Oleg Mashkov ${ }^{1}$, Viktor Mashkov ${ }^{2}$
${ }^{1}$ State Ecology Academy of Postgraduate Education and Management
${ }^{2}$ University J.E.Purkyne

# ORGANIZATION OF SELF-CONTROL PROCEDURE IN FUNCTIONALLY STABLE COMPLEXES 


#### Abstract

Intermodular exchange of diagnostic information in self-diagnosis of complex technical systems with random structure of testing links is considered. Information on the structure of testing links of the system, collected in its modules as a result of intermodular exchange, is evaluated.


Key words: self-control, self-diagnosis, inter-module exchange, information, functional stability.

Introduction. We will consider complex technical systems which can be represented as an aggregate of interconnected modules capable of testing each other. It is well known that self-diagnosis of such systems based on information on mutual tests of their modules has been given the name self-diagnosis by the principle of distributed core [1]. Self-diagnosis of this kind has been investigated in sufficient detail for the case when modules test each other's technical condition according to a preset algorithm. However, organization of tests of modules according to such algorithm is a complex problem in actual practice when the system executes its jobs [2,3]. The situation appears to be more realistic when elementary tests are performed in a random way at arbitrary instants of time, when pairs of testing and tested modules are being formed. By an elementary test (ET) is meant a test of the technical condition of an individual module of the system. In this case the structure of testing links (STL) will also be random. This means that it is impossible to determine in advance which will be the STL after the lapse of a preset diagnosis time. In this connection the problem arises of determining the time after which execution of the ET should be stopped and an analysis of the totality of results of the ET should be carried out [4]. If this time is too short, then STL of the system may prove such that it will not allow us to determine technical condition of the modules with the preset reliability, and if the time is too long, then it will result in inefficient use of modules of the system.

It can be shown that self-diagnosis with a preset reliability can be provided by the availability in testing links graph (TLG) of a system of maximum-length loops (MLL) [4]. If we take into consideration the fact that the number of performed ET is proportional to diagnosis time, then we can find a relation between the probability of formation of a MLL system in a TLG and the number of performed ETs.

Since an analysis of the totality of results of an ET will be performed by one of modules of the system, which cannot be determined in advance, it is necessary also that each module has information about MLL in the system TLG.

Hence the problem arises of relating the probability of a situation when a preset number of MLL will be formed in TLG of the system with information about it provided to each module, and the
number of performed ETs.
Estimation of Information about Maximum-Length Loops Contained in Modules of the System. After execution of an ET, the testing and the tested modules exchange information necessary to provide selfdiagnosis of the system. In this case information being transmitted is subdivided into information containing results of an ET and information containing data on the STL of the system.

Only information related to STL of the system is considered in the present paper. From the point of view of practical realization of intermodular exchange, it is desirable that this information be as simple as possible, i.e. that it be restricted to the values of local degrees of nodes of TLG.

For example, let the $i$-th module transmit to the $y$ th module the value of the local degree of the $i$-th node of TLG $\alpha_{i}$ and the values of local degrees of all nodes linked in TLG with the $j$-th node by a simple oriented path (provided that they were earlier transmitted to the $i$-th module).

Such information allows us to single out in TLG of the system the availability of structure elements playing an important role in diagnosis of system modules. A maximum-length loop is taken as such a basic element.

Let us assume that as a result of fulfillment of the totality of an ET, a MLL is formed in TLG of the system, and let us investigate which information about it is available for the modules of the system. The MLL can be formed by several methods depending on the succession of fulfillment of an ET corresponding to edges which form this MLL. In this case information about MLL in each module also depends essentially on the order of priority in fulfillment of the ET.

Since each node in a MLL is connected by edges only with two adjacent nodes, the succession in which the following four ET are fulfilled:

$$
\begin{gathered}
\tau_{i, i+1}\left(M_{i}, M_{i+1}\right), \tau_{i+1, i-1}\left(M_{i-1}, M_{i-2}\right) \\
\tau_{i-1, i}\left(M_{i-1}, M_{i}\right), \tau_{i-2, i-1}\left(M_{i-2}, M_{i-1}\right)
\end{gathered}
$$

plays an important role for the module of the system which corresponds to this node. The ET correspond to edges of the graph presented in Fig. 1.

