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Half-bare positron in the inner gap of a pulsar

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The pulsed radiation from the Crab Pulsar consists of the main pulse (MP) and inter pulse (IP), as well as of the extra pulse components appearing at certain frequencies. One of the mysteries of these data, found by Moffett and Hankins twenty years ago, is the shift of the IP at high radio frequencies compared to lower ones and return to its previous position in the higher-frequency optical and X-ray range. In previous paper we proposed the explanation of these mysterious changes with the frequency, applying the idea of the reflection of curvature radiation by relativistic positrons from the stellar surface. Presently we focus on the additional contribution of transition radiation, emitted when positron hits the surface, to the total pulse produced by the particle. It is shown that due to the 'half-bare' state of positron in the polar gap the considered contribution is significantly suppressed comparing to the one of reflected curvature radiation.

Key words: neutron stars; pulsars: PSR B0531+21; radiation mechanisms: non-thermal

INTRODUCTION

To explain the shift of the IP in the Crab Pulsar [9, 21], it was necessary to apply the radiation from returned positrons [12] in an inclined magnetic field [13, 14], reflected from the surface of the pulsar. This also opened up new possibilities for explaining additional high frequency (HF) components arising at the same frequencies as the shift of the IP and, quite probably, also related to reflection from the surface [15]. An independent treatment of the HFCs, not connected directly with the IP shift, was discussed by S. Petrova, see [22].

Thus, in contrast to the previously discussed mechanisms (see [4, 18, 19, 27] and references therein), where radiation is associated with the motion of particles and plasma flows in the direction from the surface of the star and occurs in the thickness or on the periphery of the magnetosphere, in the case of reflection from the surface the radiation of relativistic positrons occurs in the restricted internal gap and should have a number of features that are combined by the general term "half-bare effects" (or effects associated with large size of radiation formation length) discussed in [1, 5, 6, 17, 20, 25, 26, 28, 29].

The term "half-bare" particles was introduced by Feinberg [6] in connection with the phenomenon of relativistic electron scattering and radiation in crystals, considered by Ter-Mikaelyan [28]. The existence of specific interference effects associated with macroscopically large size of radiation formation length in relativistic particle radiation in substance is considered by Landau and Pomeranchuk [17], Migdal [20], Ternovsky [29], Shul'ga & Fomin [25], Shul'ga, Trofymenko & Syshchenko [26]. The review of various manifestations of large size of the formation length in relativistic particle radiation processes can be found in [1, 5].

The concept of "half-bare" particles is very effective for describing the radiation of ultrarelativistic particles in processes where the "instantaneous" change in the state of particle motion due to scattering or the influence of boundaries is substantial. In this case, in the processes of transition radiation and bremsstrahlung, the Coulomb Field of radiating particles does not have time to recover, and the radiation field can be formed on scales determined by large Lorenz-factors (macroscopic relativistic "near zone"). This leads to the observed effects due to the peculiarities of interference and coherence in such states.

In the case of pulsars under the magnetosphere of open magnetic field lines near the magnetic pole of the star, there is a so-called polar (internal) gap in which electrons are accelerated in the longitudinal (with respect to the magnetic field) electric field of the rotating star. This strong field, due to its origin in the magnetic field of the pulsar and its rapid rotation, accelerates the electrons to gamma factor values of the order of 10^7. Their curvature radiation in a (dipole or non-dipole) magnetic field generates a cascade with the production of electron-positron pairs, which creates the pulsar magnetosphere.

The radiation by electrons in the gap is considered in a large number of papers. We will focus on

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the radiation by positrons moving to the surface of the star and accelerating by the same electric field. In regions with the opposite sign of the electric field, this corresponds to radiation from electrons returning to the surface, which we will not mention further.

Significant attention in our discussion is paid to so-called transition radiation [7, 28]. It is a kind of radiation emitted by a charged particle when latter crosses the border between media with different dielectric permittivities, i. e. the border between vacuum and substance (see Fig. 1). In ultrarelativistic case it has narrow angular distribution with the centre in the direction mirror symmetrical to the velocity of the particle at the border (we will call it the mirror direction). The discussed transition radiation properties take place if the substance is conducting and radiation frequencies are lower than optical.

