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## TEMPERATURE ERRORS MODELING OF COORDINATE MEASURING MACHINES

*Coordinate measuring machines (CMMs) are commonly used to determine the parameters of a circular feature using different criteria like minimum zone, least squares, minimum circumscribed or maximum inscribed. For many years, thermal effects have been the largest single source of dimensional errors and equipment non-repeatability. Thermal error variation has a complex nonlinear nature which makes it difficult to handle. Theoretical analysis and analytical representation of almost components of the CMMs measuring uncertainty at the beginning of measurement makes possibilities to create measuring errors models in every CMMs knots, considering temperature influences and estimate there. This work presents the equations for the components of the volumetric error of a Coordinate Measuring Machine including temperature influences and the interest to the temperature effects influence determination on the CMMs uncertainty and there estimation with help of the mathematical statistics methods, for example correlation theory, factorial design and ANOVA.*

***Key worlds:*** *coordinate measuring machine, measuring uncertainty, temperature influence.*

### Introduction

The products manufacturing within smaller tolerances and in larger quantities has impelled the necessity of developing faster, more accurate, flexible and reliable quality control approaches [1]. In this case the most suitable measuring instruments are Coordinate Measuring Machines (CMMs) that can e employed in every branch of machinery industry.

The measurement uncertainty and performance of CMMs is limited by different verity of factors that take action just themselves, combining with each other in a complex combination. Geometric errors compose the most representative fraction of the volumetric error A full number of 21 errors can be determined from three axis CMMs [2]. Theoretical analysis and analytical representation of almost components of CMMs measuring uncertainty at the beginning of measurement makes possibilities to create measuring errors models in every CMMs knots, considering temperature influences and estimate there.

At the temperature of 20°C, geometric errors can be considered constant, once they vary very slowly with time. However, if the temperature is different from 20°C, these errors can change in magnitude and behaviour due to thermal deformations of the CMM structure in a general sense. Hence, the denominated thermal errors are induced, detrimental to precision and repeatability of CMMs (Bryan, 1995).

The knowledge about temperature variation makes possible determinate performance criterions for every CMMs components and temperature correction directions as goal to minimize measuring uncertainty.

### 1. Theoretical background

For many years, thermal effects have been the largest single source of dimensional errors and equipment non-repeatability. Thermal error variation has a complex nonlinear nature which makes it difficult to handle. Although not as significant as in machining, thermal error effect on CMM accuracy has been widely addressed [5]. Several studies have been developed with the aim of understanding the characteristics, amplitudes and sources of thermally induced errors so that their effects can be minimized [1]. Thermal errors, their costs and sources have remained practically unchanged (Bryan,1995). Bryan (1967) assessed the state of the art and relevance of thermally induced errors. According to Bryan, errors due to temperature variation either present the same magnitude or are greater than kinematic, static and dynamic errors. Thermally induced errors are responsible for a considerable fraction of the total error of a machine tool. Ramesh et all (2000) published a paper about thermal errors in machine tools that basically discusses the work concerning the study, measurement, modelling and compensation of thermally induced errors that were produced in the nineties [1].

One of the last research works [1, 3, 4] show the interest to the thermal effects influence determination on the CMMs uncertainty and there estimation with help of the mathematical statistics methods, for example correlation theory, factorial design and ANOVA.

Regarding coordinate measuring machines, the thermal influences issue remains even more critical, due to the poor availability of research on the theme and the insipience of the published results. The subject, although not recent, remains contemporary.

## 2. Development of the general error model

This work shows a mathematical formulation to obtain the equations for the volumetric error components in CMM considering thermal influences. All experimental runs for the acquisition of error and temperature data were conducted on a moving bridge CMM. The thermal influence problem is not simple, due to the demanding precision requirements for the CMM Using an general error model obtained by means of homogeneous transformations, each component of the volumetric error can be described as the sum of different parts that are related to the geometric errors of the machine. Geometric errors were showed as functions of position and temperature. The proposed model is based on the straightforwardness of application and adaptation of the homogeneous transformations to any kind of coordinate measuring machines and on the efficient diagnosis ability of the general error method [1]. The modelling was carried out in two stages. Firstly, the equations of the volumetric error for a reference temperature of 20°C were determined by means of homogeneous transformations. Next, the equations of the geometric errors thermally induced variations were determined using regression techniques and the least square method [1].

Hence, the equations of the volumetric error components are given in (1), (2) and (3).

$$\begin{aligned} Ex &= Pos_X + Ry_X + Rz_X + \\ &+ [Ort_{XY} + Yaw_X] Y_{34} + [Ort_{XZ} + Pitch_X + \\ &+ Yaw_Z + Roll_y](-Z - Z_{45}) + Roll_y Z_{12}, \end{aligned} \quad (1)$$

$$\begin{aligned} Ey &= Pos_Y + Rx_Y + Rz_Y + [Ort_{XY} + \\ &+ Yaw_Y](X23 + X) - Pitch_Y Z_{12} + \\ &+ [Ort_{YZ} + Roll_X + Pitch_Y + \\ &+ Pitch_Z](-Z - Z_{45}), \end{aligned} \quad (2)$$

$$\begin{aligned} Ez &= Pos_Z + Rx_Z + Ry_{ZX} - Roll_Y (X + X_{23}) - \\ &- [Roll_x + Pitch_Y] Y_{34}, \end{aligned} \quad (3)$$

where $Ex, Ey$ and $Ez$ - components of volumetric error at $20^0$C;