If elementary test $\tau_{\mathrm{i}-2, \mathrm{i}-1}$ is carried out prior to test
$\tau_{\mathrm{i}-1, \mathrm{i}}$, then the $i$-th module will receive information not only on the local degree of the node $\mathrm{v}_{\mathrm{i}-1}$ corresponding to module $\mathrm{M}_{\mathrm{i}-1}$, but also on local degrees of the node $\mathrm{v}_{\mathrm{i}-2}$ corresponding to module $\mathrm{M}_{\mathrm{i}-2}$, and on all nodes whose information was sent to module $\mathrm{M}_{\mathrm{i}-2}$ earlier.


Figure 1-Graph of testing links
Let us denote information on local degrees of nodes contained in the $i$-th module after fulfillment of the first ET by $y_{i}$. Then we can write

$$
\begin{aligned}
& y_{1}^{1}=\tau_{1.1}^{1} \alpha_{1}+\tau_{1.2}^{1} \alpha_{2}+\ldots+\tau_{1 . n}^{1} \alpha_{n}, \\
& y_{2}^{1}=\tau_{2.1}^{1} \alpha_{1}+\tau_{2.2}^{1} \alpha_{2}+\ldots+\tau_{2 . n}^{1} \alpha_{n}, \\
& \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \cdot \\
& y_{n}^{1}=\tau_{n .1}^{1} \alpha_{1}+\tau_{n .2}^{1} \alpha_{2}+\ldots+\tau_{n . n}^{1} \alpha_{n},
\end{aligned}
$$

or in matrix form $\mathrm{Y}^{1}=\mathrm{T}^{1} \mathrm{~L}$, where

$$
T^{1}=\left\lfloor T_{i, j}^{1}\right\rfloor L^{T}=\left[\alpha_{1}, \alpha_{2}, \ldots \ldots, \alpha_{n}\right]
$$

We will select and consider only ET $\left\{\tau_{\mathrm{ij}}\right\}$ to which edges of TLG forming MLL correspond. Matrix $T^{l}$ for these ETs takes the form:

$$
T^{1}=\left[\begin{array}{lllll}
1 & \tau_{1,2}^{1} & 0 & \cdots & \tau_{1, n}^{1} \\
\tau_{2,1}^{1} & 1 & \tau_{2,3}^{1} & \ldots & 0 \\
0 & \tau_{3,1}^{1} & 1 & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\tau_{n, 1}^{1} & 0 & 0 & \ldots & 1
\end{array}\right]
$$

It can be seen that matrix $T^{l}$ is symmetrical and only two of its elements other than elements of the main diagonal are equal to 1 , while the rest are equaled to 0 .

After fulfillment of the second ET, modules of the system will contain information $Y^{2}$, where $Y^{2}=T^{2} Y^{1}$, $T^{2}=\left[\tau_{i, j}^{2}\right]$. Then after fulfillment of $n$ elementary tests the modules will contain the following information:

$$
\begin{gathered}
Y^{n}=T^{n} T^{n-1} \ldots T^{1} L \text { or } Y^{n}=T_{\Sigma} L, \text { where } \\
T_{\Sigma}=T^{n} T^{n-1} \ldots T^{1} .
\end{gathered}
$$

Matrix $\mathrm{T}_{\Sigma}$ is of the form

$$
T_{\Sigma}=\left[\begin{array}{lllll}
r_{d} & r_{c} & f_{13}\left(\tau_{i j}\right) & \ldots & r_{c} \\
r_{c} & r_{d} & r_{c} & \ldots & f_{2 n}\left(\tau_{i j}\right) \\
f_{31}\left(\tau_{i j}\right) & r_{c} & r_{d} & \ldots & f_{3 n}\left(\tau_{i j}\right) \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
r_{c} & f_{n 2}\left(\tau_{i j}\right) & f_{n 13}\left(\tau_{i j}\right) & \ldots & r_{d}
\end{array}\right],
$$

where $f_{m n}\left(\tau_{i j}\right)$ is a function consisting of sums and products of elements of matrices $T^{l}, T^{2}, \ldots, T^{n}: r_{c}$ and $r_{d}$ are elements differing from zero.