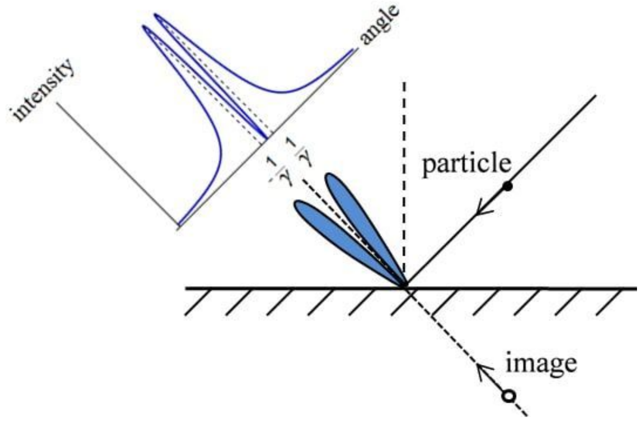

Fig. 1: Transition radiation and its angular distribution at ultrarelativistic particle incidence on conducting surface.

The radiation spectral-angular distribution can be presented in the form [7, 28]: $\mathrm{d}^2\epsilon/\mathrm{d}\omega do = e^2/(\pi^2 c)\vartheta^2/(\vartheta^2+\gamma^{-2})^2$, where ϑ is the angle between the line of sight and the mirror direction, and γ is the particle Lorenz-factor. It shows that radiation intensity can be considered as frequency-independent in the region of applicability of the presented expression.

Transition radiation pulse is nothing else than the Coulomb Field of the incident particle reflected from the border, which turns into diverging electromagnetic waves. It can be calculated (in the method of images) as radiation emitted by the particle's image, which moves inside the substance mirror symmetrically to the particle with respect to the border, when it stops on the surface and 'annihilates' with the particle (for details see [7]).

When the positron in the inner gap of a pulsar hits the surface it emits transition radiation which can propagate to the telescope. The properties of such radiation, however, will significantly differ from the ones discussed above. It is caused by the fact that before impinging on the surface the positron does not move evenly and rectilinearly and at the incidence moment has field around itself, which is different from the Coulomb one.

The point is that during the positron motion along a curved magnetic field line in the gap the long-wave part of its Coulomb Field does not have time to follow the shorter-wavelength part and the positron partly 'takes off' its Coulomb Field by the moment of impinging upon the surface, becoming half-bare. From the other point of view, such modification (suppression) of the positron proper field can be considered as the result of destructive interference of its Coulomb Field with the electromagnetic field, emitted by positron, which on large distance from the particle becomes the field of curvature radiation.

In the present case the total radiation pulse produced by the positron consists both of the Coulomb Field reflected from the surface (transition radiation) and reflected field of curvature radiation. In the present paper we calculate the spectral-angular distribution of this pulse and show that in a wide range of observation angles the average yield of such radiation coincides with the one of a single reflected curvature radiation. Significant interference of the discussed two types of radiation occurs only in the vicinity of the mirror direction for positron. Here the destructive interference of the Coulomb and curvature radiation fields (the positron half-bareness) leads to a huge suppression of transition radiation peak shown in Fig. 1. As noted, this fact allows explaining in the framework of the adopted model the observed mysterious fact of appearance of shifted interpulse at certain minimal frequency and its absence at lower ones.

RADIATION BY RETURNING POSITRONS MOVING TO THE STELLAR SURFACE

The curvature radiation from returning relativistic positrons is directed toward the surface but at sufficiently large frequencies it reflects from the surface and can arrive at the telescope having some typical peculiarities. (We assume that the positron radiation of lower frequencies, corresponding to lower values of Lorenz-factor, does not hit the pulsar surface since it dominates in the directions tangent to the positron trajectory on very high altitudes, see Fig. 2.) It is broadened comparing to the direct radiation (from electrons), outgoes it by phase (the phase associated with the star rotation), since it is created on larger altitudes and is reflected to a large angle with respect to the surface normal (see Fig. 2). Depending on the magnetic field topology in the vicinity of the pole the reflection either directly leads to the appearance of high-frequency components, or they appear as a result of nonlinear Raman scattering on the surface 'roughness' induced by the incident radiation [15].