$Ort_{XY}$, $Ort_{XZ}$ and $Ort_{YZ}$ – orthogonality errors;

$Pitch_X$, $Pitch_Y$ and $Pitch_Z$ - angular error Pitch at axis X, Y and Z;

$Pos_X$, $Pos_Y$ and $Pos_Z$ - positioning error at axis X, Y and Z;

$Rx_Y$ and $Rx_Z$ - straightness error of axis X direction Y and Z;

$Ry_X$ and $Ry_Z$ - straightness error of axis Y direction X and Z;

$Rz_X$ and $Rz_Y$ - straightness error of axis Z direction X and Y;

X, Y and Z - coordinates;

$Y_{34}$, $X_{23}$, $Z_{12}$ and $Z_{45}$ - fixed offset.

It is very important to know the amplitude of the variation experienced by the errors and steady offsets due to temperature variation. Therefore, geometric errors must be changed for different thermal states. Resulting data must be treated as functions of position and temperature and finally must be adequately introduced in the general equations.

Each geometric error was written as the sum of two parts Eq. (4). The first one represents the geometric error at 20°C and only depends on the position of the corresponding carriage. The second part represents the thermally induced error variation. It can be described as a function of temperature and position, since temperature variation may cause irregularly distributed error variation along the coordinate axes.

$$feh_i = feh_i(p) + vfeh_i(p, T), \quad (4)$$

where $feh_i$ is the error at any position i and at any thermal state;

$feh_i(p)$ is the geometric error i at position p;

$vfeh_i(p, T)$ is the thermally induced variation of geometric error i at position p.

The mathematical determination of geometric error variation due to changes in temperature is rather complex. Consequently, collected data from thermal drift at four points distributed along each evaluated axis were employed. One data set was acquired at each observation point. These sets represent the thermally induced variation of the error at a given position, until steady state. Expressions describing error variation as a function of temperature at each drift observation point were obtained by means of regression techniques. The variation can be written as:

$$vfeh_i(T) = \gamma_0 + \gamma_1 T_1 + \gamma_2 T_2 + \ldots + \gamma_j T_j, \quad (5)$$

where $vfeh_i(p, T)$ is the error variation at any point i, for i=1,…,4;

T - contains the components of the array of temperature variation from the several thermocouples j, for every j=1,…,18;

$\gamma_1, \ldots, \gamma_j$ - least squares estimators.

The selection of values that compose the temperature array at each point i was performed by means of a stepwise procedure. Therefore, thermocouples whose temperature presented a correlation greater than or equal to 99,9% were grouped. Subsequently, the effects of variables p and T were superposed for the determination of error variation at any position and at any thermal state, using regression techniques and the least squares method.

$$\begin{aligned} vfeh_i(p, T) &= \beta_0(T) + \beta_1(T) p_i + \\ &+ \beta_2(T) p_i^2 + \ldots + \beta_n(T) p_i^n, \end{aligned} \quad (6)$$

where $\beta i, (i = 0, \ldots, n)$ are the regression coefficients;

$p_i, (i = 1,...,4)$ are the positions where thermal drift was observed.

The estimation of coefficients $\beta_n$ was made possible setting nonequality relationship between Eq. (6) and error array given by (5) and applying the least squares method. The resulting system is given by (7). Least squares estimators $\beta_0, \beta_1, \beta_2, ..., \beta_n$ are determined so as to minimize Eq. (8)

$$\begin{cases} \beta_0(T)+\beta_1(T)p_1+\beta_2(T)p_1^2+\ldots+\beta_n(T)p_1^n = \\ \qquad = vfeh_1(T), \\ \beta_0(T)+\beta_1(T)p_2+\beta_2(T)p_2^2+\ldots+\beta_n(T)p_2^n = \\ \qquad = vfeh_2(T), \\ \beta_0(T)+\beta_1(T)p_3+\beta_2(T)p_3^2+\ldots+\beta_n(T)p_3^n = \\ \qquad = vfeh_3(T), \\ \beta_0(T)+\beta_1(T)p_4+\beta_2(T)p_4^2+\ldots+\beta_n(T)p_4^n = \\ \qquad = vfeh_4(T) \end{cases} \qquad (7)$$

and

$$S(\beta_0, \beta_1, \ldots, \beta_n) = \sum_{i=1}^{m} \varepsilon_i^2 = = \sum_{i=1}^{m} (vfeh_i - \beta_0 - \beta_1 x_{1i} - \cdots - \beta_n x_{ni})^2, \qquad (8)$$

where $vfeh_i(T)$ = error variation at any point i for i=1,…,4;

$vfeh_i(p,T)$ = thermally induced variation of geometric error i at position p.

## 3. Experimental Results

A temperature gradient was introduced, which varied from 20°C to 26°C during the warming up period and from 26°C to 20°C at cooling down. As a result, four data sets that describe error variation at each observation point were obtained.

