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SUBJECTIVE ENTROPY EXTREMIZATION PRINCIPLE AS A TOOL OF AN AIRCRAFT MAXIMAL DURATION HORIZONTAL FLIGHT CONTROL

This paper continues researches of optimal control in active systems started before. In application to an aircraft maximal duration horizontal flight control problem solution, it is proposed to use the individual's subjective preferences entropy extremization principle as a helpful tool. Basing oneself upon that principle alone, one can reveal the optimal mode of operational control even without knowing the extremal one. Since the subjective entropy extremization principle allows; independently on the conditions of transversality, Weierstrass-Erdmann, principle of maximum by L.S. Pontryagin (USSR), as well as principle of optimality by R. Bellman (USA); finding the extremals, their optimal conjunctions of all kinds: either breaks with shifts, or both at smooth and corner points, for closed and restricted areas; stipulated by obeying the only a priory condition of the Euler-Lagrange equations; it is suggested to call this principle by the name of its author, professor Vladimir Aleksandrovich Kasianov, National aviation university (Kyiv, Ukraine). The necessary calculations are fulfilled. Plotted corresponding diagrams.

Key words: *maximal duration, horizontal flight, subjective entropy, individual preferences, multi-alternativeness, active systems control, active element, variational problem, operational functional.*

Introduction

The presence of an aircraft power plant of any kind on board an airplane is dictated by the reasons of the balanced combinations for a certain flight tasks fulfillment, reliability of the designed structures, flight safety motivations, and a thought of economical efficiency [1].

Problem formulation in the general view. The choice made by the aircraft operators (active elements of the system) is somewhat optimal. To discover that optimality is the concept of this paper in general.

The problem relation to important scientific and practical tasks. The theme of this article relates with the important scientific tasks of analytical researches in the field of the optimal control theory regarding control in the active systems.

Its practical significance touches the issues of fuel-energy saving modes of operation of aircrafts and their powerplants.

Analysis of the latest researches and publications. In our researches we base ourselves upon the literature sources briefly listed as [1 – 12].

This paper continues the discussions on the optimality of the active system element's (individual's, subject's) controlling behavior initiated in our previous publications [6-11].

It deals with the unsolved part of the general problem in the aspects of mathematical modeling of the active element control.

The task setting. The objectives of this paper are to justify the use of the entropy concept for explanations of the individual's optimal controlling decisions choice.

The main content of the researches

Having considered objectively existing optima for an aircraft maximal duration horizontal flights obtained in the previous publications and achieved on the basis of the results of the variational problems solutions of the changeable mass material point dynamics we come to the variational problem in terms of subjective analysis.

1. The variational problem in terms of subjective analysis

1.1. Maximal duration horizontal flight with the possible extremal

Accordingly to the theoretical statements of [2, 3] we have got the extremal speed of the aircraft maximal duration horizontal flight in the view of a function of a flying object changeable mass:

$$v_T(m) = \sqrt[4]{\frac{4}{3}\frac{bm^2g^2}{C_{x_0}\rho^2S^2}}\,, \tag{1}$$

where b – some stable value which is being determined from the blowing in the wind tunnels;

m – mass of the flying apparatus;

$g = 9{,}81$ m/s^2;

C_{x_0} – value of the head resistance force coefficient at the value of the lifting force when it equals zero;

ρ – density of the air at the given altitude;

S – character square area of the flying object [3, § 5, P. 199].

All derivations are performed in the supposition of the parabolic polara relations between coefficients of the aerodynamic forces.

The duration of the flight is determined by the value of the corresponding functional. In the case (1) the expression is the function of the aircraft changed mass

$$T(m)_{max} = \left(\frac{3}{4}\right)^{\frac{3}{4}} \frac{\eta Q\sqrt{\rho S}}{\sqrt[4]{C_{x_0}}\left[bg^2\right]^{\frac{3}{4}}}\left(\frac{1}{\sqrt{m}} - \frac{1}{\sqrt{M_0}}\right), \quad (2)$$

where η – efficiency of the propulsive complex, considered to be a constant for the rough problem setting;

Q – low calorific value of the fuel;

M_0 – mass of the flying apparatus at the initial moment in time [3, § 5, P. 201, 202].

Corresponding distance at that maximal duration:

$$L\big|_{T_{max}} = \frac{\eta Q}{4g}\sqrt{\frac{3}{bC_{x_0}}}\ln\left(\frac{M_0}{M_E}\right),$$

where M_E – final mass of the flying apparatus.

