

On the comparison of fundamental numerical ephemerides

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We present the results of our comparison of three main numerical ephemerides (DELE, INPOP, EPM) for the determination of precision and errors of their dynamical coordinate systems. It was shown that all of them have comparable levels of precision, however the EPM demonstrates an unusual shift of the coordinate origin. Systematic errors were estimated as well, and mutual shifts of coordinate centres were found.

Key words: astrometry, ephemerides, precision estimation

INTRODUCTION

Fundamental astronomical ephemerides serve as a basis for dynamical astronomy tasks. They not only provide us with precise planetary positions, but define the coordinate axes of a fundamental coordinate system. That fundamental system is as close as possible to the true inertial coordinate system: the only system in which Newton's (or Einstein's) equations of motion take place.

Our interest to planetary ephemerides is motivated by our activities in space geodynamics. The Ukrainian Earth Orientation Parameters Laboratory owns *Juliette/KG++* [4] and *SteelBreeze* [1] software. Their algorithms are explained in [3, 5]. Both software products utilize precise planetary positions. Our question is: how will the replacement of the planetary ephemeride influence the geodynamic results?

Currently there are three well-known numerical ephemerides. These are: DELE¹ [8], created and maintained by JPL; INPOP² [7] created and maintained by Paris Observatory, and EPM³ [10], owned by Institute of Applied Astronomy of RAS.

As each ephemeride makes use of its own calculation method, set of observational data, constants, etc., some differences in processing and calculating results are to be expected. Comparison of the ephemerides will then help us to estimate the errors of the ephemerides as a whole. This method is often employed in astronomy and geodesy, as a reliable method of estimating errors.

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¹<ftp://ssd.jpl.nasa.gov/pub/eph/planets/ioms/de421.iom.v1.pdf>

²<http://www.imcce.fr/inpop/>

³<http://quasar.ipa.nw.ru>

THE METHOD

Method of comparison is based upon the Helmert transform [11]. It is geodetic method, but it was used in [2] for the comparison of different realizations of the Earth coordinate frames, and has shown realistic results. The same approach is used by astronomical community to compare stellar catalogues, see for example [6] and the bibliography therein.

The ephemerides were compared for example in [9], where the precision of single planetary positions was in question. It is difficult to compare our results with the results of other authors, as our article is, perhaps, the first one where the Helmert transform was used to compare dynamical coordinate systems of the fundamental ephemerides.

Let us introduce a list of N objects in the sky (stars, quasars, planetary position) with measured coordinates in two distinct, but similar, coordinate systems. Here and below the coordinate system is marked with lower index, while the object is marked with upper index in parenthesis. The transformation from system 2 to system 1 should then be written as:

$$\mathbf{r}_1^{(i)} = \hat{A}\mathbf{r}_2^{(i)} + \mathbf{b}, \quad (1)$$

$\mathbf{r}_1^{(i)}$ and $\mathbf{r}_2^{(i)}$ are the barycentric position of the same point (i) at the same moment, measured in two (lower index) coordinate systems, \mathbf{b} is the vector of translation of the coordinate centres of two systems.

Matrix \hat{A} is a rotation matrix:

$$\begin{pmatrix} \mu + 1 & \gamma & -\beta \\ -\gamma & \mu + 1 & \alpha \\ \beta & -\alpha & \mu + 1 \end{pmatrix}. \quad (2)$$

$$\begin{cases} \sigma_{12}^2 = \sigma_1^2 + \sigma_2^2, \\ \sigma_{23}^2 = \sigma_2^2 + \sigma_3^2, \\ \sigma_{31}^2 = \sigma_3^2 + \sigma_1^2. \end{cases}$$

The vector \mathbf{b} and the matrix \hat{A} accounts for a systematic part of the shift between the 2nd and the 1st coordinate system. This matrix is the product of scale factor $1 + \mu$ and three rotation matrices:

$$\hat{A} = \begin{pmatrix} \mu + 1 & 0 & 0 \\ 0 & \mu + 1 & 0 \\ 0 & 0 & \mu + 1 \end{pmatrix} \times \mathbb{R}(\gamma)\mathbb{Q}(\beta)\mathbb{P}(\alpha). \quad (3)$$

