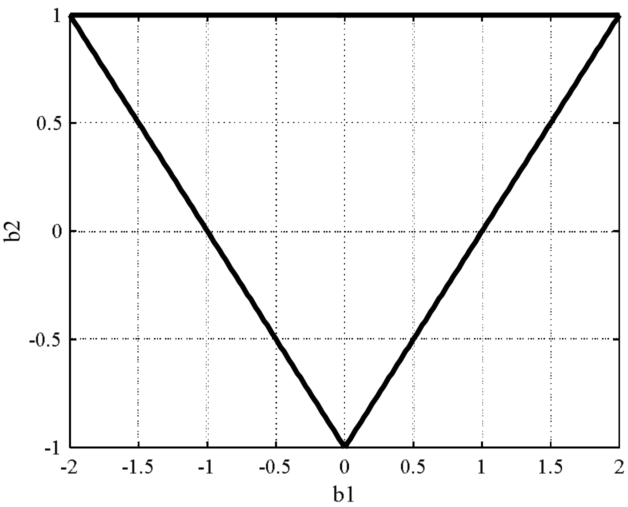


Fig. 1. Stability triangle for second order transfer function

The stability triangle limits the denominator coefficients' values at transfer function of the second order [11, 13]. This means that the transfer function characteristic equation roots

$$D(z) = z^2 + b_1 z + b_2 = 0,$$

will be located inside the unit circle on the Z-plane.

To assess the stability of tunable transfer functions above the second order, necessary is to solve the corresponding algebraic equations. However, we thus suggest the following procedure. Let we consider the 5th order characteristic equation, which has the following form:

$$D(z) = z^5 + b_1 z^4 + b_2 z^3 + b_3 z^2 + b_4 z + b_5.$$

Then by the Juri criterion we can write

$$\begin{cases} 1 + b_1 + b_2 + b_3 + b_4 + b_5 > 0; \\ 1 - b_1 + b_2 - b_3 + b_4 - b_5 > 0. \end{cases}$$

Combining the odd and even coefficients respectively

$$\begin{cases} d_1 = b_1 + b_3 + b_5; \\ d_2 = b_2 + b_4. \end{cases}$$

We get that lines forming the stability triangle will have the form

$$\begin{cases} 1 + d_1 + d_2 > 0; \\ 1 - d_1 + d_2 > 0 \end{cases} \quad \text{or} \quad \begin{cases} d_2 > -1 - d_1; \\ d_2 > -1 + d_1. \end{cases}$$

In coordinates (d_1, d_2) we again do not observe the stability upper limit, which by analogy can be found from the Viet theorem and Euler formulas, taking into account the roots $z_0 = \alpha$, $z_{1,2} = \beta e^{\pm j\phi}$, $z_{3,4} = \gamma e^{\pm j\psi}$:

$$\begin{cases} \alpha + 2[\beta\cos\phi + \gamma\cos\psi] = -b_1; \\ \beta^2 + \gamma^2 + 2\alpha(\beta\cos\phi + \gamma\cos\psi) + 4\beta\gamma\cos\phi\cos\psi = b_2; \\ \alpha\beta^2 + \alpha\gamma^2 + 4\alpha\beta\gamma\cos\phi\cos\psi + \\ \quad + 2\beta\gamma[\beta\cos\psi + \gamma\cos\phi] = -b_3; \\ 2\alpha\beta^2\gamma\cos\psi + 2\alpha\beta\gamma^2\cos\phi + \beta^2\gamma^2 = b_4; \\ \alpha\beta^2\gamma^2 = -b_5. \end{cases}$$

Based on these equations, we can find limiting inequations for both the coefficients and the sum of even and odd coefficients:

$$\begin{cases} -3 < -b_1 < 5; \\ -6 < b_2 < 10; \\ -2 < -b_3 < 10; \\ -3 < b_4 < 5; \\ 0 < b_5 < 1 \end{cases}$$

and

$$\begin{cases} -9 < d_1 < 16; \\ -6 < d_2 < 15. \end{cases}$$

The results obtained allow us to construct a stability triangle in the coordinates (d_1, d_2), Fig.2 .