At a constant flight speed we obtain the change of the mass of the airplane from the differential equation which has a well known solution [12, P. 24, # 15]:

$$m(t) = \sqrt{\frac{a_1}{b_1}}tg\left[arctg\left(M_0\sqrt{\frac{b_1}{a_1}}\right) - t\sqrt{a_1 b_1}\right],$$

where a_1, b_1 – corresponding constants:

$$a_1 = \frac{C_{x_0}\rho S v^3}{2\eta Q}, \qquad b_1 = \frac{2bg^2}{\eta Q\rho S v}.$$

After compilation, for the set of some two reachable alternatives of the considered operational modes control, the so-called operational functional (the prototypes of such a kind of functionals are, for instance, in the publications of [9, P. 57, (1), (2)], [7, P. 119, (3.38)]), which includes H_π – active element's (individual's) subjective preferences entropy, we can find, from the necessary conditions for the extremals to exist, written in the view of the system of the equations by Euler-Lagrange, the corresponding expressions of the canonical distributions of the preferences, likewise in [9, P. 58, (4)], [7, P. 115-135]:

$$\pi_j = \frac{e^{-\beta F_j}}{\sum_{i=1}^{2} e^{-\beta F_i}}, \quad (3)$$

where β – structural parameter;

$$F_1 = \frac{2\eta Q\rho v_1 S}{C_{x_0}\left(\rho v_1^2 S\right)^2 + b(2mg)^2}, \quad (4)$$

$$F_2 = \frac{2\eta Q\rho v_2 S}{C_{x_0}\left(\rho v_2^2 S\right)^2 + b(2mg)^2}, \quad (5)$$

where v_1, v_2 – alternative functions of speed.

For the extremal speed we get the expression absolutely one and the same to (1).

Fulfilling the necessary mathematical calculations with the methods of (1)-(5), as a sort of experimenting work, we are getting convinced in preferring the "right" (optimal, the best) alternative for delivering the extremal (maximal) value to the objective functional.

1.2. Duration of the horizontal flight with the application of subjective entropy of unextremal operational modes preferences

Herein, we particularly have to call attention to the fact that the entropy extremization principle alone makes it possible choosing the best reachable combined alternative in that case when the extremal speed of the horizontal flight is unachievable.

If there are two unextremal, generally speaking changeable depending functionally upon the aircraft mass, speeds (modes) of horizontal flight, there can be three possibilities in the general case:

1. The first speed delivers the longest duration of the horizontal flight;
2. The second one; or
3. A certain combination of the two speeds (modes of operation).

The solution ("right" choice) implies the longer duration of the flight as the bigger value of the corresponding flight segment functional of the type:

$$T(m) = \int_{M_0}^{m} -\frac{2\eta Q\rho v S}{C_{x_0}\left(\rho v^2 S\right)^2 + b(2xg)^2}dx, \quad (6)$$

where x – technical variable;
being found on the basis of the bigger differential:

$$dt = -\frac{2\eta Q\rho v S}{C_{x_0}\left(\rho v^2 S\right)^2 + b(2mg)^2}dm, \quad (7)$$

or derivative of the duration, correspondingly to (4), (5).

However, we do not impose that artificial, for the given problem formulation, condition of the bigger value of the integral (6), differential (7) or derivative of the time (duration) in the view of the expressions (4) and (5) with the related speeds of the horizontal flight in accordance with their segments of the flight. Instead, we compile the functional and find the canonical distributions of the controlling system active element's (individual's) subjective preferences functions in the view of likewise (3).

2. Practical application of the problem solution

Suppose an aircraft and its flight parameters are:
M_0 = 10,000 kg; M_E = 8,000 kg; η = 0,3;

$$Q = 42{,}700 \cdot 10^3 \text{ J/kg}; \ \rho = 1 \text{ kg/m}^3; \ S = 50 \text{ m}^2;$$
$$C_{x_0} = 0{,}02; \ b = 0{,}045.$$

There are presumably two possible unextremal speeds of horizontal flight $v_1(m)$ and $v_2(m)$; corresponding depictions of them in the style of $v1(m)$ and $v2(m)$ are demonstrated in fig. 1.