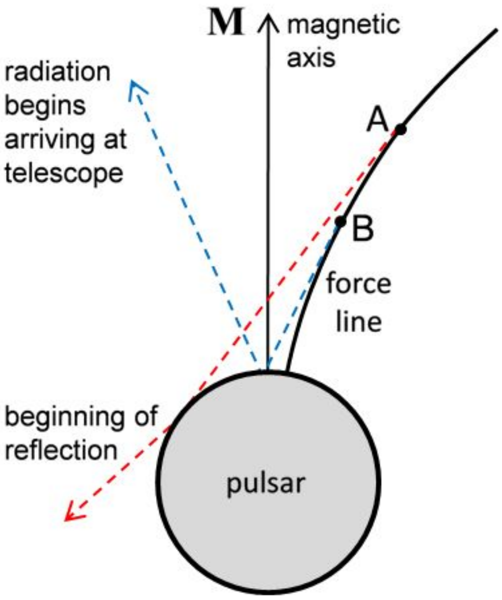

Fig. 2: At the point A of the positron trajectory its radiation begins being reflected from the surface. At lower altitude (point B) the emitted radiation (having higher frequency) can reflect at small angle to the magnetic axis and arrive at the telescope. Its characteristic frequency is the minimal one at which the shifted IP appears. The figure is schematic. The points are much closer to the surface of the star than it is shown.

An important role is played by peculiarities of radiation in relativistic motion. The distance responsible for radiation process (coherence or formation length), proportional to the square of Lorentz-factor, becomes a macroscopic value. This fact significantly influences upon radiation. It can be approximately considered that certain region of the positron trajectory is responsible for radiation emission at certain frequency $\omega \sim c\gamma^3/R$, where R is the radius of the particle trajectory, γ is its Lorenz-factor in the region under consideration and c is the speed of light. In this case with the increase of the frequency the radiation region moves along the magnetic field force line toward the surface where the coherence length becomes larger than the height of the polar gap due to significant increase of γ. The total radiation emitted by positron in this case (interfering curvature and transition radiations) can be considered as transition radiation by half-bare particle. We do not consider here the hard radiation of positron annihilation on the surface, see [30]. The "softest" part of this radiation corresponds to backward emission direction and forms a line in the gamma range (at half value of the annihilation frequency 511 keV).

On small altitudes the surface of the star can be considered flat which allows using the method of images for calculation of the positron radiation even at relativistic energies of the particle. (The positron's image in this case substitutes real electron currents induced by the particle's field on the surface. The component of such currents along the strong magnetic field of the star remains intact by this field while the transversal one becomes suppressed. Nevertheless, frequent collisions of electrons due to extremely high density of the surface matter keep transversal electron motion 'alive' reducing somehow the magnetic field impact on it. Therefore, presently we do not take the influence of magnetic field upon radiation characteristics into account.) The present mechanism of radiation takes place at rather high frequencies. Further for simplicity we assume that the positron moves along a segment of a circle. In the case of a dipole magnetic field it is a legitimate simplification for the motion geometry within the gap. Indeed, for a dipole (following [11, 18] we consider here the field of a stationary non-rotating dipole) the equation for the limiting force line of the magnetic field touching the light cylinder, has the form $r = r_0 \sin^2\theta$, where $r_0 = 27r_{\mathrm{LC}}/[(9-\sin^2\beta)(\sin^2\beta+\cos\beta\sqrt{9-\sin^2\beta})]$, is the parameter of the force line (its diameter) and θ is the polar angle counted from the magnetic axis (see [11, 18]). Here r_{LC} is the radius of the light-cylinder and β is the angle between the magnetic and rotation axes of the pulsar. At low altitude for the line curvature radius it gives: $4\sqrt{rr_0}/3$. Thus, along the field line its curvature changes not significantly on the scale of the gap. Therefore on the part of the trajectory important for radiation it can be assumed to be constant. It means the possibility to replace the real trajectory by a circular arc. Of course, it is not precise within a large area of space, but we are interested foremost in a qualitative picture. In the present work we are far from the intention to consider the accurate and self-consistent models in connection with illustration of effects which are interesting for us. It is important that such models exist (see [3] and references therein) testifying consistency of the whole scheme. However, greatly complicating the calculations, they cannot shed further light on the discussion of our problems of a shifted interpulse in the centimetre range.