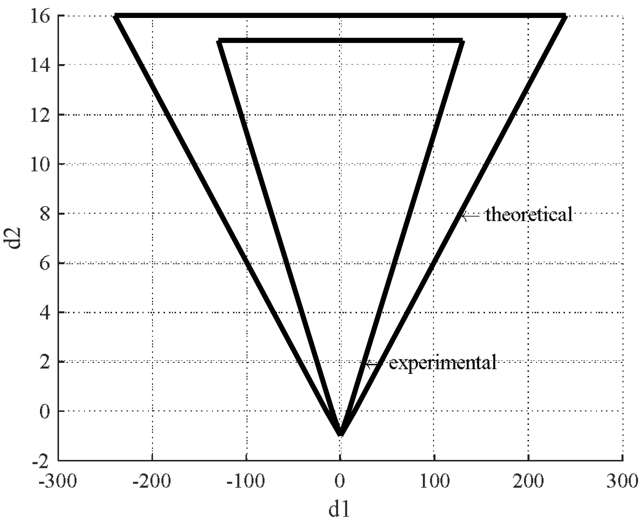


Fig. 2. Stability triangle for the fifth order transfer function

The resulting ratio allow generalisation. Based on the analysis of the characteristic equation coefficients' maximum values in the worst case, we can specify that these values are determined by the combination formulae [12]:

$$C_n^m = \frac{n!}{m!(n-m)!}, \quad (4)$$

where the characteristic equation order (or the frequency-dependent component transfer function order),

n, m – coefficient number,

C_n^m – coefficient value, for example, for the fifth order equation coefficient

$$b_3 = C_5^3.$$

Thus, it is possible to calculate the coefficients' maximum values for all characteristic equation coefficients at any order of the transfer function.

Studies and analysis of the stability triangle upper boundary of the, determined by the sum of even coefficients, showed that it can be calculated on the basis of relation

$$d_{2\max} = 2^{n-1} - 1,$$

where n – the characteristic equation order (or the frequency-dependent component transfer function order).

Having analyzed the computer system frequency-dependent components transfer functions of orders above second we revealed that the obtained estimates are higher than the coefficients' possible values in the entire frequency range, as demonstrated with Fig. 2, representing the stability triangles of theoretical calculations using formula (4) and their experimental verification.

In this case the theoretical assessment can be considered as the upper limit of the characteristic equation coefficients, which is overrated, but can be easily calculated.

The study of correlation between the largest values of coefficients obtained experimentally and the maximum theoretical values of various computer system frequency-dependent components above the second order, using the digital filters example, showed that there exists some dependency on the order, Fig. 3-7.

These dependencies can be considered as correction factors depending on the order n.

The dependences obtained appearing on the graphs Fig. 3-7 are well approximated by power equations:

– for Butterworth filters:

$$K_{but}(n) = Ae^{Bn},$$

where $A = 0,6516$; $B = -0,2$;

– for Chebyshev filters, Chebyshev inverse and elliptic filters:

$$K(n) = Cn^2 + Dn + E,$$

where the coefficients C , D and E values are summarized in Table 1.

Based on Table 1 data, some coefficients values can be neglected and the resulting equations are approximated by dependencies:

– for Chebyshev, Chebyshev inverse and odd-order elliptic filters:

$$K(RP) = F \cdot RP^2 + G \cdot RP + H.; \quad (5)$$

- for even order elliptic filters:

$$K_1(RP) = J \cdot \ln(RP) + L. \quad (6)$$

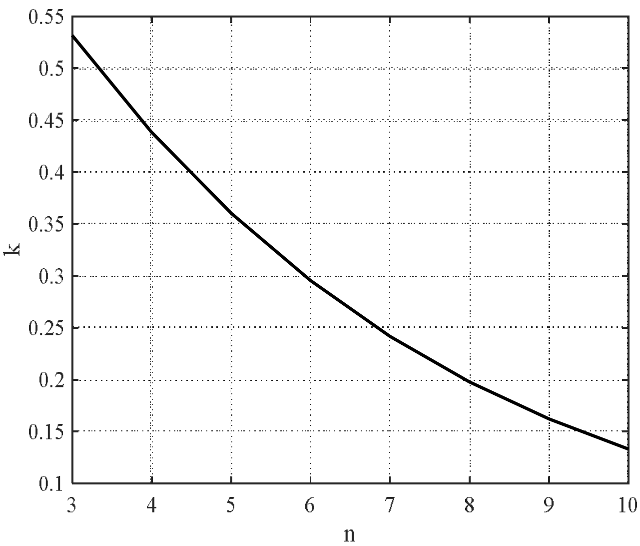


Fig. 3. Graph of the correlation between the coefficients' largest values and the maximum values obtained for the digital Butterworth filters