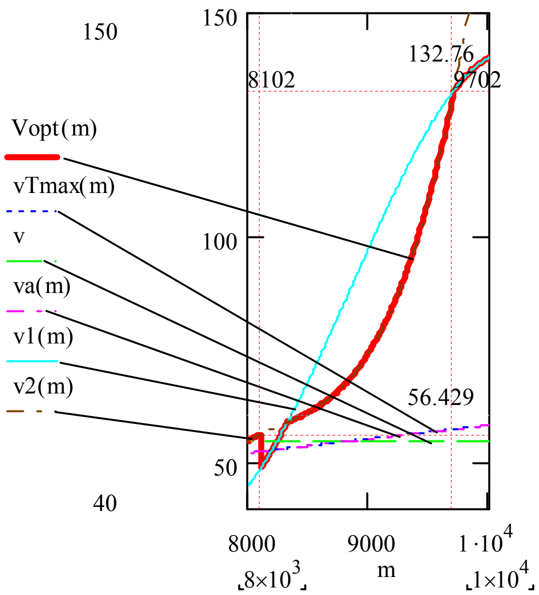

Fig. 1. Variants of the horizontal flight speeds

In fig. 1 it is also depicted: $Vopt(m)$ – optimal speed combined out of the two alternative speeds; for comparison $vT\max(m)$ – the extremal (optimal) speed of the horizontal flight; v – a constant flight speed from the optimal diapason; $va(m)$ – the approximated optimal speed (it is the closest to the extremal, but not optimal although); $vl(m)$, $v2(m)$ – the changeable unextremal speeds for (4), (5) respectively.

If the speeds $v_1(m)$ and $v_2(m)$ can arbitrary (subjectively) be chosen at any moment of the flight time, in general case as that was mentioned above, one or another combination of them will be delivering the maximal value to the functional (6).

For the given data we find the canonical distributions of the making decisions active element's preferences, calculated in this experiment through the expressions (3)-(5).

The preferences of $\pi_1(m)$ and $\pi_2(m)$ of the corresponding alternative speeds of $v_1(m)$ and $v_2(m)$, related to the corresponding modes of operational control, are shown in fig. 2 in the stylized manner as $\pi l(m)$ and $\pi 2(m)$.

The calculated subjective entropy of $H(m)$ is also represented in fig. 2 in the corresponding scales f and n.

The same scales is used for the subjective entropy maximal value which is for this given case equaled to $\ln(2)$, see fig. 2.

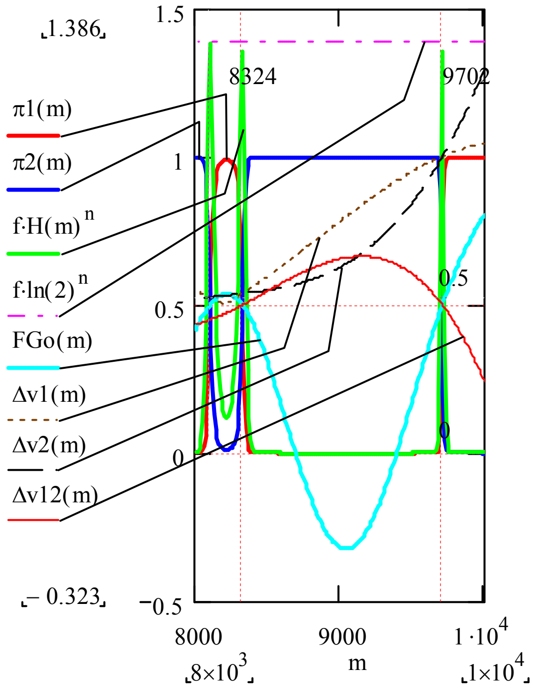

Fig. 2. Individual's preferences functions as the operational modes control and their subjective entropy formed by the effectiveness functions

In fig. 2 it is also illustrated: the function $FGo(m)$ – represents the relations between the functions of derivatives (4) and (5), it has the similar shape with π_1 in the corresponding scale; the functions $\Delta vl(m)$, $\Delta v2(m)$, and $\Delta v12(m)$ – they take into account the differences between the extremal speed of the flight and the first not extremal one determined from (4), the second speed of the flight also the one not extremal from (5), and between these not extremal speeds of the flight from (4) and (5) respectively.

Corresponding integrals of the flight duration found from (6) are illustrated in fig. 3, 4.

In fig. 3, 4, in the corresponding scale, it is represented: $Topt(m)$ – flight duration for the optimally combined of the two alternative speeds mode; $Tvl(m)$ – duration of the flight performed with the first speed of $v_1(m)$; $Tv2(m)$ –duration of the flight performed with the second speed of $v_2(m)$; $Tmopt(m)$ – extremal (maximal) flight duration obtained from (6) with the extremal of (1), it equals (2), or the maximal possible value of (2) when the upper end of the integration in (6) $m = M_E$.