We imply that the angles α, β, γ are so small, that we can neglect trigonometric function and set $\sin x \rightarrow x$ and $\cos x \rightarrow 1$. μ is a scale factor, generally $\mu \ll 1$. There are 7 unknowns in (2) and (3): $\mu, \alpha, \beta, \gamma, \mathbf{b}$. To solve (1) for them one should generate a lot of (1) for different points or for the same point at different moments in time, and then solve them simultaneously with the Least Squares method.

In our work there are no catalogues to compare. Our catalogues are the planetary positions generated with the ephemerides software. For every ephemeride we used their native software shipped with the ephemeride. For every planet we generated a list of positions once a day. In the calculation of $\mu, \alpha, \beta, \gamma, \mathbf{b}$, the data for one planetary orbiting period is used.

Having $\mu, \alpha, \beta, \gamma, \mathbf{b}$ found, one can then apply \hat{A} and \mathbf{b} to the known point $\mathbf{r}_2^{(i)}$:

$$\mathbf{r}'_2^{(i)} = \hat{A}\mathbf{r}_2^{(i)} + \mathbf{b}, \quad i \in [1, N]. \quad (4)$$

The systematic portion of the difference between two ephemerides is eliminated with \hat{A} and \mathbf{b} . The rest part of the difference is interpreted as total random error. In the general case:

$$\mathbf{r}'_2^{(i)} \neq \mathbf{r}_1^{(i)}, \quad \forall i,$$

and $\sum (\mathbf{r}'_2^{(i)} - \mathbf{r}_1^{(i)})^2$ characterizes a total random error of both ephemerides.

As an initial approximation let us postulate that the random errors are uncorrelated. Then, one can write:

$$\sigma_{12}^2 = \sigma_1^2 + \sigma_2^2, \quad (5)$$

where

$$\sigma_{12}^2 = \frac{1}{N} \sum_{i=1}^N (\mathbf{r}'_2^{(i)} - \mathbf{r}_1^{(i)})^2,$$

and σ_i^2 is the random error dispersion of the i -th ephemeride. From the cross-comparison of three ephemerides one can find $\sigma_1^2, \sigma_2^2, \sigma_3^2$ which characterize random errors of the ephemerides:

Transformations between reference points of ephemerides become visible when analysing \mathbf{b} ; mutual orientation of the axes is given by elements of \hat{A} .

THE DATA

In our work we used DELE421, INPOP10a and EPM2008 (see references and footnotes above). All of them are accompanied with their own (native) software to calculate planetary positions. Those positions are expressed in a dynamical ephemeride coordinate system, built by integrating of the equations of planetary motions in Post-Newtonian metrics with some lists of additional bodies included.

We made the comparison of barycentric planetary positions calculated once a day for the time interval equal to orbital period of the planet. Then, we shifted the time by one half of the orbital period and repeated the calculations. We repeated these calculations for a time period until the end of 2050. For example, for Mercury, we used 88 day-to-day positions, and the shift was 44 days.

RESULTS AND CONCLUSIONS

Figs. (1)–(6) present σ_i versus time. Every point on the graphs represents the σ_i 's calculated on one planetary revolution from the time moment given at X axis. Uranus and Neptune are not shown, as there are no sufficient ephemeris data in EPM2008 for their full revolution. Fig. 7 shows general view of \mathbf{b} vectors for Venus for three ephemeris pairs. The behaviour of \mathbf{b} is quite similar for all planets.

Precision values for DELE421 and INPOP10a are quite similar. In contrary to them EPM2008 precision values demonstrate uncommon behaviour versus time. At times the formal EPM2008 error is 2-5 time higher. A promising explanation for these values are possible correlations between the ephemerides errors in (5).

