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The shape and the size of the Universe according to the Poincaré dodecahedral space hypothesis

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One of the best models of the Universe is the Poincaré Dodecahedral Space (PDS) model, which has the best fit to the data of the cosmic microwave background (CMB) sky maps from the Wilkinson Microwave Anisotropy Probe. The present work increased the falsifiability of the PDS model by finding a better estimate of the size of the Poincaré space using parameters previously found by Roukema et al. (2008). The improved size of Poincaré space model is 18.2 $\pm 0.5 h^{-1}$ Gpc for matter density parameter $\Omega_{\rm m} = 0.28 \pm 0.02$. This gives the lowest redshift of multiply imaged objects as $z = 106 \pm 18$.

Key words: cosmic background radiation, cosmological parameters, large-scale structure of Universe

INTRODUCTION

In 2003 J. P. Luminet et al. [11] gave Poincaré dodecahedral space topology as possible explanation for weak wide-angle temperature correlations in the cosmic microwave background (CMB) which were detected by the COsmic Background Explorer (COBE) and later by the Wilkinson Microwave Anisotropy Probe (WMAP) [7]. These wide-angle temperature correlations correspond to the largest scales in the observable Universe, thus if the temperature correlations are weak it implies a lack of structures in the biggest scales (about 10 h^{-1} Gpc) which would be observed in a flat infinite space (R^3) or in a big enough positively curved space (S^3). Such lack of largest structures may suggest more sophisticated topology of the Universe.

The locally homogeneous 3-manifolds that have recently been studied as the best candidates to fit the WMAP data include not only the Poincaré Dodecahedral Space (PDS) S^3/I^* [3, 4, 8, 10, 11, 16, 17] but also the 3-torus T^3 [1, 2, 5, 6, 19]. However PDS seems to be better balanced than other spaces [14, 15] and thus more attractive as a topological model of the Universe.

Briefly speaking, topology is a science about geometrical properties of objects, for example spaces (or more general manifolds). Object (space) keeps its topological properties when it is stretched, bent or crumpled. This is unlike cutting or splitting, when the topology of the object changes. If we consider a loop $\gamma \in \mathcal{M}$ and basic properity of space $\mathcal{M}$ is that every γ can be shrink to a point, then such space is called *simply–connected* (e.g. euclidian spaces: $\mathbf{M^1}, \mathbf{M^2} \ldots \mathbf{M^n}$ or spheres: $\mathbf{S^2}, \mathbf{S^3} \ldots \mathbf{S^n}$) Otherwise, if $\exists \gamma \in \mathcal{M}$ which cannot be shrink to a point (e.g. because of some holes in $\mathcal{M}$), the space $\mathcal{M}$ is *multi–connected*.

One way of thinking about *cosmic topology*[1] is to consider space as a polyhedron whose opposite faces are identified. Such polyhedron is called *fundamental domain* (FD). For cosmologists the most importatnt effect of multi–connectness are multiply imaged objects ("copies" of objects). If we live in multi–connected space and it is small enough, i.e. the size of FD is smaler than observable universe, we should see multiple copies of objects. This mean that in principle it should be possible to find an optimal orientation and size of the FD in astronomical coordinate system, by discovering those copies and correlation between them.

POINCARÉ DODECAHEDRAL SPACE MODEL

From mathematical point of view Poincaré dodecahedral space is a 3-sphere (hypersphere) S^3 quotiented by icosahedral holonomy group I^* (so it can be written as S^3/I^* [12]). Because of its geometry

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[1] In general there are three ways of thinking about topology of n-space: (i) as a n-space placed in $(n+1)$-space, (ii) as a *fundamental domain* with glued opposite faces and (iii) as a tiling of the full covering space by multiple copies of the *fundamental domain* [13, 12].

(and its topological properties) PDS model requires positively curved space ($k > 0$), which means a hypersphere S^3 covering space. This gives a big constraint on this model because if space is confirmed to be flat (zero curvature) or hyperbolic (negative curvature), then the Universe cannot have PDS topology. Nevertheless, the data from WMAP spacecraft gives us $\Omega_{\rm tot} \approx 1.013 \pm 0.02$. This uncertainty on $\Omega_{\rm tot}$ parameter still allows space to be positively curved. Fundamental polyhedron (fundamental domain) for the Poincaré space is a regular dodecahedron. Its opposite faces (which are pentagons) are identified (glued) after $\pm 36°$ twist (Fig. 1).