Conditional equations system for the two alternative speeds will be

$$v(m)_{opt} = \begin{cases} v_1(m), \text{ if } \pi_1(m) \geq \pi_2(m); \\ v_2(m), \text{ otherwise.} \end{cases} \quad (8)$$

Accordingly to the alternatives (8) the integral of the flight duration will be formed as the linear combination from (6):

$$T(m) = \int_{M_0=10{,}000}^{9{,}702} -\frac{2\eta Q\rho v_1(m)S}{C_{x_0}\left(\rho[v_1(m)]^2 S\right)^2 + b(2mg)^2} dm +$$

$$+ \int_{9{,}702}^{8{,}324} -\frac{2\eta Q\rho v_2(m)S}{C_{x_0}\left(\rho[v_2(m)]^2 S\right)^2 + b(2mg)^2} dm +$$

$$+ \int_{8{,}324}^{8{,}102} -\frac{2\eta Q\rho v_1(m)S}{C_{x_0}\left(\rho[v_1(m)]^2 S\right)^2 + b(2mg)^2} dm +$$

$$+ \int_{8{,}102}^{M_E=8{,}000} -\frac{2\eta Q\rho v_2(m)S}{C_{x_0}\left(\rho[v_2(m)]^2 S\right)^2 + b(2mg)^2} dm . \quad (9)$$

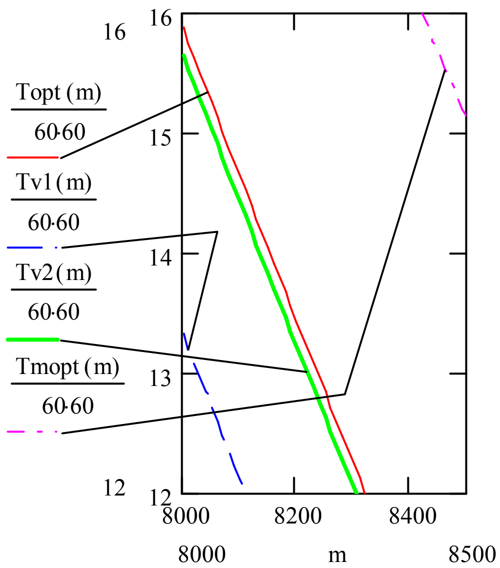

Fig. 3. Integral of the flight duration in the flying object mass change diapason of 8,500 … 8,000

For comparison, if the airplane can perform the flight either on the first mode or on the second mode only, that is without switching them, then the functional of the considered class will be

$$\Phi_\pi = H_\pi - \beta\left\{\pi_1 \int_{M_0}^{M_E} \frac{2\eta Q\rho v_1 S}{C_{x_0}\left(\rho v_1^2 S\right)^2 + b(2mg)^2} dm + \right.$$

$$\left. + \pi_2 \int_{M_0}^{M_E} \frac{2\eta Q\rho v_2 S dm}{C_{x_0}\left(\rho v_2^2 S\right)^2 + b(2mg)^2} \right\} + \gamma\left[\sum_{i=1}^{2} \pi_i - 1\right]. \quad (10)$$

The preferences from (10) will comprise their corresponding integrals from (10).

Then conditional system of (6) not likewise in the case of (9) will be

$$T(m)_{max} = \begin{cases} T_1[v_1(m)], \text{ if } \pi_1(m) \geq \pi_2(m); \\ T_2[v_2(m)], \text{ otherwise,} \end{cases} \quad (11)$$

where $T_1[v_1(m)]$ and $T_2[v_2(m)]$ – corresponding integrals of the type of (6).

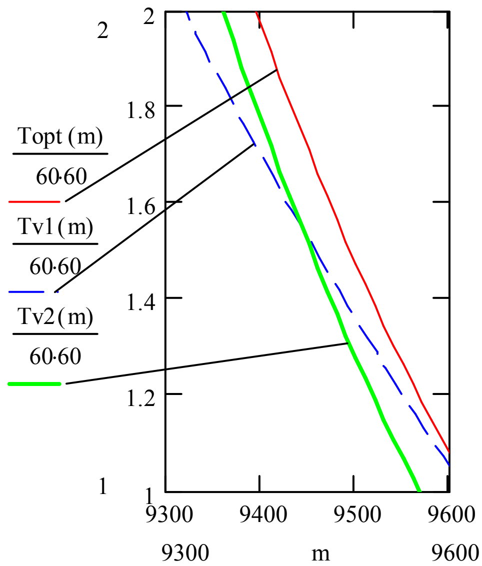

Fig. 4. Integral of the flight duration in the flying object mass change diapason of 9,600 … 9,300

