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SUBSIDY DIFFERENTIATION AND THE ROUTING PARTIAL COMPATIBILITY IN A DUOPOLY MARKET

The paper states that elderly and impaired people require a barrier-free transport in Taiwan. The current transport modes should be adjusted to meet financial limits in social welfare supports. Furthermore, based on some protection laws, government needs to subsidy public and private bus firms. In this paper, we build a model considering subsidy differentiation and the routing partial compatibility to investigate supporting services in a duopoly transport market. We find that subsidy and the degree of partial compatibility can significantly influence bus firms' routing profiles. Practical implications of the findings are also discussed.

Keywords: *subsidy differentiation; partial compatibility; supporting services; demand responsive transport services.*

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ДИФЕРЕНЦІАЦІЯ СУБСИДІЙ І ЧАСТКОВА СУМІСНІСТЬ МАРШРУТІВ НА ДУОПОЛЬНОМУ РИНКУ

У статті обгрунтовано, що на Тайвані необхідно зняти обмеження на проїзд у транспорті літнім людям і людям з інвалідністю. Наявні види транспорту мають бути скориговані з урахуванням фінансових обмежень у соціальних програмах. Крім того, на основі деяких законів про соціальний захист уряд повинен ефективно субсидувати державні і приватні автобусні фірми. Для цього з урахуванням диференціації субсидій і часткової сумісності маршрутів побудовано модель з метою дослідження допоміжних послуг на дуопольному транспортному ринку. Доведено, що субсидії і ступінь часткової сумісності можуть істотно вплинути на види маршрутів автобусної фірми. Також обговорено практичну значущість результатів.

Ключові слова: *диференціація субсидій; часткова сумісність; допоміжні послуги; попитозалежні транспортні послуги.*

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ДИФФЕРЕНЦИАЦИЯ СУБСИДИЙ И ЧАСТИЧНАЯ СОВМЕСТИМОСТЬ МАРШРУТОВ НА ДУОПОЛЬНОМ РЫНКЕ

В статье обосновано, что на Тайване необходимо снять ограничения на проезд в транспорте пожилым людям и людям с инвалидностью. Имеющиеся виды транспорта должны быть скорректированы с учетом финансовых ограничений в социальных программах. Кроме того, на основе некоторых законов о социальной защите правительство должно эффективно субсидировать государственные и частные автобусные фирмы. Для этого с учетом дифференциации субсидий и частичной совместимости маршрутов построена модель с целью исследования вспомогательных услуг на дуопольном транспортном рынке. Доказано, что субсидии и степень частичной совместимости могут существенно повлиять на виды маршрутов автобусной фирмы. Также обсуждена практическая значимость результатов.

Ключевые слова: *дифференциация субсидий; частичная совместимость; вспомогательные услуги; спросозависимые транспортные услуги.*

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Introduction. Public transportation is essential for economic development of a country. From 1946, bus service has played a prominent role in the economic growth of Taiwan. However, Wang et al. (2009) illustrated that the population of elderly persons (more than 65 years old) in Taiwan is estimated to be 2 mln. in 2018 (see Table 1). Furthermore, in recent years, the accessibility and mobility to transport of impaired persons are gradually emphasized in the society of Taiwan, this is why elderly and impaired people require a barrier-free transport in Taiwan. As a result of the announcement of the Act of people with disabilities rights protection in 2009, public and private bus firms have put efforts to setup a system of demand responsive transport services (DRTS) in Taiwan. DRTS, also named dial-a-ride, flexible transportation or special transportation system, presents multi-applied transport service of paratransit mode (Kirby et al., 1974). Cervero (1997) sorted the service configuration of DRTS into the following 4 routing profiles: many-to-many, many-to-few/few-to-many, few-to-few, and many-to-one/few-to-one. Many-to-many routing profile means that a provider places no constraints on the type of trips it handles; i.e. origins and destinations are random and can happen anywhere. Many-to-few routing profile means that a provider has random origins and only regular destinations (hospitals, sightseeing places etc.). DRTS in Taiwan, also named Fu-Kang bus service (i.e., medical transportation), usually makes use of few-to-few or few-to-one routing profile carrying impaired people to hospitals. Origin and destination points of impaired people are usually from home to hospitals or clinic centers. Kittelson et al. (2004) stated that DRTS services can be explained and defined in terms of both "modes of firm" and "types of market".