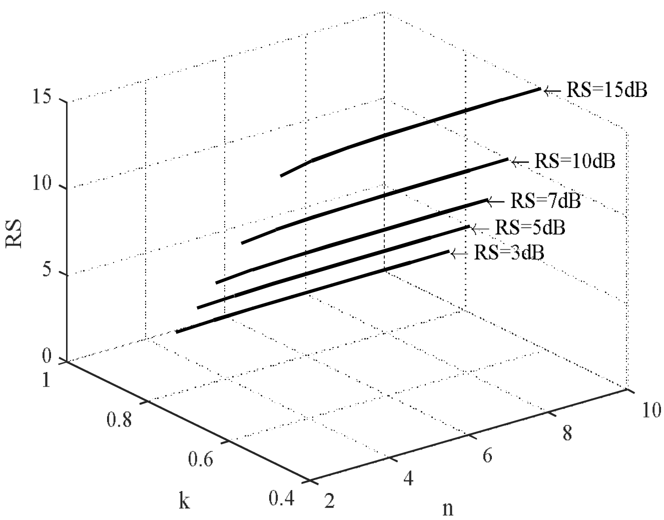
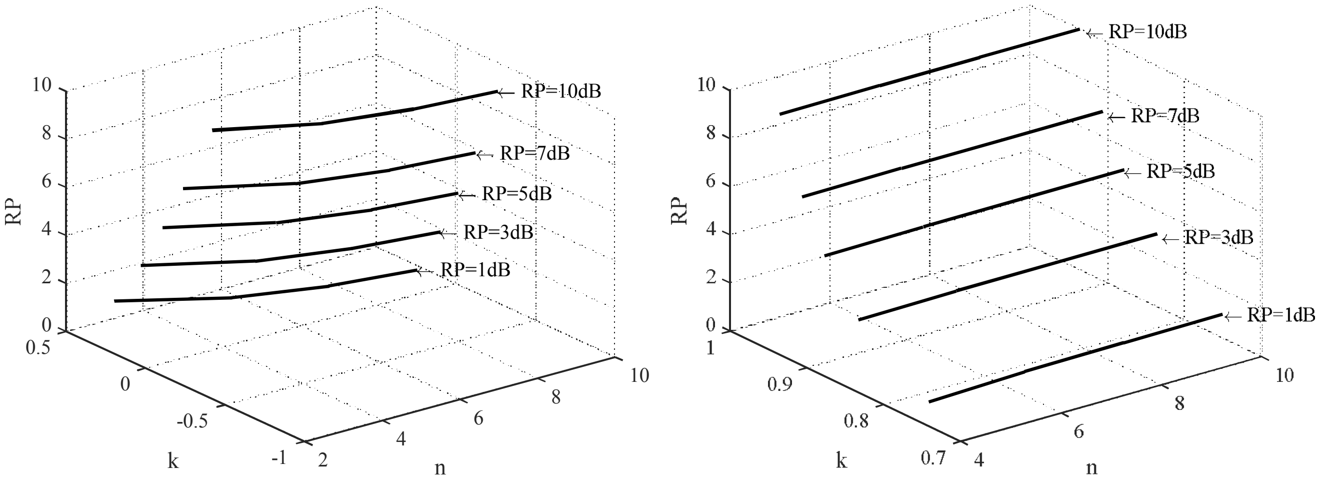
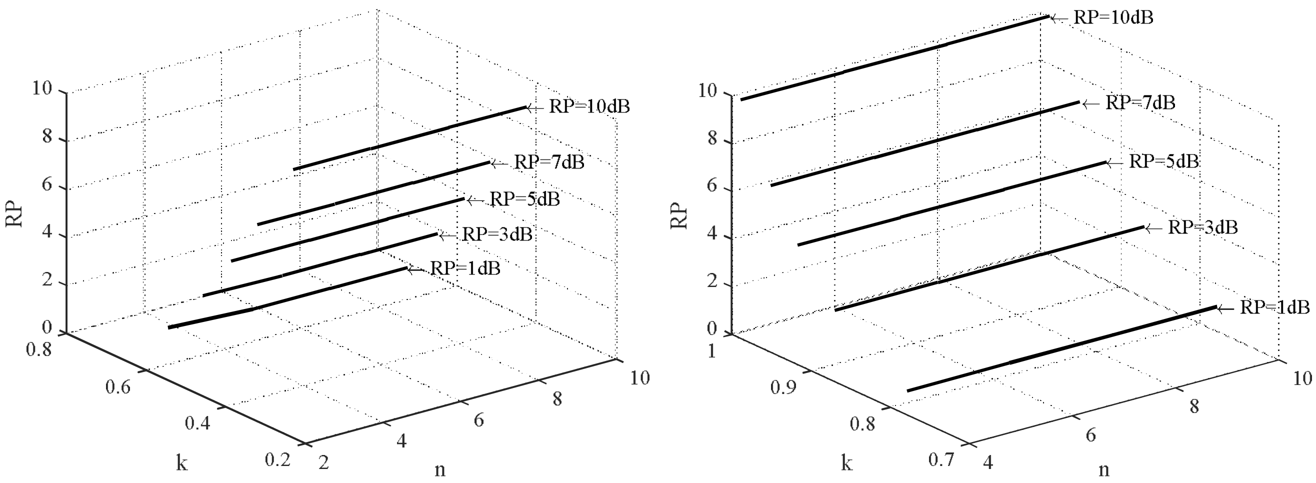