It can be shown that the following propositions are true for matrix $T_{\Sigma}$ [4]:

1. A row will be found without fail in matrix $T_{\Sigma}$, which has no less than five elements differing from zero.
2. A row will be found without fail in matrix $T_{\Sigma}$, which has precisely $n-3$ zero elements (with the odd number of modules in the system).
3. Total number of zero elements $k_{0}$ in matrix $T_{\Sigma}$ satisfies relations

$$
\begin{aligned}
& \left(k_{0}\right)_{\max }=n^{2}-4 n \text { at even } n \text { and } n \leq 5, \\
& \left(k_{0}\right)_{\max }=n^{2}=4 n-1 \text { at } \text { odd } n, \\
& \left(k_{0}\right)_{\max }=n^{2}-5 n+3 \text { at } n \leq 5, \\
& \left(k_{0}\right)_{\max }=n^{2}-6 n+8 \text { at } n>5 .
\end{aligned}
$$

Sequence of fulfillment of the ET affects the values $f_{m n}\left(\tau_{i j}\right)$ which can take values 0,1 or 2 .

Thus, depending on the succession of formation of MLL edges, formation of $n!$ matrices $T_{\Sigma}$ is possible, which are characterized by a definite number of zero elements.

Since formation of matrices $T_{\Sigma}$ is of a random nature, we will consider the probability of formation of definite groups of matrices. To do this, we break down the totality of all possible matrices $T_{\Sigma}$ into groups $B_{k 0=i}\left(T_{\Sigma}\right)$ with the number of zero elements $k_{0}$ in each being equal to $i$, where $\left(k_{0}\right)_{\min } \leq l \leq\left(k_{0}\right)_{\max }$.

Therefore the probability of formation of each group of matrices $B_{i}\left(T_{\Sigma}\right)$ can be considered as the probability of the presence in matrix $T_{\Sigma}$ of precisely $i$ zero elements, i.e. $P\left\{k_{0}=l\right\}$.

The method of determining probability $P\left\{k_{0}=l\right\}$ is as follows. Intermodular information exchange is presented as a graph (Fig. 2) with nodes corresponding to modules of the system, and edges to elementary tests and to information exchange between modules of the system. The edges are numbered depending on the order of their formation. Arrows in Fig. 2 show direction of the
increase of numbers of two adjacent edges.
Information transmitted from one module of the system to another is represented on the TLG as an arrow directed from the edge having smaller number to the edge having greater number. Since information is transmitted in the direction of increasing number of edges, the number of arrows directed one after the other in the same direction is representative of the distance of information transmission and of the number of modules which receive this information.


Figure 2. Graph of intermodular exchange of information.

The cases are considered successively when takes a concrete value $i$ from $k_{0}=\left(k_{0}\right)_{\text {min }}$ to $k_{0}=\left(k_{0}\right)_{\max }$. Arrangement of arrows in Fig. 2 is determined for each $k_{0}=l$, at which the number of information exchanges between modules of the system is equal to $\left(k_{0}\right)_{\max }-l$. It should be noted that only information exchanges between modules corresponding to nonadjacent nodes of MLL are taken into account in this case. Then the number of indexings of edges corresponding to each arrangement of arrows in Fig. 2 is determined. The obtained number of indexings of edges is equal to the total number of different matrices $T_{\Sigma}$ having $k_{0}=l$.

We determine probability $P\left\{k_{0}=l\right\}$ as the ratio between the number of matrices $T_{\Sigma}$ having $k_{0}=l$ and the total number of all possible matrices $T_{\Sigma}$, i.e. $n!$.

Thus the estimation of information on maximumlength loops contained in modules of the system can be performed with the aid of proposed matrix $T_{\Sigma}$. For example, in the case under consideration when one MLL is formed in a TLG of the system, zero elements will be found without fail in matrix $T_{\Sigma}$, and this means that modules of the system will not have full information about this MLL.

In order for modules of the system to possess such information, it is necessary that additional (with respect to MLL) edges are present in the TLG of the system. This represents the situation when information exchange takes place between modules of the system, corresponding to nodes connected by additional edges. Such information exchange can result in a situation when all modules of the system receive full information about MLL and about local degrees of nodes of TLG.

We will investigate the possibility of such situation.