In recent years, the studies of Fang-Kang bus and taxi services have focused on the routing planning characteristics and optimized solutions analysis used to improve the operational efficiency of vehicle fleets for bus firms and the service qualities for users. Huang and Hsu (2009) and Shyr et al. (2009) studied the optimization of operational costs, service quality, riders demand and efficiency under the carpool condition. In addition, Chang (2004) analyzed the integration of the Fu-Kang bus service and Mass Rapid Transit (MRT) service. Moreover, political economy and behavior choice were also discussed in the above issue. For instance, Chang (2008) used the DEMATEL and the cluster analysis models to explain the taxi market policies. Lin (2005) and Gi (2010) studied the vehicle choice of the elderly people and analyzed the transport demand of DRTS. Chang and Chu (2008) discussed the relationship between the choice behavior of riders who are willing to pay fee and operational costs of taxi.

According to the Act of people with disabilities right protection, Ministry of transportation and communications (MOTC) is giving new impetus to promote a perfect barrier-free transport service and door-to-door service in order to benefit accessibility and mobility of impaired people. For some reasons there are difficulties still needed to be overcome. Firstly, impaired people are a social exclusion group, government usually does not support them with sufficient social welfare budgets. Secondly, impaired people are a minority group; i.e., they have no "economics of agglomeration" effect, transport providers do not want to enter this poor market. Thirdly, most of costs of Fu-Kang bus are supported by the social welfare budgets and the patronage of the social welfare organization, the riders or policy makers easily neglect the need to use vehicle resources efficiently and to optimize these vehicle resources. In addition, according to the Law of people with disabilities having a discount rate to ride public transport vehicle, people with disabilities favor a 50% discount rate to ride a domestic transport vehicle and a discount rate of one-way trip based on regulated taxi rate from 50% to 66.7% to ride a Fu-Kang bus, however, the operational incentive of providers is still needed to be increased.

We observe that product and transport service compatibility have the analogous structure in differentiated industry. Farrell and Saloner (1987) indicated that compatibility was significantly studied in industrial economics. For instance, in the industrial revolution period, the use of interchangeable parts of railroad gauges was an essential step. Nowadays, with a rapid growth of information technologies, academic economists have noticed the importance of product compatibility. Chou and Shy (1993) found that compatibility with other brands varies among brands in the same industry. For example, some Apple machines can read DOS diskettes but DOS machines cannot read Apple format. Economides (1996) analyzed the economic features of networks. He illustrated the structure of a network, that many components of a network are demanded for the provision of a representative service. Moreover, he gave an example of information superhighway network to explain that services demanded by consumers consist of many complementary components.

The aim of this paper is to establish a model for the DRTS with the partial compatibility and supporting service (e.g., Chou and Shy, 1993) in a duopoly transport market, in particular, for vanpool taxi and the Fu-Kang bus service. This both services provide a differentiated routing profile. In addition, accessibility and mobility of riders with routing profiles and subsidy are considered. Moreover, we assume that same total budget is spent on a vehicle and routing profile of DRTS for the sake of simplicity. By doing this, we are able to analyze the issue mentioned above.

The paper is organized as follows. In the next section, we use the concept of partial compatibility that includes the element of differentiated subsidy and routing profiles to present the relations between the partial compatibility of routing profiles, a degree of compatibility and subsidy. In addition, we use sub-game perfect equilibrium method to solve the problem. Relationships and implications in subsidy, partial compatibility of routing profiles and the degree of compatibility are discussed in Section 3. The major findings and suggestions are in Section 4.

Table 1. **Forecast on the population over the age of 65 in Taiwan**

Year	Over the age of 65, %	The structure of 65 age (%)		Over the age of 65 (thousand)	
		65-74	Over 75	65-74	Over 75
2008	10.4	56.9	43.1	1,365	1,032
2013	11.6	54.6	45.4	1,478	1,231
2018	14.7	58.3	41.7	2,028	1,452
2023	18.5	62.7	37.3	2,758	1,644
2028	22.5	58.7	41.3	3,147	2,215
2031	27.9	50.0	50.0	3,271	3,271
2041	34.0	45.3	54.7	3,399	4,111
2051	37.5	40.3	59.7	3,069	4,547

Source: Wang, L. et al. (2009).