Fig. 4. Graph of the correlation between the highest coefficient values and the maximum values obtained for inverse digital Chebyshev filters at various oscillation levels in the attenuation band RS



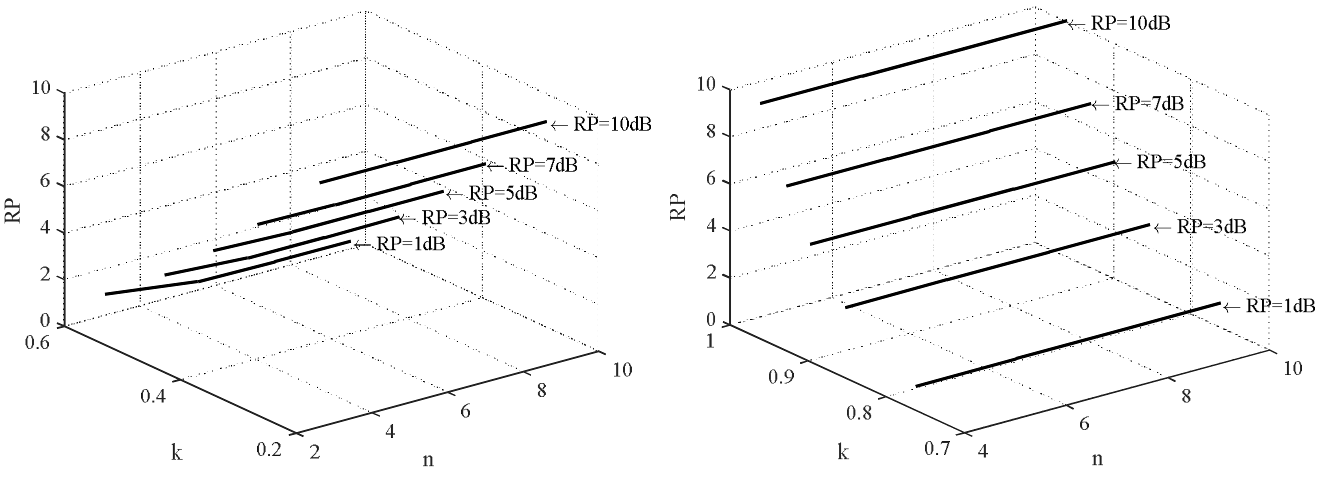
a b

Fig. 5. Graph of the correlation between the highest coefficient values and the maximum values obtained for digital Chebyshev filters a) odd order sequence, b) even order sequence at different oscillation levels in the RP passband



a b

Fig. 6. Graph of the correlation between the highest coefficient values and the maximum values obtained for the elliptic digital filters a) odd order sequence, b) even order sequence at different oscillation levels in the RP passband and attenuation band RS = 11 dB



a b

Fig. 7. Graph of the correlation between the highest coefficient values and the maximum values obtained for the elliptic digital filters a) odd order sequence, b) even order sequence at different oscillation levels in the RP passband and attenuation band RS = 15 dB