Estimation of Completeness of Information about Maximum-Length Loops. The problem can be
subdivided into two parts:

1. The set of all possible matrices $T_{\Sigma}$ is divided into groups each being characterized by its value of $\mathrm{k}_{0}$. All probabilities $P_{i, j}$ of a situation are determined for each group of matrices $B_{i}\left(T_{\Sigma}\right)$ when after exchange of information between the $i$-th and the $j$-th module, the $i$-th and the $j$-th row in matrix $T_{\Sigma}$ of the system will have no zero elements. In this case only such $i$ and $j$ are considered when edge ( $i, j$ ) does not belong to MLL
2. Based on probabilities $P_{i, j}$, probabilities $P^{B}{ }_{U}$ are determined of the situation when all rows of matrix $T_{\Sigma}, T_{\Sigma} \in B i\left(T_{\Sigma}\right)$ will have no zero elements.

Probabilities of formation of each group of matrices $P\left(B_{i}\right)$ are determined earlier as the probabilities of availability in matrix $T_{\Sigma}$ of exactly f zero elements. In this case probability $P_{U}$ can be determined with regard to the fact that after formation of MLL, arbitrary matrix $T_{\Sigma}$ will take place and

$$
P_{U}=\sum_{i=1}^{s} P\left(B_{l}\right) \cdot P_{U}^{B}
$$

where $s$ is the number of groups of matrices $T_{\Sigma}$.
Determining Probability $P_{i j}$. It should be pointed out that formation of additional (with respect to MLL) edges is represented in matrix $T_{\Sigma}$ in elements other than $r_{c}$ and $r_{d}$. If an additional edge $(i, j)$ has appeared after formation of MLL, then the $i$-th and the $j$-th rows of matrix $T_{\Sigma}$ are added element-wise. If edge ( $i, j$ ) appeared before formation of MLL, then elements $\tau_{i j}$ and $\tau_{j i}$ in the $i$-th and $j$-th rows of matrix $T_{\Sigma}$ will differ from zero. If additional edge ( $i, j$ ) was formed after some edges of MLL already existed, and a part of them did not, then random variable $\omega$ is introduced to take this fact into account, which takes "0" or "1" values with equal probability. Elements of the $i$-th and $j$-th rows which are marked as $r$, are multiplied by $\omega$, whereupon the $i$-th and the $j$-th rows are added element-wise.

Probalilities $\mathrm{P}_{\mathrm{i}, \mathrm{j}}$ are determined for each of groups $B_{i}\left(T_{\Sigma}\right)$. To do this, one of matrices $T_{\Sigma}^{\prime}$ is selected where $T_{\Sigma}{ }^{\prime} \in B i\left(T_{\Sigma}\right)$. Then the number of zero elements is determined in each row of matrix $T_{\Sigma}{ }^{\prime}: \beta_{1}$, $\beta_{2}, \ldots, \beta_{n}$.

Since the other matrices $T_{\Sigma}, T_{\Sigma} \in B_{i}\left(T_{\Sigma}\right)$ can be formed from matrix $T_{\Sigma}{ }^{\prime}$ by means of a cyclic substitution of rows, the $i$-th row of matrix $T_{\Sigma}$ can have the following number of zero elements, respectively: $\beta_{1}, \beta_{2}, \ldots, \beta_{n}$. This is also true for the $j$-th row. There is a strict correspondence between the number of zero elements in the $i$-th and the $j$-th rows, which can be determined from the initial distribution of the number of zero elements over rows of matrix $T_{\Sigma}{ }^{\prime}$. To do this, we will consider cyclic substitution

$$
\begin{array}{llll}
\beta_{1}, & \beta_{2}, & \ldots, & \beta_{n} \\
1, & 2, & \ldots, & n
\end{array}
$$

where $\beta_{1}, \beta_{2}, \ldots, \beta_{n}$ are the numbers of zero elements in rows of matrix $T_{\Sigma}: 1,2, \ldots, n$ are numbers of rows of matrix $T_{\Sigma}$ belonging to group $B_{i}\left(T_{\Sigma}\right)$. Thus, at each shift of the lower row of substitution (2), there is a strict correspondence between the $i$-th and the $j$-th row, which is defined by the upper row of substitution (2). At each shift of the lower row of substitution (2) we determine the existence of the fact of joint covering of the $i$-th and the $j$-th row of matrix $T_{\Sigma}$ by nonzero elements of all columns. We sum up all cases of complete covering of columns.

The value of the ratio of the number of cases of complete covering $m_{i, j}$ to the number of all cases being considered $M_{j j}$ represents probability $P_{i, j}$.