The Model. Consider riders who can freely choose a public or a private bus firm (Fu-Kang bus or Vanpool taxi). Each rider is endowed with Y dollars to be spent on a vehicle and routing profile of DRTS. We denote by the vehicle price of public and the private bus firm i, i=A,B. Hence, given a total budget of Y, a rider choosing a bus firm i spends E_i=Y—p_i on i's routing profile of DRTS. In addition, when people with disabilities ride a domestic transport vehicle, they are favored a discount rate based on the aforementioned law. We denote by θ_k the government's subsidies, the routing profiles of the public and the private bus firm k, k=1,2 and the subsidy rate is $0<\theta_k<1$. Hence, a rider choosing a bus firm i spends $\theta_k E_i$=θ_k(Y—p_i) on i's routing profile of DRTS. We denote by N_i the total number of routing profile that can be chosen by the firm A or firm B. The utility of a rider choosing a bus firm is defined as an increasing function of the number of routing profile compatible with two bus firms i, i=A,B. Riders are uniformly indexed by δ on the interval [0,1] according to their relative preference towards firm — B . We define the utility of a rider type δ as

$$U^{\delta} = \begin{cases} (1-\delta)(\theta_1 N_A)^2 \\ \delta(\theta_1 N_B)^2 \end{cases} \tag{1}$$

Thus, the utility function (1) explains preferences display "riders desire which routing profiles to use". That is, a rider's preferences towards a particular routing profile are affected by a fixed parameter, (δ or (1—δ)), and by the number of routing profiles available as public or private. Moreover, the government gives them differentiated subsidies that is $0<\theta_k<1$,k=1,2, and $\theta_1>\theta_2$. The more routing profiles that the rider can choose, the more the utility of a bus firm increases.

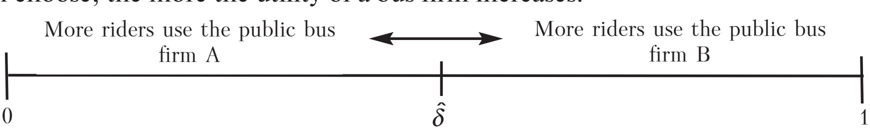

Figure 1. **The distribution of riders' tastes**

The riders who is indifferent to the choice between firm A and firm B is denoted by δ, which is found from equation (1) by solving

$$(1—\delta)(\theta_1 N_A)^2=\delta(\theta_1 N_A)^2 \tag{2}$$

Thus, in equilibrium, a rider indexed by $\delta<\hat{\delta}$ is a rider of using firm A. On the contrary, a rider indexed by $\delta_A \equiv \hat{\delta}$ is a rider of using firm B. The total number of riders is denoted by $\delta_A \equiv \hat{\delta}$, and the total number of riders is denoted by $\delta_B \equiv (1-\hat{\delta})$. Totally,

$$\delta_B/\delta_A=(1- \hat{\delta})/\hat{\delta}=(\theta_2 N_B/\theta_1 N_A)^2 \tag{3}$$

Hence, Proposition 1. The firm with the higher market share is supported by various routing profiles and subsidies. Formally, $\delta_B<\delta_A$ if and only if $\theta_2 N_B<\theta_1 N_A$.

Proposition 1 indicates wide relations between subsidies and routing profiles. For instance, the public bus firm has the largest market share and is supported by a variety of routing profiles and government subsidies.

The Routing Profiles. Now, the market share of DRTS is not so large due to medical transport for impaired people. The bus firms require as less as possible expenditure of operational costs. They face the dilemma of their operation sustainability and less social welfare budgets. We conjecture that the number of routing profiles supported by bus firms should be proportional to the aggregate amount of money spent on each of the profiles, and therefore we make the following assumption:

Assumption 1. The number of differentiated routing profiles supported by public and private bus firms is proportional to the aggregate expenditure of the riders pricing for one routing profile type.

In addition, both bus firms and riders acquire subsidy from government when riders choose the routing profile type.

Formally,

$$\theta_1 N_A = \theta_1 \hat{\delta} E_A = \theta_1 \hat{\delta}(Y - P_A). \tag{4}$$

$$\theta_2 N_B = \theta_2 (1-\hat{\delta}) E_B = \theta_2 (1-\hat{\delta})(Y - P_B). \tag{5}$$

Substituting equations (4) and (5) into eq. (3) yields

$$\hat{\delta} = \frac{\theta_1 E_A}{\theta_1 E_A + \theta_2 E_B} = \frac{\theta_1 (Y - P_A)}{\theta_1 (Y - P_A) + \theta_2 (Y - P_B)} = \frac{\theta_1 Y - \theta_1 P_A}{(\theta_1 + \theta_2) Y - \theta_1 P_A - \theta_2 P_B}. \tag{6}$$

Network Effects. Proposition 2 demonstrates the network effect without network externalities.