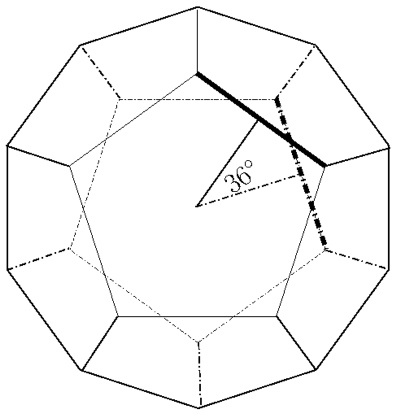

Fig. 1: A regular dodecahedron being a fundamental domain of Poincaré space model is a one way of thinking about topology of PDS model. Opposite faces of these FD are identified after $\pm 36°$ twist. Such topology requires positively curved space ($k > 0$).

We need at least five parameters describing orientation and size of PDS fundamental domain (FD). Three for describing orientation of the dodecahedron (l, b, θ) one for *twist angle*[2] (ϕ), and one for describing size of dodecahedron (α). The relation between α and the curvature radius $R_{\rm C}$ (hence $r_{\rm inj} = (\pi/10) R_{\rm C}$, which gives the size of the FD) is:

$$\tan \frac{r_{\rm SLS}}{R_{\rm C}} = \frac{\tan \pi/10}{\cos \alpha} \qquad (1)$$

(i.e. Eq. (15) of [17]), where $r_{\rm SLS}$ is a radius to the *surface of last scattering* (SLS). Fig. 2 shows how the size of FD depends on α parameter.

THE METHOD OF CALCULATIONS

The basic principle for finding the size and orientation of the FD is called the *matched circles* principle [9]. The principle and its corollary *matched discs* [18] method are shown in Fig. 2. This shows multiple copies of the observable Universe, each bordered by a copy of the SLS, in the covering space. The multiple copies of the sub-SLS Universe intersect with themselves on matched discs. See Sec. 2.1 of [18] for more discussion.

An improved estimate of α is obtained using the optimal cross-correlation method [16, 17] over α, and the previous solution for the fundamental domain axes (Table 1). $N_{\rm p} = 2 \times 10^5$ points are used per correlation calculation, so that the $r \lesssim 1\ h^{-1}$ Gpc scale is usable. The optimisation criterion is defined as the mean cross-correlation below a given length scale, normalised by $r_{\rm SLS}$ so that it has only a weak dependence on $\Omega_{\rm m}$ and Ω_Λ,[3] i. e.:

$$\bar{\xi}_\beta(\alpha) := \frac{1}{\beta r_{\rm SLS}} \int_0^{\beta r_{\rm SLS}} \xi_{\rm C}(r, \alpha) {\rm d}r \qquad (2)$$

(Eq. (6) of [18]). Cross-correlations at the scales $\beta = 0.033$ and 0.1, i. e. $\beta r_{\rm SLS} \approx 0.33$ and $1\ h^{-1}$ Gpc, respectively, are checked in the 7-year WMAP ILC map.[4]

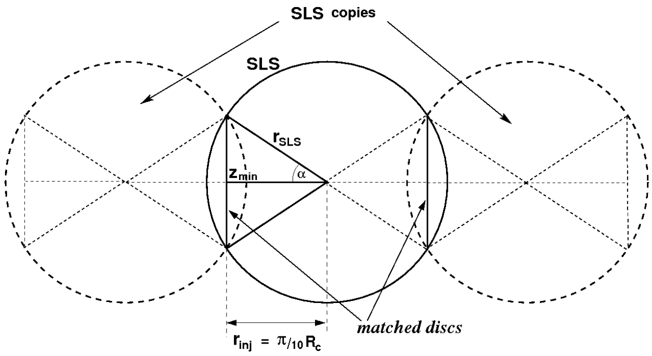

Fig. 2: Matched discs for the Poincaré dodecahedral space, with injectivity radius $r_{\rm inj} = (\pi/10) R_{\rm C}$, shown in the universal covering space $\rm S^3$ of radius $R_{\rm C}$. Any object in a matched disc (orthogonal to the plane of the page) has a topologically lensed copy in the second matched disc of a matched disc pair. The matched-disc angular radius is α, redshift to the centres of matched discs is $z_{\rm min}$ and SLS radius is $r_{\rm SLS}$.