This probability differs from probability $P_{i, j}$ in that the order of priority of formation of edges of MLL and of edges corresponding to additional tests is not taken into account. To take into account the order of priority of formation of graph edges, we will consider separately the $i$-th and the $j$-th rows of matrix $T_{\Sigma}$ :

$$
\begin{gathered}
T_{\Sigma i}=\left[\begin{array}{l}
\tau_{i, 1}, \tau_{i, 2}, \ldots r_{c} \omega, r_{d}, r_{c} \omega, \ldots, \tau_{i, j-1}, \tau_{i, j}, \\
\tau_{i, j+1}, \ldots, \tau_{n}
\end{array}\right], \\
T_{\Sigma j}=\left[\begin{array}{l}
\tau_{j, 1}, \tau_{j, 2}, \ldots \tau_{j, i-1}, \tau_{j, i}, \tau_{j, i+1}, \ldots, r_{c} \omega, r_{d}, \\
r_{c} \omega, \ldots, \tau_{j, n}
\end{array}\right] .
\end{gathered}
$$

We will introduce into consideration random events $b_{1}, b_{2}, b_{3}$ and $b_{4}$, where $b_{1}=\left\{\tau_{i, i-1} \neq 0\right\}, b_{2}=$ $\left\{\tau_{i, i+1} \neq 0\right\}, b_{3}=\left\{\tau_{j, j-1} \neq 0\right\}, b_{4}=\left\{\tau_{j, j+1} \neq 0\right\}$.

Probabilities of these events are equal to 0.5 since by definition random quantity $\omega$ takes with equal probability the values 0 or 1 . Since in element-wise summation of the $i$-th and the $j$-th rows, summation of elements $\tau_{i, i+l}$ and $\tau_{j, j+l}$ is possible, we introduce into consideration complex events $A_{i}$, where $A_{i}=\left\{b_{t l} \oplus\right.$ $\left.b_{t 2}\right\}, t_{1}, t_{2} \in[1,4]$. Then probability of event $A_{i}$ will be equal to $P\left(A_{i}\right)=P\left(b_{t 1}\right)+P\left(b_{t 2}\right)-P\left(b_{t 1}\right) \cdot P\left(b_{t 2}\right)=0.75$. We will define event $\quad B_{k}$, $k={ }^{-1} \square, \square \square 4 \square$ as $B_{k}=\left\{b_{t 1} \oplus b_{t 0}\right\}$, where $t_{1} \in[1,4]$ $b_{t 0}=\left\{\tau_{q} \neq 0\right\}, q \in[(j, i+1) ;(i, j+1)]$. Then probabilities of the events will be equal to

$$
\begin{aligned}
& P\left(B_{1}\right)=\left\{\begin{array}{l}
0.5 . \text { if } \tau_{j, i-1}=0 . \\
1 . \text { if } \tau_{j, i-1}=1 .
\end{array}\right. \\
& P\left(B_{2}\right)=\left\{\begin{array}{l}
0.5 . \text { if } \tau_{j, i-1}=0 . \\
1 . \text { if } \tau_{j, i-1}=1 .
\end{array}\right. \\
& P\left(B_{3}\right)=\left\{\begin{array}{l}
0.5 . \text { if } \tau_{i, j-1}=0 . \\
1 . \text { if } \tau_{i, j-1}=1 .
\end{array}\right. \\
& P\left(B_{4}\right)=\left\{\begin{array}{l}
0.5 . \text { if } \tau_{i, j-1}=0 . \\
1 . \text { if } \tau_{i, j-1}=1 .
\end{array}\right.
\end{aligned}
$$

Thus, consideration of the order of priority in
formation of edges of the graph is reduced to determination of the following probability:

$$
P_{S}=P(A) \prod_{k=1}^{4} P\left(B_{k}\right)
$$

Therefore, the $i$-th and the $j$-th rows of matrix $T_{0}$ at each cyclic shift of the lower row of substitution (2) overlap jointly all its columns with probability $P_{s}$. It should be noted that probability $P_{s}$ is determined for the $i$-th and the $j$-th rows separately for each new matrix $T_{\Sigma}$ formed from the initial matrix $T_{\Sigma}{ }^{\prime}$ through a cyclic substitution of rows.