We use a model expression for accelerating field in the form

$$\begin{aligned} E(\alpha) = {} & E_1\alpha\theta(\alpha_1-\alpha)+ \\ & +\big(E_1+E_2(\alpha-\alpha_1)\big)(1-\alpha)\theta(\alpha-\alpha_1), \end{aligned} \qquad (1)$$

which is a simplified synthetic version of the fields discussed in [2, 4, 8, 23, 24]. It takes into account such derived properties of the field as its quick parabolic increase in the upper part of the gap and subsequent vanishing on the stellar surface, as well as the field penetration into magnetospheric plasma. In this region we substitute its exponential dependence on the altitude [23, 24] by a linear one, which does not significantly affect the results but simplifies calculations. Here r is the pulsar radius and α_0 is the opening angle corresponding to the "effective" region of the positron trajectory (which has radius R), the radiation emitted from which reflects from the surface of the star. By its definition, for a given field line this value is defined from pure geometrical considerations (see Figs. 2, 3) and can be expressed as $\alpha_0 \sim \arccos(1 - r/R) - r/R$. For more detailed study of the "effective" spatial region for positron radiation (involving the account of radiation coherence) see [14]. It is natural to use the angular variable α (here in units of α_0) as the coordinate of the positron (and its image). The parameters E_1 and E_2 define the absolute magnitude of the field. Their values (which result in the following magnitudes of the dimensionless parameters f_1 and f_2: $f_1 = 4 \cdot 10^3, f_2 = 3 \cdot 10^7$) are chosen from the requirement that curvature radiation emitted by a positron in the region of the linear growth of the field belongs to radio band and positrons in the vicinity of the surface have Lorenz-factors of the order of 10^7 (for details see [14]). In the method of images the radiation from a positron reflected from the stellar surface is the radiation (direct) by the positron's image moving mirror symmetrically to the positron with respect to the surface. Therefore, further we will concentrate on the motion and radiation of the positron's image.

Equation defining the dependence of the positron Lorenz-factor γ on α is

$$\mathrm{d}\gamma/\mathrm{d}\alpha = eE(\alpha)R/mc^2,$$

which is the same as the well-known equation for a particle energy gain in external electric field:

$$\mathrm{d}\epsilon/\mathrm{d}t = e\mathbf{E}\mathbf{v}.$$

Dependence of the image Lorentz-factor γ upon α is the solution of this equation and in the case of accelerating field (Eq. (1)) has the following form:

$$\begin{aligned}\gamma(\alpha) = \theta(\alpha_1 - \alpha)\{\gamma(0) + f_1\alpha^2/\alpha_1^2\}+\\ + \theta(\alpha - \alpha_1)\{\gamma(\alpha_1) + \delta\gamma(\alpha)\},\\ \delta\gamma = (f_1/\alpha_1 - f_2)(\alpha - \alpha_1)+\\ + \big(f_2(1+\alpha_1) - f_1\big)\big(\alpha^2 - \alpha_1^2\big)/2\alpha_1-\\ - f_2(\alpha^3 - \alpha_1^3)/3\alpha_1,\end{aligned}$$

where $\theta(x)$ is the step function which is equal to zero for $x < 0$ and to unit for $x > 0$, $f_{1,2} = e\alpha_0 E_{1,2} R\alpha_1/mc^2$. In the model of the electric field used here α_1 is some value of the angular variable α at which the character of the electric field dependence on α changes from slow linear growth to rapid parabolic one (with subsequent vanishing on the stellar surface), see the schematic dependence in Fig. 4.

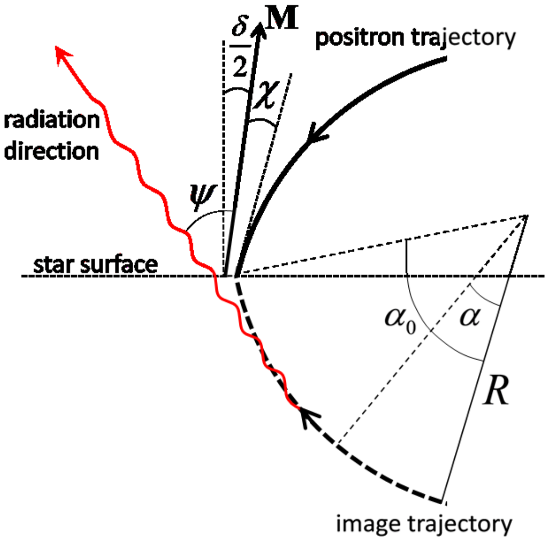

Fig. 3: The geometry of a positron and its image motion used for calculations. The reflected radiation direction angle ψ approximately corresponds to a definite value of Lorentz-factor and radiated frequency. χ is the angle between the magnetic axis and the positron trajectory on the pulsar surface. $\delta/2$ is the magnetic axis inclination angle with respect to the surface normal (for details see [13, 14]). For convenience the scale of the trajectory radius R is significantly decreased.