Proposition 2. An increase in the subsidy of the firm A (θ_1)

(1) increases the number of A riders(δ_A);

(2) decreases the number of B riders(δ_B);

(3) increases the variety of routing profiles (N_A) used for the firm A and decrease the variety of routing profiles (N_B);

(4) increases the welfare of A riders and decrease the welfare of B riders.

Proof. Proposition 2-(1) follows form equation (6) since $\partial\hat{\delta}/\partial\theta_1 > 0$. Proposition 2-(2) immediately follows since δ_B=1—δ_A. Proposition 2-(3) follows from equation (4) since as $\hat{\delta}$ increases and θ_1 increases, it is implied that N_A must increase while N_B must decrease. Proposition 3-(4) follows from equation (1), since an increase in N_A increases the utility of an A rider, however, decrease in N_B decreases the utility of a B rider.

When θ_1 increases, equations (4) and (5) implies that two factors exist that cause the variety of N_A expanded: First, the direct effect that is $Y—P_A$ increased, riders spend less on vehicle and spend more on routing profiles; and second, the indirect effect via increases in the number of A riders that are $\hat{\delta}$ increase. Equation (6) also implies that N_B decreases since there are less B users.

Proposition 2-(3) demonstrates the network effect generated by an increase in the firm A subsidy (θ_1) on the welfare of B riders, that is

$$\theta_1 \uparrow \Rightarrow \delta_A \uparrow \Rightarrow \delta_B \downarrow \Rightarrow N_B \downarrow \Rightarrow U^{B-riders} \downarrow.$$

An increase in the number of A riders causes a decrease in the number of B riders, which in turn decreases the routing profile of N_B, which decreases B riders.

Partial Compatibility. Shy(1993) cited an instance that a computer manufacturer affirms his computer that is DOS compatible, there are always some packages of software that can operate on one machine, but reject to operate on another computer. He concluded that 100 % compatibility is not a benefit for a computer manufacturer. Partial compatibility of package software is possible on one machine. We adopt the concept of partial compatibility and use the supporting-services approach to model network and interpret for modeling the routing profiles of DRTS of partial compatibility. The number of routing profiles chosen specifically for the bus firm *i* is denoted by n_i, $i=A,B$. The main feature of this model is that bus firms are able to be partially compatible in the sense that in addition to its routing profile, a public or private bus firm can also run a selected number of routing profiles operated by its rival firm. We set λ_i, $0<\lambda_i<1$ and measure the exogenously given degree of compatibility of firm *i* with respect to *j's* routing profile. That is, λ_i measures the proportion of *j* routing profile that can be run on the *i* firm, $i, j=A,B$ and $i \neq j$. In addition, we consider that subsidies $0<\theta_k<1$, $k=1,2$ of routing profile when a rider uses any bus firm. Therefore, the total number of routing profiles available to an *i*-firm rider is equal to

$$\theta_1 N_A = \theta_1(n_A + \lambda_A n_B). \tag{7}$$

$$\theta_2 N_B = \theta_2(n_B + \lambda_B n_A). \tag{8}$$

Equations (4) and (5) substitute into equation (7) and (8) yields

$$\theta_1 n_A = \theta_1((Y - P_A) - \lambda_A n_B). \tag{9}$$

$$\theta_2 n_B = \theta_2((Y - P_B) - \lambda_B n_A). \tag{10}$$

Competition Perspective. Norman and Thisse (2000) illustrated that the maintaining competition is a right thing which warrants protection between individual freedom, rights and limits the power of agents. On the other hand, competition is as a mechanism for allocating resources which promotes economic efficiency.