With regard to probabilities $\mathrm{P}^{\gamma}{ }_{\mathrm{s}}, \gamma=\overline{1, n}$, where y is the number of the cyclic shift of the lower row of substitution (2), we can define more exactly the value of $m_{i j}$ decreasing to $m^{\omega}{ }_{i j}$. To determine $\mathrm{m}^{\omega}{ }_{\mathrm{ij}}$, we find first probabilities $P_{m i j}(k)$ of situations when among $m_{i j}$ substitutions for which the $i$-th and the $j$-th rows of matrix $T_{\Sigma}$ overlap jointly all its columns (without regard for $\omega$ ), $k$ such substitutions will be found for which the $i$-th and the $j$-th rows of matrix $T_{S}$ overlap jointly all its columns (with regard for $\omega$ ). These probabilities can be found from an expression corresponding to the Bernoulli formula for unequal probabilities in each outcome

$$
\begin{aligned}
& P_{m_{i j}}(k)=\sum_{i_{1}, i_{2}, \ldots i_{n}=\overline{1, n}} P_{i_{1}} P_{i_{2}} \ldots P_{i_{k}} \bar{P}_{i_{k-1}} \ldots \cdot \bar{P}_{i_{n}}, \\
& i_{1} \neq i_{2} \neq \ldots=i_{n} \\
& P_{i_{1}}=P_{S}^{1}, P_{i_{2}}=P_{S}^{2}, \ldots, P_{i_{n}}=P_{S}^{n} .
\end{aligned}
$$

In this case quantity $m_{i j}$ is determined as the most probable number of substitutions for which the $i$-th and the $j$-th rows of matrix $T_{\Sigma}$ overlap jointly all its columns with regard for $\omega$. Then we find $m^{\omega}{ }_{i j}$ from the system of two inequalities

$$
\left\{\begin{array}{l}
P_{m_{i j}}\left(m_{i j}^{\omega}\right) \geq P_{m_{i j}}\left(m_{i j}^{\omega}+1\right) \\
P_{m_{i j}}\left(m_{i j}^{\omega}\right) \geq P_{m_{i j}}\left(m_{i j}^{\omega}-1\right)
\end{array}\right.
$$

Probability $P_{i, j}$ sought can be found with regard to $m^{\omega}{ }_{i j}$ as

$$
P_{i, j}=\frac{m_{i j}^{\omega}}{M_{i j}}
$$

After exchange of information between the $i$-th and the $j$-th modules, a situation is possible when these modules will not possess complete information about MLL. Now let us assume that the $i$-th module will then exchange information with the $j_{2}$-th module. It is evident that in this case there exists a difference as to whether ET ( $i, j$ ) was performed or not. An ET performed earlier can be taken into account by means
of respective conditional probabilities.
Probability $P\left(i, j_{2} / i, j_{l}\right)$ is the probability of the situation when after exchange of information between the $i$-th and the $j$-th modules, the $i$-th and the $j$-th rows in corresponding matrix $\mathrm{T}_{\Sigma}$ of the system will be free from zero elements provided that exchange of information between the $i$-th and the $j$-th modules was carried out earlier. This probability is determined from consideration of three rows of matrix $\mathrm{T}_{\Sigma}$. As in consideration of only two rows, probabilities $\mathrm{P}_{\mathrm{S}}$ are determined. But in this case as distinct from expression (3), events $\mathrm{A}_{1}$ and $\mathrm{A}_{2}$ are possible. Then based on probabilities $P^{\gamma}$ s, $\gamma=\overline{1, n}$ we find successively $P_{m i j, j 2}(k), m^{\omega}{ }_{i j, j 2}$ and $P\left(i, j_{2} / i, j_{1}\right)$. All other conditional probabilities $P\left(i, j_{n} / i, j_{1} ; \ldots ; i, j_{n-1}\right)$ can be found in a similar manner.

Determination of Probability $P_{U}$. Based on probabilities $P_{i, j}$ and conditional probabilities $P\left(i, j_{n} / i\right.$, $\left.j_{1} ; \ldots ; i, j_{n-1}\right)$, complete information about MLL in each module of the system can be formed as the Markovian chain with irreversible states and one absorbing state. The states in this case are the states of the system with concrete additional tests. After completion of the next additional test, the system changes to the state when corresponding matrix of the system $T_{\Sigma}$ has no rows with zero elements or changes to the state in which there are rows with zero elements in matrix $T_{\Sigma \text {. This state is marked by additional tests. }}^{\text {. }}$ It is assumed that a restriction is imposed on the multiplicity of performance of elementary tests.