Table 1 – **Value of the approximation dependencies' coefficients**

order	**coefficient**	**Chebyshev filter**					**Inverse Chebyshev**					**Elliptic filter**									
		RP					**RS**					**RS**									
												11					**15**				
												RP									
		1	**3**	**5**	**7**	**10**	**3**	**5**	**7**	**10**	**15**	**1**	**3**	**5**	**7**	**10**	**1**	**3**	**5**	**7**	**10**
odd	C	0,0434	0,0425	0,0422	0,0414	0,0387	-0,0004	-0,0006	-0,0008	-0,0012	-0,0019	0,0039	0,0013	0,0005	0,0001	-0,0004	0,0074	0,0034	0,0018	0,0009	0,0002
even		-0,0012	-0,0006	-0,0004	-0,0003	-0,0002						-0,0002	$-3e^{-5}$	0	0	$9e^{-16}$	-0,0004	$-8e^{-5}$	$-2e^{-5}$	$-2e^{-5}$	0
odd	D	-0,3618	-0,3473	-0,3398	-0,3289	-0302	0,0043	0,0063	0,0087	0,0129	0,0216	-0,0246	-0,0078	-0,0028	-0,0008	0,0015	-0,0471	-0,021	-0,011	0,0054	0,0014
even		0,0086	0,0044	0,0028	0,0022	0,0014						0,0011	0,0002	$-7e^{-15v}$	0	$-7e^{-15}$	0,0022	0,0005	0,0002	0,0002	0
odd	E	0,757	0,577	0,4314	0,2916	0,085	0,8263	0,7717	0,7237	0,6577	0,5546	0,6634	0,562	0,4866	0,4197	0,327	0,6333	0,5094	0,4179	0,3367	0,2265
even		0,7342	0,8294	0,8748	0,9041	0,9338						0,778	0,8698	0,9173	0,9509	0,9892	0,7608	0,8519	0,8973	0,928	0,9614

Table 2 – **Dependencies approximation coefficients' values**

Filter type	**Filter order**	**Coefficient**				
		F	**G**	**H**	**J**	**L**
Chebyshev	odd	-0,0034	-0,1426	0,8935	-	-
	even	0,1429	1,3429	-0,4	-	-
Inverse Chebyshev	-	0,0082	0,0165	0,8465	-	-
Elliptic	odd, RS=11	0,0018	-0,0926	0,7492	-	-
	odd, RS=15	0,0027	-0,1148	0,7395	-	-
	Even, RS=11	-	-	-	0,1283	0,7782
	Even, RS=15	-	-	-	0,1223	0,7627

Conclusion

Thus, to estimate the coefficients variation boundaries for frequency-dependent components above the second order, necessary is, based on formula (4), to determine the characteristic equation coefficients' boundary values and multiply those values by the correction factors determined by equations (5-6) depending on the type of frequency-dependent components (filters). The obtained values represent the components' stability boundary at denominator coefficients' change. This greatly facilitates the stability evaluation when rebuilding the component transfer function's coefficients.

REFERENCES

1. Dorf, R.C. and Bishop, R.H. (2017), *Modern Control Systems*, Pearson Education Inc., ISBN-13: 978-0134407623, 831 p.
2. Tuzlukov, V. (2013), *Signal Processing in Radar Systems*, CRC Press, 596 p.
3. Popov, D.I. (2016), "The digital signal processing systems analysis", *Izvestija vysshikh ychebnykh zavedenij. Tekhnicheskie nayki*, No 2, pp. 83-92.
4. Ukhina, A.V., Bilenko, A.A. and Sytnikov, V.S. (2016), "Improving the efficiency software and hardware complexes in NPP APCS", *Jadernaja i radiatsionnaja bezopasnostj*, No 3, pp. 70-76.
5. Ljapun, D.A. (2016), "Investigation of a computer platform management system on a mobile object", *Naukovi pratsi. Serija: Komp'juterni tekhnologiji. Chornomorskyi nats. un. im. P. Mogily*, No. 275, Vol. 287, pp. 62-68.
6. Zhengyou, He. (2016), *Wavelet Analysis and Transient Signal Processing Applications for Power Systems*, Chine Electric Power Press, 252 p.
7. Li, M., Duan, Y., Wang, Y. and Zhang, L. (2016), Time-Frequency Analysis Method in the Transient Power Quality Disturbance Analysis Application, *Electronic Devices*, No. 1, pp. 9-13.
8. Dzhigan, V.A. (2013), Adaptive signals filtering: theory and algorithms, Tekhnosfera, Moscow, 528 p.
9. Zhuravlev, A.Ju., Pavlov, A.V. and Zhurba, V.O. (2012), "The stability overall increasing technique and correction the transient processes quality direct indicators in digital control systems", *Kompressionnoe i energeticheskoe mashinostroenie*, No. 1, pp. 36-40.
10. Astrom, K.J. and Witternmark, B. (1987), *Computer controlled systems: Theory and design*, Prentice-Hall.Inc, 480 p.
11. Sergienko, A.B. (2006), Digital signal processing, Piter, St. Petersburg, 751 p.
12. Bronshhtejn, I.N. and Semendjaev, K.A. (1980), *Mathematics handbook*, Nauka, Moscow, 976 p.
13. Sytnikov, V.S. and Stupen, P.V. (1999), "Checking the stability of digital filters by transfer function", *Trudy Odesjk. politech. un-ta*, Odessa, No 1(7), pp. 160-162.