We assume the game that a rider chooses A or B individually. Afterward we search for the subgame perfect equilibrium in both games with respect to between partial compatibility of routing profile and subsidies. Without loss of generality, we neglect the cost, bus firm i(i=A,B) maximizes

$$\Pi_A^* = (1-\hat{\delta})\theta_1 N_A P_A. \tag{11}$$

$$\Pi_B^* = \hat{\delta}\theta_2 N_B P_B. \tag{12}$$

with respect to P_i, we solve the reaction functions:

$$P_A = R^p{}_A(P_B) = Y/2 + \theta_2(Y - P_B)/2\theta_1 \tag{13}$$

and

$$P_B = R^p{}_B(P_A) = Y/2 + \theta_1(Y - P_A)/2\theta_1 \tag{14}$$

Nash equilibrium of the subgame satisfies $P^*_i = P^p_i(P^*_j)$ which implies that:

$$P_A^* = Y(1/3 + \theta_2/3\theta_1). \tag{15}$$

$$P_B^* = Y(1/3 + \theta_1/3\theta_2). \tag{16}$$

Hence, we have the following proposition.

Proposition 3.

(1) The vehicle price P^*_A depends on subsidy θ_1, θ_2.

(2) Also, the vehicle price P^*_B depends on subsidy θ_1, θ_2.

Proposition 3-(1) interprets that when θ_1 increases; i.e. firm A obtains subsidies from government, then P^*_A decreases; i.e. riders can pay less price for riding a bus. On the other hand, when θ_2 increases; i.e. firm B obtains subsidies from government, then P^*_B increases; i.e. riders are willing to pay more price for riding the firm A.

Similarly, proposition 3-(2) highlights that when θ_2 increases; i.e. firm B obtains subsidies from government, P^*_B decreases; i.e. the riders can pay less price for riding a bus. On the other hand, when θ_1 increases; i.e. firm A obtains subsidies from government, then P^*_A increases; i.e. riders pay more price for riding private buses. The reason why riders prefer to pay more prices for riding a private bus is because they feel importance of their privacy.

Finally, in order to show the above relationship more precisely, we plot the subsidies range (grey area) between the prices of 2 bus firms in the following figures.

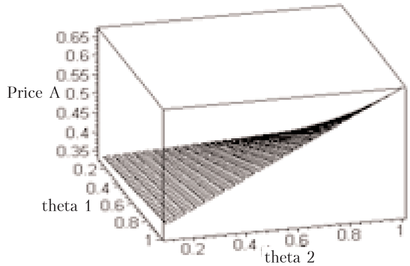

Figure 2. **0<θ_1<1, 0<θ_1<1, the range of P^*_A**

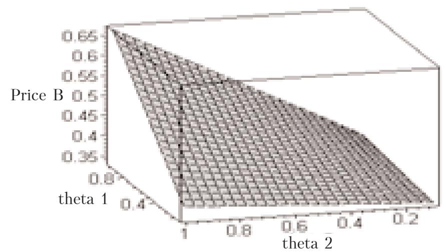

Figure 3. **0<θ_1<1, 0<θ_1<1, the range of P^*_B**

Figure 2 shows the range of P^*_A is approximately $0.35<P^*_A<0.65$ that is based on the subsidy of bus firm $0<\theta_1<1$ and $0<\theta_2<1$. Similarly, Figure 3 illustrates the range of P^*_B is approximately $0.35<P^*_A<0.65$ that is based on the subsidy of bus firm $0<\theta_1<1$ and $0<\theta_2<1$. Due to the symmetric relationship between P^*_A and P^*_B, we obtain that $P_A^* = P_B^* \approx 0.67$ is the equilibrium value when the subsidy is $\theta_1=\theta_2$. These findings are similar to the discount rate which is from 50% to 66.7% when a rider uses a Fu-Kang bus in Taiwan.

Subsidy (θ_k) and Partial Compatibility of Routing Profiles (n_i). We consider further network effects among subsidy θ_1, θ_2 and degree of compatibility λ_A, λ_B and the number of routing profiles chosen n_A, n_B. We substitute equations (9) and (10) into equations (13) and (14) yielding

$$2\theta_1 P_A = \theta_1 Y + \theta_2 (n_B + \lambda_B n_A). \tag{17}$$

$$2\theta_2 P_B = \theta_2 Y + \theta_1 (n_A + \lambda_A n_B). \tag{18}$$

From equations (17) and (18), it is easy to obtain n_A, n_B as follow:

$$n_A = \frac{Y(\theta_1^2 \lambda_A - \theta_2^2 + 2\theta_1\theta_2(1-\lambda_A))}{3\theta_1\theta_2(1-\lambda_A\lambda_B)}. \tag{19}$$

$$n_B = \frac{Y(\theta_2{}^2\lambda_B - \theta_1{}^2 + 2\theta_1\theta_2(1-\lambda_B))}{3\theta_1\theta_2(1-\lambda_A\lambda_B)}. \qquad (20)$$