Fig. 7 shows some unexpected and unexplained movement of EPM2008 barycenter ~ 1 km/year in the direction perpendicular to the ecliptic. In contrast to EPM2008, the mutual movement of DELE421 and INPOP10a origins (shown scaled 100 times in the selected area of the Fig. 7) look chaotic and do not exceed 200 m in amplitude. The movement of the EPM2008 barycenter lies in the plane, nearly perpendicular to the Y axis. That is why it might influence ecliptic to equator inclination angle and result in wrong nutations deduced from EPM2008.

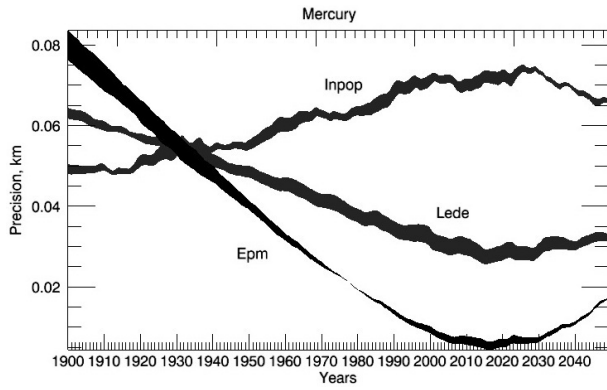


Fig. 1: σ for Mercury.

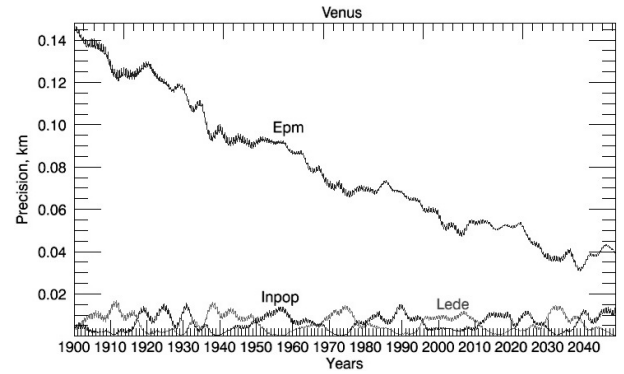


Fig. 2: σ for Venus.

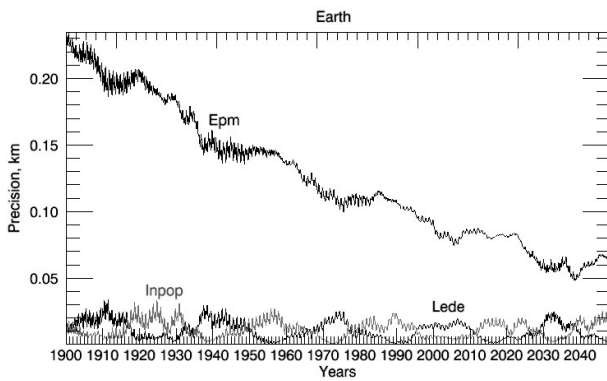


Fig. 3: σ for Earth.

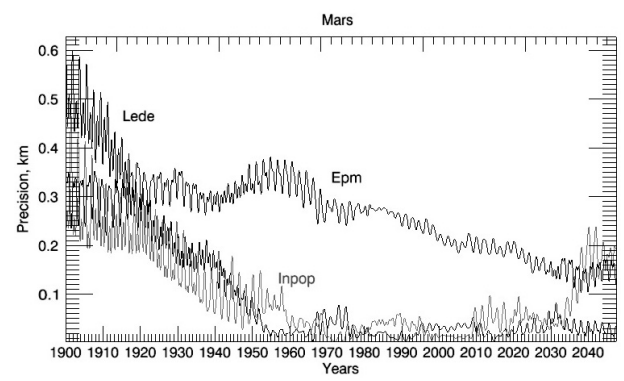


Fig. 4: σ for Mars.

The explanation of this issue needs additional analysis. Possibly it originated from different list of asteroids, or from some problems with connections between EPM2008 and ICRF, or perhaps it is an uncompensated Moon influence. All of these possibilities should be investigated prior to reaching a final conclusion. In any case, until then we cannot recommend the EPM2008 as a main ephemeris for space geodynamics tasks.