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Оцінка стабільності на основі застосування трикутника стабільності для передавальних функцій вище другого порядку

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Предметом дослідження є передавальна функція системи вище другого порядку. **Мета роботи**: спрощення процесу оцінки стійкості системи для передавальних функцій вище другого порядку на основі трикутника стійкості і нерівностей допустимих значень коефіцієнтів передавальних функцій. **Завдання**: спростити оцінку стійкості системи; дослідити характеристичне рівняння і визначити обмежуючі нерівності. **Методи,** які використовувались: критерій стійкості Джурі, аналітичний метод для визначення стійкості при використанні графічного представлення на основі трикутника стійкості, аналітичне рішення характеристичного рівняння, математичне моделювання. Отримані наступні **результати**. Знайдено максимально допустимі значення коефіцієнтів, а також нерівності, які обмежують коефіцієнти передавальної функції. Виявлено коефіцієнти відповідності між максимальними теоретичними значеннями коефіцієнтів, що легко визначаються і практичними їх значеннями, отримані функціональні залежності коригувальних коефіцієнтів. **Висновки**. На основі проведеної роботи для оцінки меж зміни коефіцієнтів частотно-залежних компонент вище другого порядку необхідно визначити максимальні значення коефіцієнтів характеристичного рівняння і помножити їх на коригувальні коефіцієнти, отримані в результаті проведених досліджень. Отримані значення представляють границю стійкості компонентів при зміні знаменних коефіцієнтів. Це значно полегшує оцінку стабільності при переробці коефіцієнтів функції передачі компонентів.

Ключові слова: критерій Джурі; характеристичне рівняння; межі стійкості; нерівності, що обмежують; частотно-залежний компонент вище другого порядку.

Оценка устойчивости на основе применения треугольника устойчивости для передаточных функций выше второго порядка

А. В. Ухина, В. С. Ситников, К. А. Пуц

Предметом исследования является передаточная функция системы выше второго порядка. **Цель работы:** упрощение процесса оценки устойчивости системы для передаточных функций выше второго порядка на основе треугольника устойчивости и неравенств допустимых значений коэффициентов передаточных функций. **Задание:** упростить оценку устойчивости системы; исследовать характеристическое уравнение и определить ограничивающие неравенства. Используемыми **методами** являются: критерий устойчивости Джури, аналитический метод к определению устойчивости при использовании графического представления на основе треугольника устойчивости, аналитическое решение характеристического уравнения, математическое моделирование. Получены следующие **результаты**. Найдены максимально допустимые значения коэффициентов, а также ограничивающие неравенства для коэффициентов передаточной функции. Выявлены коэффициенты соответствия между легко определяемыми максимальными теоретическими значения коэффициентов и практическими их значений, получены функциональные зависимости корректирующих коэффициентов. **Выводы**. На основе проведенной работы для оценки границ изменения коэффициентов частотно-зависимых компонент выше второго порядка необходимо определить максимальные значения коэффициентов характеристического уравнения и умножить их на корректирующие коэффициенты, полученные в результате проведенных исследований. Полученные значения представляют границу устойчивости компонентов при изменении коэффициентов знаменателя. Это значительно облегчает оценку стабильности при восстановлении коэффициентов передаточной функции компонента.

Ключевые слова: критерий Джури; характеристическое уравнение; границы устойчивости; ограничивающие неравенства; частотно-зависимый компонент выше второго порядка.