According to equations (19) and (20), we can obtain effects of θ_1, θ_2 on n_A, n_B as follow:

$$\partial n_A/\partial\theta_1 = 2/3\theta_2(1-\lambda_A\lambda_B). \qquad (21)$$

$$\partial n_A/\partial\theta_2 = -2/3\theta_1(1-\lambda_A\lambda_B). \qquad (22)$$

$$\partial n_B/\partial\theta_1 = -2/3\theta_2(1-\lambda_A\lambda_B). \qquad (23)$$

$$\partial n_B/\partial\theta_2 = 2/3\theta_1(1-\lambda_A\lambda_B). \qquad (24)$$

We compare equations (21) and (23). Given θ_2 and $1—\lambda_A\lambda_B>0$, we have

$$\partial n_A/\partial\theta_1 > \partial n_B/\partial\theta_1. \qquad (25)$$

Similarly, we compare equations (22) and (24). Given θ_1 and $1—\lambda_A\lambda_B>0$, then we have

$$\partial n_A/\partial\theta_2 < \partial n_B/\partial\theta_2.$$

From equation (25), we can see that the effect of θ_1 on n_A is larger than that of n_B. On the other hand, we can see that the effect of θ_2 on n_B is larger than that of n_A. The intuition behind this result is that subsidy has different effects on the partial compatibility of routing profiles. If subsidy θ_1 increases then the partial compatibility of firm A's routing profiles increases that would lead to increase of n_A. However, if the rival subsidy θ_2 increases then the partial compatibility of firm A's routing profiles decreases.

Effects of partial compatibility of routing profiles (n_i), degree of compatibility (λ_i) and subsidy (θ_k). According to equations (19) and (20), we can obtain the following the effects of λ_A, λ_B on n_A, n_B:

$$\frac{\partial n_A}{\partial\lambda_A} = \frac{\theta_1(\theta_2\lambda_B + \theta_1 Y - 2\theta_2 Y)\lambda_A + \theta_2(2\theta_1 Y - \theta_1 - \theta_2 Y)}{3\theta_1{}^2\theta_2{}^2(1-\lambda_A\lambda_B)^2}. \qquad (27)$$

$$\frac{\partial n_A}{\partial\lambda_B} = Y * \frac{\theta_1(\theta_1 - 2\theta_2)\lambda_A + \theta_2(2\theta_1 - \theta_2)}{3\theta_1{}^2\theta_2{}^2(1-\lambda_A\lambda_B)^2}. \qquad (28)$$

$$\frac{\partial n_B}{\partial\lambda_A} = Y * \frac{(\theta_2{}^2 - 2\theta_1\theta_2)\lambda_B + 2\theta_1\theta_2 - \theta_1{}^2}{3\theta_1{}^2\theta_2{}^2(1-\lambda_A\lambda_B)^2}. \qquad (29)$$

$$\frac{\partial n_B}{\partial\lambda_B} = \frac{\theta_2\lambda_B(\theta_1\lambda_A - 2\theta_1 Y + \theta_2 Y) + \theta_1(2\theta_2 Y - \theta_2 - \theta_1 Y)}{3\theta_1{}^2\theta_2{}^2(1-\lambda_A\lambda_B)^2}. \qquad (30)$$

From equations (27) to (30), we establish the following proposition:

Proposition 4.

(1) when $1/2\ \theta_2<\theta_1<2\theta_2$, λ_B is sufficiently small, λ_A is sufficiently large, $\partial n_A/\partial\lambda_A > 0$.

(2) when $1/2\ \theta_2<2\theta_2<\theta_1$, λ_B, is sufficiently large, λ_A is sufficiently small, $\partial n_A/\partial\lambda_A > 0$.

(3) when 1/2 $\theta_2<\theta_1<2\theta_2$, λ_A is sufficiently large, $\partial n_A/\partial\lambda_B>0$.

(4) when 1/2 $\theta_2<2\theta_2<\theta_1$, λ_A is sufficiently small, $\partial n_A/\partial\lambda_B>0$.

(5) when 1/2 $\theta_2<\theta_1<2\theta_2$, λ_B is sufficiently large, $\partial n_B/\partial\lambda_A>0$.

(6) when 1/2 $\theta_2<2\theta_2<\theta_1$, λ_B is sufficiently small, $\partial n_B/\partial\lambda_A>0$.