The without shifting integrals in (11) will be

$$T_1(m) = \int_{M_0}^{m} -\frac{2\eta Q\rho v_1(x)S}{C_{x_0}\left(\rho[v_1(x)]^2 S\right)^2 + b(2xg)^2} dx , \quad (12)$$

$$T_2(m) = \int_{M_0}^{m} -\frac{2\eta Q\rho v_2(x)S}{C_{x_0}\left(\rho[v_2(x)]^2 S\right)^2 + b(2xg)^2} dx . \quad (13)$$

The results of the calculation experiments for the case of (10)-(13) are shown in fig. 5.

In fig. 5, in the corresponding scales, it is depicted: flight durations – $\mathrm{Topt}(m)$, $\mathrm{Tv1}(m)$, and $\mathrm{Tv2}(m)$, calculated by (2), (12), and (13) respectively, all derived from (6) with the related speeds; entropy – $H(m)$, of the related preferences functions of $\pi_1(m)$ and $\pi_2(m)$, obtained by the formulas (3) and (12) and (13) instead of (4) and (5) with the corresponding speeds and denoted with the marks of $\pi 1(m)$ and $\pi 2(m)$ respectively; the maximal value of the entropy: $\ln(2)$; and the value of the preferences intersections: 0,5.

3. The researches results

Analysis of the researches results will be made with the help of the illustrations fig. 1 – 5.

Optimal combined speed $\mathrm{Vopt}(m)$, fig. 1, makes us be inclined to assert, although scientifically substantiated before, achieved in this research on the basis of the other postulates of subjective entropy extrimizing principle:

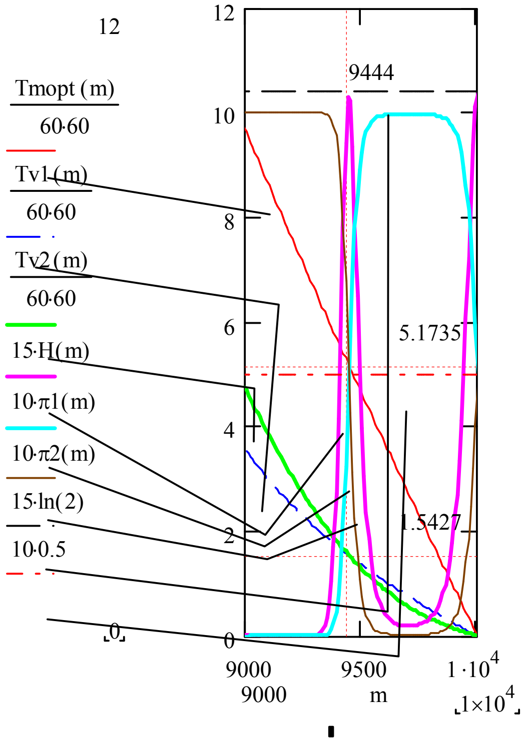

Fig. 5. Individual's preferences functions as the operational modes control and their subjective entropy formed by the effectiveness functions

– *"The closer the possible alternative mode of operation to the extremal one, the better"*;

– *"The optimal transitions from one operational mode to another one can be realized at the smooth or corner points of the intersections of the alternative operational modes, when they are positioned on the same side from the extremal; or these shifts have a skip character if the alternatives lie on the opposite sides from the extremal"*.

Analysis of the researches results illustrated in fig. 2 allows us asserting:

– *"The individual's preferences, showing the "right" alternative choice, react on the difference in the effectiveness functions regarding corresponding alternatives"*;

– *"The crossing of the levels of the indifference by the effectiveness functions difference is the necessary and sufficient conditions for the change of the operational modes of the both types (the corner points as well as the leaps)"*;

– *"The crossing of the levels of the indifference by the alternative functions difference itself is the necessary and sufficient conditions for the corner points change of the operational modes only"*.

Concerning the traces in fig. 3-5 and in the whole diapason of the independent variable possible values change it shows the expediency of that or another mode of operation. For instance, in compliance with the special case of the problem formulation (10)-(13), fig. 5. For example, at the mass value of 9,444, there is no difference between the alternative modes. For comparison, at this value with the extremal speed of the horizontal flight of the aircraft, the duration would be more than three times longer, the absolute values are 5,1735 instead of 1,5427.