Note, that the radio emission in pulsars is coherent and the main contribution to the coherent emission is made by positrons with maximal value of their energy on the effective part of the trajectory [14], the radiation from which is received by telescope. Such situation is quite different from the case of synchrotron radiation of the electron component of cosmic rays in radio galaxies, quasars, supernova remnants etc. Due to the decreasing energy spectrum of cosmic ray particles the main contribution to the radiation flux is made by electrons with minimal Lorenz-factor. It leads to the well-known relation between the index of electron energy spectrum and spectral index of cosmic radio emission. In calculation in this case only contribution from a region in the vicinity of the spectral maximum depending on the electron energy [7, 10, 16] is essential.

Positron radiation spectral-angular density is calculated with the use of the well-known expression [16, 10], see the scheme of angular diagram in Fig. 5 :

$$\frac{\mathrm{d}^2\epsilon}{\mathrm{d}\omega do} = \frac{e^2\omega^2}{4\pi^2c^3}\left|\int\limits_{\infty}^{\infty} \mathrm{d}t\,[\mathbf{n}, \mathbf{V}(t)] \times \right.$$
$$\left. \times \exp\left\{i\omega\left(t - \frac{\mathbf{n}\mathbf{r_0}(t)}{c}\right)\right\}\right|^2, \quad (2)$$

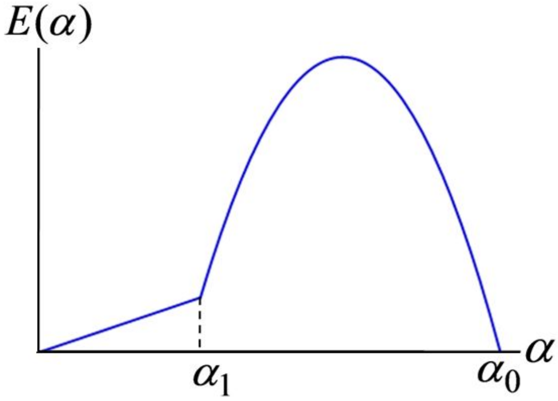

Fig. 4: The linear section corresponds to smooth penetration of accelerating electric field into the lower magnetosphere [23, 24], the parabolic – to a simplified version of known models of the electric field in the gap [2, 8]. This field returns positrons from pairs in magnetosphere to the star and accelerates them along magnetic field lines.

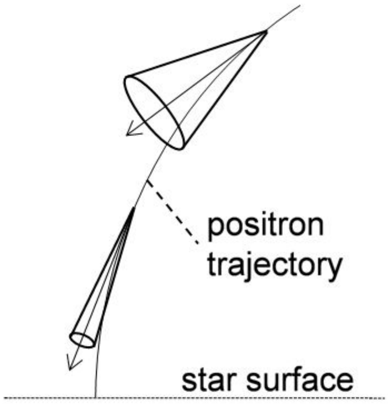

Fig. 5: Gradual narrowing of the positron curvature radiation angular diagram with the increase of its Lorenz-factor. This narrowing of the cone plays important role, leading to the increase of the transversal region responsible for coherent radiation emission with the growth of the positron energy [14].

in which $\mathbf{r_0(t)}$ is the law of the positron's image motion, $\mathbf{n}$ is a unit vector along the radiation direction and $\mathbf{v(t)}$ is the image velocity. The radiation spectral distribution (averaged with respect to rapid oscillations analogous to the ones shown in Fig. 6 presenting radiation angular distribution) is similar to the one of synchrotron radiation and has rather distinct maximum. This allows (for qualitative analysis) considering radiation in each direction (defined by angle ψ) as approximately monochromatic with frequency $\omega(\psi) \sim c\gamma^3/R$ (where ψ and α are connected by relation $\psi = \alpha_0 - \alpha + \delta + \chi$ (see Fig. 3) and γ is the Lorentz-factor).