All coloured figures, including those showing the dependence of $\alpha, \beta, \gamma, \mu$ on time are be provided in the electronic version of the article.

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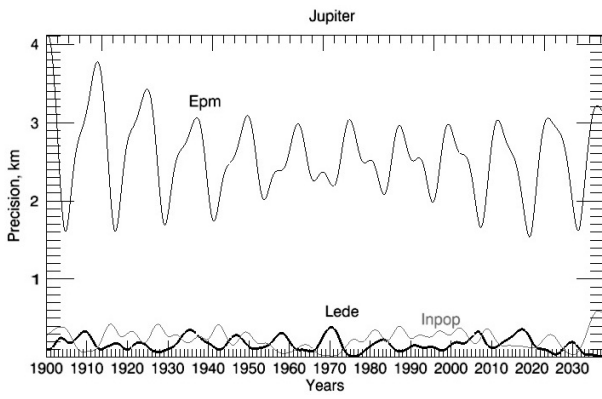


Fig. 5: σ for Jupiter.

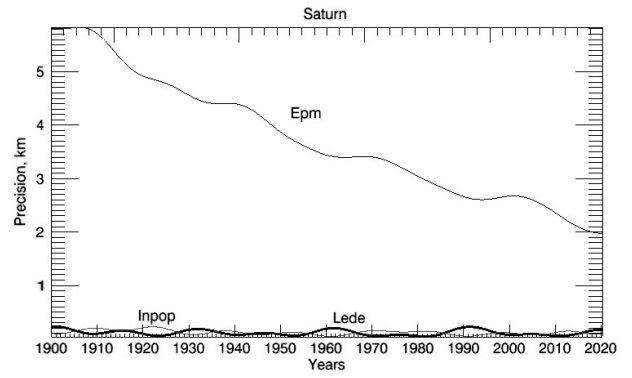


Fig. 6: σ for Saturn.

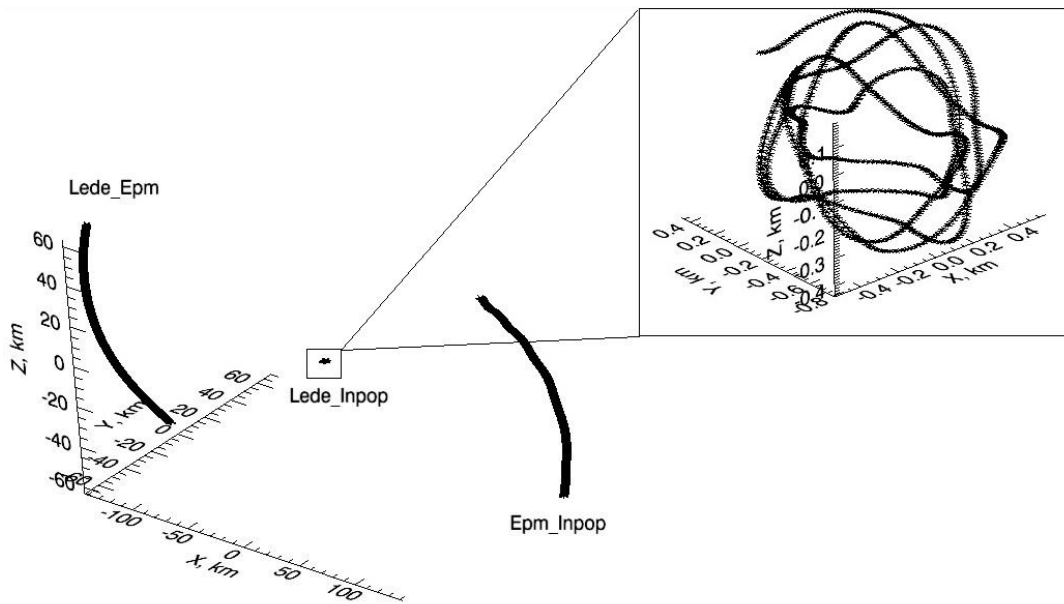


Fig. 7: General view of vector \mathbf{b} .