(7) when 1/2 $\theta_2<\theta_1<2\theta_2$, λ_A is sufficiently small, λ_B is sufficiently large, $\partial n_B/\partial\lambda_B>0$.

(8) when 1/2 $\theta_2<2\theta_2<\theta_1$, λ_A is sufficiently large, λ_B is sufficiently small, $\partial n_B/\partial\lambda_B>0$.

Propositions 4-(1) and (7) interpret that under the given subsidy constraint and $\lambda_B(\lambda_A)$ is sufficiently small and $\lambda_A(\lambda_B)$ is sufficiently large, each bus firm's degree of partial compatibility has a positive effect on its own routing profiles. By contrast, propositions 4-(2) and (8) interpret that each bus firm's degree of partial compatibility has a negative effect on its rival bus firm's routing profiles under different partial compatibility degrees and subsidy levels. On the contrary, propositions 4-(3) and (5) highlight that under the given constraint of subsidy and $\lambda_A(\lambda_B)$ is sufficiently large, each firm's degree of partial compatibility has a positive impact on the rival's routing profiles. By contrast, propositions 4-(4) and (6) highlight that each firm's degree of partial compatibility has a negative impact on the rival bus firm's routing profiles under different partial compatibility degrees as above. Finally, we show the above relationships more clearly, we use the Maple 6 software to plot their subsidies range (grey area) between partial compatibility of routing profiles in the following figures.

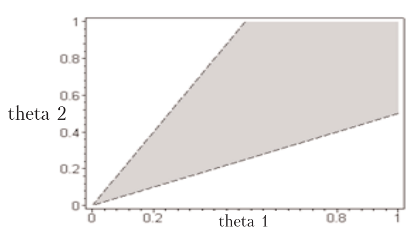

Figure 4. **The subsidy range: 1/2 $\theta_2<\theta_1<2\theta_2$**

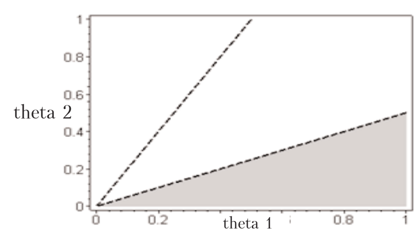

Figure 5. **The subsidy range: 1/2 $\theta_2<2\theta_2<\theta_1$**

As can be seen in Figures 4 and 5, when the subsidy range is 1/2 $\theta_2<\theta_1<2\theta_2$, $\partial n_A/\partial\lambda_A>0$, $\partial n_A/\partial\lambda_B>0$, $\partial n_B/\partial\lambda_A>0$ and $\partial n_B/\partial\lambda_B>0$. When the subsidy range is 1/2 $\theta_2<\theta_1<2\theta_2$, $\partial n_A/\partial\lambda_A>0$, $\partial n_A/\partial\lambda_B>0$, $\partial n_B/\partial\lambda_A>0$ and $\partial n_B/\partial\lambda_B>0$.

Difference of Partial Compatibility of Routing Profiles (n_i), degree of compatibility (λ_i) and subsidy (θ_i). Now we investigate the difference of n_A/λ_A between n_B/λ_B under the different scopes of subsidy θ_1,θ_2 we obtain

$$\frac{\partial n_A}{\partial\lambda_A}-\frac{\partial n_B}{\partial\lambda_A}=\frac{Y\theta_1^2(\lambda_A+1)-Y\theta_2^2(\lambda_B+1)+2Y\theta_1\theta_2(\lambda_B-\lambda_A)-\theta_1\theta_2}{3\theta_1^2\theta_2^2(1-\lambda_A\lambda_B)^2}. \tag{31}$$

$$\frac{\partial n_A}{\partial\lambda_B}-\frac{\partial n_B}{\partial\lambda_B}=\frac{Y\theta_1^2(\lambda_A+1)-Y\theta_2^2(\lambda_B+1)+2Y\theta_1\theta_2(\lambda_B-\lambda_A)+\theta_1\theta_2}{3\theta_2^2\theta_1^2(1-\lambda_A\lambda_B)^2}. \tag{32}$$

From Equations (31) to (32), we establish the following proposition:

Proposition 5

(1) when 1/2 $\theta_2<2\theta_2<\theta_1$, λ_A and Y are sufficiently large, λ_B is sufficiently small, $\frac{\partial n_A}{\partial \lambda_A}-\frac{\partial n_B}{\partial \lambda_A}>0$.