The curves that describe thermally induced variation of positioning error Y and can be observed that at certain positions, in the early beginning of cooling down process, error variation presents an increasing tendency. This fact is ascribed to system inertia and to the environmental conditions in the room at the moment of data acquisition.

If humidity is high, heating system will work harder, causing cyclic temperature elevations.

Correlation coefficients for each curve were 99,89%, 99,83%, 99,7% and 99,8% respectively. Analysis of residuals showed random behaviour, following an approximately normal distribution with mean value close to zero and constant variance.

The expression for the variation of axis Y positioning error was obtained through the sum of the solution of system (9) with the error at reference state, so that the function describing the behaviour of the referring error was obtained.

$$\begin{cases} vfeh(0,T) = vfeh_0(T), \\ vfeh(75,T) = vfeh_{75}(T), \\ vfeh(200,T) = vfeh_{200}(T), \\ vfeh(300,T) = vfeh_{300}(T). \end{cases} \qquad (9)$$

An analysis performed on the results obtained during the positioning thermally induced error variations for all axes allowed for the conclusion that the assembly principle between slide and scale influences magnitude and behaviour of these errors when the machine is submitted to environmental temperature variation.

The collected data sets were adequately introduced in the mathematical equations to allow for the development of the proposed model.

## Conclusions

The ring gauge was measured at several positions and at different temperatures. The coordinates of 10 randomly distributed points on the surface that defines the ring diameter were collected.

The values of the components of the volumetric error were synthesized at the coordinates of the collected points by means of the proposed model.

Next, the error correction at the coordinates of the measured points was performed, and the corrected coordinate values were obtained. Subsequently, the diameter was estimated and the difference between calculated and standard diameter values was determined.

Finally, the difference between calculated and standard diameter values was determined. Standard diameter was obtained by means of calibration. The ring gauge calibrated diameter was 181,0124mm ±1µm at 20°C, whereas at 26°C, the value was 180,0137mm ±0,7µm.

Having known calculated and calibrated diameters, residual error difference was determined.

$$Eres = D_{Calculated} - D_{calibrated},$$

where $D_{Calculated}$ -- gauge calculated diameter;

$D_{calibrated}$ -- gauge calibrated diameter.

From residual error values, an analysis was executed to decide whether the model is adequate or not. Model adequacy is conditioned to residual error values relatively small and normally distributed.

The thermal behaviour of straightness and angular errors of all axes do not depend on the position of the corresponding moving carriages. Hence, during thermal drift data collection, error values can be evaluated at one point only.

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## МОДЕЛЮВАННЯ ТЕМПЕРАТУРНИХ ПОХИБОК НА КООРДИНАТНО-ВИМІРЮВАЛЬНИХ МАШИНАХ

***Т.М. Хаейн***

Координатно-вимірювальні машини (КВМ) широко використовуються для визначення геометричних параметрів вимірювальних об’єктів за допомогою різних критеріїв, наприклад, методом найменших квадратів. Протягом багатьох років температурні впливи були одним з найбільших джерел просторових похибок. Варіація температурних похибок має нелінійну природу, що ускладнює проведення вимірювальних експериментів і обробку результатів вимірювання. Теоретичний аналіз і аналітичне представлення більшості складових вимірювальної невизначеності КВМ на етапі планування вимірювального експерименту дає можливість побудувати математичні моделі похибок в кожній ланці КВМ, враховуючи вплив температурних факторів і оцінити їх. В роботі представлені аналітичні моделі складових об’ємних похибок КВМ, визначені температурні впливи на вимірювальну невизначеність КВМ, які можуть бути оцінені за допомогою методів математичної статистики, на прикладі кореляційної теорії і дисперсійного аналізу.

**Ключові слова:** Координатно-вимірювальна машина, вимірювальна невизначеність, температурний вплив.

## МОДЕЛИРОВАНИЕ ТЕМПЕРАТУРНЫХ ПОГРЕШНОСТЕЙ НА КООРДИНАТНО-ИЗМЕРИТЕЛЬНЫХ МАШИНАХ

***Т.М. Хаейн***

Координатно-измерительные машины (КИМ) широкого используются для определения геометрических параметров измерительных объектов с помощью различных критериев, например, метода наименьших квадратов. На протяжении многих лет исследований температурное влияние было одним из важнейших источников пространственных погрешностей. Вариация температурных погрешностей имеет нелинейную природу, что усложняет проведение измерительных экспериментов и обработку результатов измерений. Температурный анализ и аналитическое представление основных составляющих измерительной неопределённости КИМ на этапе планирования измерительного эксперимента даёт возможность построить математические модели погрешностей в каждом узле КИМ при влиянии температурных факторов и оценить их. В работе представлены аналитические модели составляющих объемных погрешностей КИМ, определены температурные влияния на измерительную неопределённость КИМ, которые могут быть оценены с помощью методов математической статистики, например, корреляционной теории и дисперсионного анализа.

**Ключевые слова:** Координатно-измерительная машина, измерительная неопределённость, температурное влияние.

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