Transitions of the system from one state to another actually take place at random instants of time. However we can assume that these transitions occur at strictly fixed instants since in this model we are not interested in time characteristics of the process.

Then Markovian chain with discrete states and discrete time for our case can be illustrated in Fig.3.
It is apparent that $S_{0}$ stands for the state of the system without additional tests, and $S_{\Sigma}$ for the state when rows with at least one zero element are absent in matrix corresponding to the system. We can assume in the simplest case that all probabilities of transition from state $S_{i}$ to state $S_{j}$ (except for state $S_{\Sigma}$ ) are equal, i.e. if transitions from state $S_{m 1+2}$ to $r$ other states from the group of states with three additional tests are
possible, then all probabilities of transitions from state $S_{m l+2}$ are equal to $1 / r$.

The following quantity is determined for each state of the system:

$$
\begin{gathered}
Q S_{m_{l}-\ldots-m_{k-1}-i=} \\
P\left(S_{\left.m_{1}+\ldots+m_{k-1}+i\right)} P\left(S_{m_{1}+\ldots+m_{k-1}+i} ; S_{\Sigma}\right),\right. \\
i=\overline{1, m_{k}},
\end{gathered}
$$

where $P\left(S_{m I+\ldots+m k-I+i}\right)$ is probability of the fact that after performance of $k$ additional tests the system will prove to be in state $S_{m l+\ldots+m k-1+i} ; P\left(S_{m l+\ldots+m k-1+i} ; S_{\Sigma}\right)$ is probability of transition of the system from state ( $S_{m l+\ldots+m k-1+i}$ to state $S_{\Sigma}$.

This probability is equal to conditional probability $P\left(i, j_{n} / i, j_{1} ; \ldots, i, j_{n-1}\right)$, where tests $\left(i, j_{1}\right),\left(i, j_{2}\right), \ldots$, ( $i, j_{n-l}$ ) determine state $S_{m l-\ldots-m k-1-i}$.

To obtain probability $P\left(S_{m l-\ldots-m k-l-i}\right)$, we will take advantage of correspondence between the number of additional tests in the system and index of step in the discrete Markovian chain.

This correspondence implies that the index of step is equal to the number of additional tests and, correspondingly, to the number of the group of states with the fixed number of additional tests. In this case probability $P\left(S_{m l-\ldots-m k-1}-i\right)$ is determined by the formula of total probability

$$
P_{k+1}^{(j)}=\sum_{i=1}^{L} P_{k}^{(i)} \cdot P_{k, k+1}^{(i j)}, \quad j=1, \ldots, M,
$$

where $L$ is the number of possible states of the system at the $k$-th step; $M$ is the number of possible states at the $(k+1)$-th step; $P^{(j)}{ }_{k+1}$ is probability of the situation that at the $(k+1)$-th step (with $k+1$ additional tests) the system will prove to be in state $S_{j} ; P^{(i)}{ }_{k}$ is probability of the fact that at the $k$-th step the system will be in the state $S_{i} ; P^{(i)}{ }_{k, k-1}$ is probability of the situation that the system at the $(k+l)$-th step will prove to be in state $\mathrm{S}_{\mathrm{j}}$ if it was at the $k$-th step in the state $\mathrm{S}_{\mathrm{i}}$.


Fig. 3. System state and Transition graph

Expression (4) allows us to determine step-by-step, one after the other, probabilities $P\left(S_{j}\right), \quad P\left(S_{m l-l}\right), \ldots$, $P\left(S_{m l \ldots-m k-1-i}\right)$ with regard to the fact that at the initial instant of time the system was in the state $\mathrm{S}_{0}$. Then quantity $M \quad\left(\Gamma_{k}, \quad S_{\Sigma}\right)$, where $M\left(\Gamma_{k}, S_{\Sigma}\right)=\sum_{S_{i} \in \Gamma_{k}} Q_{S_{i}}$ should be determined for each group of states (i.e. states with a definite number of additional tests).