Conclusions

On the basis of the researches results, (1)-(13), illustrated in fig. 1-5, we can come to the conclusions that: *"Subjective entropy extremization principle allows; independently on the conditions of* ***transversality****,* ***Weierstrass-Erdmann****, from the calculus of variations, as well as independently on the principle of maximum by* ***L.S. Pontryagin*** *(USSR), also principle of optimality by* ***R. Bellman*** *(USA); finding the extremals, their optimal conjunctions of all kinds: either breaks with shifts, or both at smooth and corner points, for closed and restricted areas; stipulated by obeying the only a priory condition of the* ***Euler-Lagrange*** *equations"*.

There is no objections to call this newly invented *"Subjective entropy extremization principle"* by the name of its author, professor **Vladimir Aleksandrovich Kasianov**, National Aviation University (Kyiv, Ukraine).

Prospects of further researches. It is important to investigate other types of functionals of the kind of (9), (10) as well as with the different functions of the effectiveness of the sort of (4), (5), also research operational modes of control for horizontal flights of maximal distance subject to multi-alternativeness and conflicts.

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ПРИНЦИП ЭКСТРЕМИЗАЦИИ СУБЪЕКТИВНОЙ ЭНТРОПИИ КАК ИНСТРУМЕНТ УПРАВЛЕНИЯ МАКСИМАЛЬНОЙ ПРОДОЛЖИТЕЛЬНОСТЬЮ ГОРИЗОНТАЛЬНОГО ПОЛЕТА САМОЛЕТА

А.В. Гончаренко

Эта статья продолжает ранее начатые исследования оптимального управления в активных системах. В приложении к решению проблемы управления горизонтальным полетом самолета максимальной продолжительности предложено использовать принцип экстремизации энтропии индивидуальных субъективных предпочтений, в качестве удобного инструмента. Опираясь лишь на этот принцип, можно обнаружить оптимальный режим эксплуатационного управления, даже не зная экстремального. Поскольку принцип экстремизации субъективной энтропии позволяет; независимо от условий трансверсальности, Вейерштрасса-Эрдманна, принципа максимума Л.С. Понтрягина (СССР), а также принципа оптимальности Р. Беллмана (США); нахождение экстремалей, их оптимальных соединений всех типов: либо разрывных со смещением, либо, как гладких, так и угловых, для закрытых и ограниченных областей; обусловленное единственно априорным условием уравнений Эйлера-Лагранжа; предлагается назвать этот принцип именем его автора, профессора Владимира Александровича Касьянова, Национальный авиационный университет (Киев, Украина). Выполнены необходимые расчеты. Построены соответствующие диаграммы.

Ключевые слова: максимальная продолжительность, горизонтальный полет, субъективная энтропия, индивидуальные предпочтения, многоальтернативность, управление активными системами, активный элемент, вариационная задача, эксплуатационный функционал.

ПРИНЦИП ЕКСТРЕМІЗАЦІЇ СУБ'ЄКТИВНОЇ ЕНТРОПІЇ ЯК ІНСТРУМЕНТ КЕРУВАННЯ МАКСИМАЛЬНОЮ ТРИВАЛІСТЮ ГОРИЗОНТАЛЬНОГО ПОЛЬОТУ ЛІТАКА

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Ця стаття продовжує раніше розпочаті дослідження оптимального керування в активних системах. У застосуванні до розв'язання проблеми керування горизонтальним польотом літака максимальної тривалості запропоновано використовувати принцип екстремізації ентропії індивідуальних суб'єктивних переваг, у якості зручного інструменту. Спираючись лише на той принцип, можливо виявити оптимальний режим експлуатаційного керування, навіть не знаючи екстремального. Оскільки принцип екстремізації суб'єктивної ентропії дозволяє; незалежно від умов трансверсальності, Вейерштрасса-Ердманна, принципу максимуму Л.С. Понтрягіна (СРСР), а також принципу оптимальності Р. Беллмана (США); знаходження екстремалей, їхніх оптимальних з'єднань усіх типів: або розривних зі зміщенням, або, як гладких, так і кутових, для закритих та обмежених областей; обумовлене єдине апріорною умовою рівнянь Ейлера-Лагранжа; пропонується назвати цей принцип іменем його автора, професора Володимира Олександровича Касьянова, Національний авіаційний університет (Київ, Україна). Виконано необхідні розрахунки. Побудовано відповідні діаграми.

Ключові слова: максимальна тривалість, горизонтальний політ, суб'єктивна ентропія, індивідуальні переваги, багатоальтернативність, керування активними системами, активний елемент, варіаційна задача, експлуатаційний функціонал.

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