DISCUSSION. THE EFFECT OF POSITRON HALF-BARENESS

Figure 6 represents angular distributions of radiation by positron for $\omega \sim 10^9 c^{-1}$ calculated with the use of Eq. (2). The solid line shows the result which takes into account the interference between transition and curvature radiations (the effect of positron 'half-bareness'). For its calculation we have to consider the positron's image which accelerates along an arc of a circle (up to velocity v_f corresponding to Lorenz-factor $\gamma \sim 10^7$) and stops on the surface 'annihilating' with positron (see introduction section). The other distributions correspond to imaginary cases and are shown for comparison.

Particularly, the dot-dashed line shows the peak of transition radiation from a 'dressed' positron (reflection of only Coulomb Field), which is the same as the one in Fig. 1. In this case the image, before stopping and 'annihilating' on the surface, moves with constant speed v_f. (Due to extremely narrow angular size of the considered peak ($\gamma^{-1} \sim 10^{-7}$) its two-humped structure (see Fig. 1) is not presented here.)

Dashed line shows the distribution for hypothetical case when image, after accelerating along the arc, does not stop at the surface but continues its motion with constant speed, equal to its speed on the surface at the end of the circular motion. It is pure curvature radiation without taking into account the transition one (which is associated with the image stop on the surface). The wide plateau corresponds to radiation emitted during circular motion (which is curvature radiation itself). The peak on the left side is just the ordinary peak of bremsstrahlung emitted in the final direction of motion of the scattered (deflected) particle. It has nearly the same magnitude as the peak of a 'pure' transition radiation (dot-dashed line) but opposite direction of field strength vector. (It is due to the fact that this peak coincides with the one produced by a charged particle at its rapid acceleration from the rest state to the velocity v_f ('appearance' of particle with velocity v_f), while the transition radiation peak, as noted before, corresponds to the opposite process of particle stopping — 'disappearance' of particle with such velocity.) This leads to mutual cancellation of these peaks (solid line) if take into account interference of curvature and transition radiation fields (i. e. include the stop of the image at the surface).

Thus, transition radiation peak of 'half-bare' positron is considerably suppressed comparing to the one of the 'dressed' positron. Fig. 6 also shows that the effect of radiation suppression occurs only in the vicinity of the mirror direction. In a wide range of angles (plateau area) the radiation intensity is close to the one of the reflected curvature radiation if not take into account rapid oscillations, which are expected to vanish after averaging over the whole polar cap of the pulsar.

As noted (see introduction), the intensity of 'pure' transition radiation (dot-dashed line in Fig. 6) does not depend on frequency. Therefore, the suppression of this peak plays important role in application of the considered model for explanation of IP shift. Indeed, as shown in [14], the observed spectral properties of the shifted IP can be (at least qualitatively) explained just by means of curvature radiation of the positrons moving toward the stellar surface (plateau area in Fig. 6), for which the approximate correspondence between radiation direction and its frequency exists. In this case the minimum frequency at which shifted IP appears (around $\omega/2\pi \sim 5\ GHz$) can be considered as the one, corresponding to the direction of radiation (at point B in Fig 2) which after reflection from the surface begins arriving at telescope. The intense frequency-independent transition radiation would have created the analogous shifted interpulse at lower frequencies had it not been suppressed by the effect of positron half-bareness.

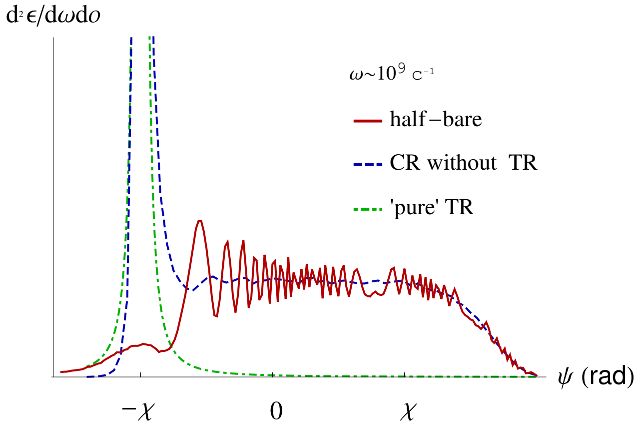

Fig. 6: Intensity angular distribution of: radiation by half-bare positron (solid line), transition radiation (TR) by 'dressed' positron (dot-dashed line), curvature radiation (CR) without taking into account TR (dashed line). χ see in Fig. 3.

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