This quantity is in essence the average value of probability of the event that after performance of $k$ additional tests, rows with at least one zero element will be absent in the matrix of the system $\mathrm{T}_{\Sigma}$, i.e.

$$
M\left(\Gamma_{k}, S_{\Sigma}\right)=\tilde{P}_{U}^{B_{1}}
$$

We will also determine probability of the event that after performance of $\omega$ ET, MLL will be formed in TLG of the system and each module will have information about this MLL and about local degrees of all nodes of TLG.

$$
\begin{gathered}
P_{0}=P_{M L L} P_{U} \quad \text { or with regard to }(1) \\
P_{0}=P_{M L L} \sum_{l=1}^{s} P\left(B_{l}\right) \tilde{P}_{U}^{B_{l}}=P_{M L L} \sum_{l=1}^{s} P\left(B_{l}\right) M\left(\Gamma_{k}, S_{\Sigma}\right), \\
\text { where } k=\omega-n .
\end{gathered}
$$

It should be mentioned here that determination of probability of formation of the maximum-length loop $P_{M L L}$ is an independent problem which is not considered in the present paper.

Furthermore, we will take into consideration the fact that several MLL can be formed in a TLG of the system after fulfillment of $\omega$ ET. Because of this, probability $P_{0}$ is considered with respect to a particular MLL. In this connection, notation $P_{0}{ }^{M L L i}$ (where $i=\overline{1, \bar{m}}, m$ is the maximum number of MLL for the system of $N$ modules) is introduced for this probability. Based on this, probability $P_{\Sigma}$ of situation that after fulfillment of $\omega$ ET $m$ MLL will be formed in TLG of the system and each module will have
complete information about them, is equal to $P_{\Sigma}=\left(P_{0}{ }^{M L L i}\right)^{m}$, since $P_{0}{ }^{M L L 1}=P_{0}{ }^{M L L 2}=P_{0}{ }^{M L L m}$, and probability for $t$ arbitrary MLL

$$
P_{\Sigma, t}=C_{m}^{t}\left(P_{0}^{M L L_{i}}\right)^{t}\left(1-P_{0}^{M L L_{i}}\right)^{m-t}
$$

In closing, the following conclusions can be made. One of the main features of self-diagnosis of complex technical systems with the random structure of testing links is the intermodular information exchange. This exchange is carried out in order that each module of the system should have sufficient information for the subsequent diagnosis of the system with preset probability.

At a random fulfillment of ET and, therefore, at a random exchange of information between modules of the system, it is impossible to determine exactly which information will be possessed by modules after a preset time of operation and. therefore, it is impossible to say reliably whether a particular module will be capable of performing diagnosis of the system.

In this connection probabilistic characteristics are determined in the present paper, which allow us to establish a relation between diagnostic information possessed by a module and the number of elementary tests performed in the system.

The procedures for determining probability $P_{\Sigma, t}$ can be presented in the form of computer programs. These procedures allow us to avoid processing of considerable bodies of information in order to provide probabilistic characteristics (relations) sought. For example, to investigate only one MLL without application of these procedures, one would have to consider and simulate N , different situations of ET distribution. There will be about one hundred billion such situations for a system consisting of 14 modules.

Relation between probability $P_{\Sigma, t}$ and the number of performed ET determined in the present paper is one of the main characteristics required for development of the procedure of self-diagnosis of complex technical systems.

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# ОРГАНИЗАЦИЯ ПРОЦЕДУРЫ САМОКОНТРОЛЯ В ФУНКЦИОНАЛЬНО УСТОЙЧИВЫХ КОМПЛЕКСАХ 

Машков О.А., Машков В.А.

Рассмотрен межмодулярный обмен диагностической информацией при самодиагностике сложных технических систем со случайной структурой тестовых элементов. Оценивается информация о структуре тестовых элементов системы, собранных в ее модулях в результате межмодульного обмена.

Ключевые слова: самоконтроль, самодиагностика, межмодульный обмен, информация, функциональная устойчивость.

# ОРГАНІЗАЦІЯ ПРОЦЕДУРИ САМОКОНТРОЛЮ У ФУНКЦІОНАЛЬНО СТІЙКИХ КОМПЛЕКСАХ. 

Машков О.А., Машков В.А.

Розглядається міжмодульний обмін діагностичної інформації при самодіагностуванні складних технічних систем 3 випадковою структурою тестових елементів. Здійснюється оцінка інформації про структуру тестових елементів, які зібрані у її модулях за результатами міжмодульного обміну.

Ключові слова: самоконтроль, самодіагностика, міжмодульний обмін, інформація, функціональна стійкість.