RESULTS AND CONCLUSIONS

In previous work of Roukema et al. (2008) [17] orientation of dodecahedron in galactic coordinates was found. Coordinates to the centres of faces are shown in Table 1. Using these coordinates a new estimate of α parameter (matched circle angular size) and thus size of Poincaré space was found: $\alpha = 23° \pm 1.4°$. Fig. 3 show that mean cross-correlation function

[2] Twist angle is needed for gluing/identifying FD faces — we can only identify them after $\pm 36°$ twist.
[3] See Sec. 2 in [18] for details.
[4] `http://lambda.gsfc.nasa.gov/data/map/dr4/dfp/ilc/wmap_ilc_7yr_v4.fits`

$\bar{\xi}_\beta(\alpha)$ [see Eq. (2)] is maximized near $\alpha = 23°$, even for $\beta = 0.4$, i. e. $\beta r_{\rm SLS} \approx 4\ h^{-1}$ Gpc. Value $\alpha = 23 \pm 1.4°$ corresponds to $2r_{\rm inj} = 18.2 \pm 0.5\ h^{-1}$ Gpc for $\Omega_{\rm m} = 0.28 \pm 0.02$ which is the "inner" size of the Universe (i. e. size of FD). The redshift to the matched disc centres and thus the lowest redshift of expected multiply-imaged objects is $z_{\rm min} = 106 \pm 18$.

The estimate of α from [17] ($\alpha = 21°$) was suspected to have significant systematic error (about 10°), but the smaller scale cross-correlations from Fig. 3 clearly overcome this risk.

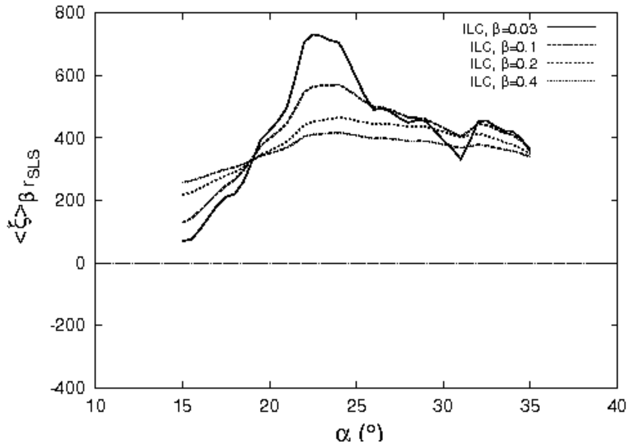

Fig. 3: Gpc-scale cross-correlation $\bar{\xi}_\beta(\alpha)$ from Eq. (2) in μK^2 against matched-circle radius α, for the WMAP7 ILC map, for $\beta = 0.03, 0.1, 0.2, 0.4$ (from top to bottom at $\alpha \sim 23°$) adopting $|b| > 10°$ as an approximate galactic mask.

Table 1: Orientation of the FD of Poicaré space found by Roukema et al. (2008) [17]. For each face, $\{l, b\}_i$ shows centre of i face in FD ($i + 6$ is opposite). Coordinates are estimated with 2° accuracy.

i	l [°]	b [°]
1	184	62
2	305	44
3	46	59
4	117	20
5	176	−4
6	240	13

This result implies that if very high density peaks collapse and form stars so that they are visible in the redshift range $200 > z > 106$, then matched discs, with objects at $z = 106 \pm 18$ at the centres and successively higher redshifts radially outwards up to $z = 200$, should cover about 20% of the full sky. A pair of topologically lensed objects in a pair of matched discs would be seen at identical redshifts. This provides a potential sub-SLS way of testing the Poincaré dodecahedral space hypothesis through photometry and spectroscopy of very high redshift objects.

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