(2) when 1/2 $\theta_2<\theta_1<2\theta_2$, λ_B is sufficiently large, λ_A and Y is sufficiently small, $\frac{\partial n_A}{\partial \lambda_A}-\frac{\partial n_B}{\partial \lambda_A}>0$.

(3) when 1/2 $\theta_2<2\theta_2<\theta_1$, λ_A is sufficiently large, λ_B and Y is sufficiently small, $\frac{\partial n_A}{\partial \lambda_A}-\frac{\partial n_B}{\partial \lambda_A}>0$.

(4) when 1/2 $\theta_2<2\theta_2<\theta_1$, λ_B and Y are sufficiently large, λ_A is sufficiently small, $\frac{\partial n_A}{\partial \lambda_A}-\frac{\partial n_B}{\partial \lambda_A}>0$.

Propositions 5-(1) and (4) interpret that under the limited scope of subsidy, the degree of partial compatibility of own bus firm is more than the rival's one, the own routing profiles are bigger than the rival's ones. Moreover, total budget Y with respect to E is a rider choosing bus firm i spends $\theta_k E_i \equiv \theta_k (Y - p_i)$ on i's routing profile of DRTS. A bus firm plans more routing profiles, riders are willing to spend more money for riding them. On the contrary, propositions 5-(2) and (3) highlight that under the constraint of subsidy, the degree of partial compatibility of the rival's bus firm is more than the own one, moreover, Y is diminished, it means that a rider is willing to spend less money for riding the bus firm which plans less routing profiles.

Numeral Experiments. In this subsection, we further confirm our findings above precisely so that our model can have more practical implications. The description of number experiments is the following. Firstly, in aforementioned section, impaired people are favored a 50% discount rate to ride a domestic transport vehicle and 50% to 66.7% taxi-rate-based discount to ride a Fu-Kang bus. In addition, we calculated the range of price value of equations (15) and (16) in terms of subsidy of a bus firm $0<\theta_1<1$ and $0<\theta_2<1$. P^*_A, P^*_B are approximately $0.35<P^*_A<0.65$, $0.35<P^*_B<0.65$. The equilibrium value is $P_A^* = P_B^* \approx 0.67$. Secondly, we obtained the result of

$$\partial n_A / \partial \lambda_A > 0,$$
$$\partial n_A / \partial \lambda_B > 0,$$
$$\partial n_B / \partial \lambda_A > 0,$$
$$\partial n_B / \partial \lambda_B > 0.$$

caused by the relation 1/2 $\theta_2<\theta_1<2\theta_2$ and the result of

$$\partial n_A / \partial \lambda_A < 0,$$
$$\partial n_A / \partial \lambda_B < 0,$$
$$\partial n_B / \partial \lambda_A < 0,$$
$$\partial n_B / \partial \lambda_B < 0.$$

caused by the relation 1/2 $\theta_2<2\theta_2<\theta_1$. Finally, according to these range, we assume that the adaptive values of θ_1, θ_2, λ_A and λ_B in equations (31) and (32) and list below as Tables 2 and 3. The value θ_1, θ_2, λ_A and λ_B are in equation (31) among $0.175>\theta_1>0.690$, $0.179>\theta_2>0.204$, $0.54>\lambda_A>0.515$, $0.416>\lambda_B>0.441$ and are in equation (32) among $0.29<\theta_1<0.32$, $0.670<\theta_2<0.673$, $0.296<\lambda_A<0.326$, $0.71<\lambda_B<0.68$.

According to the adaptive values of equations (31) and (32), we carried out the numeral experiments and showed the results in the following Figures 5, 6, 7 and 8.

Table 2. **θ_1, θ_2, λ_A, λ_B of equation (31)**

θ_1	θ_2	λ_A	λ_B	Equation (31)
0.715	0.179	0.540	0.416	0.99803442
0.714	0.18	0.539	0.417	0.95253265
0.713	0.181	0.538	0.418	0.90761358
0.712	0.182	0.537	0.419	0.86326501
0.711	0.183	0.536	0.420	0.81947508
0.710	0.184	0.535	0.421	0.77623224
0.709	0.185	0.534	0.422	0.73352525
0.708	0.186	0.533	0.423	0.69134314
0.707	0.187	0.532	0.424	0.64967523
0.706	0.188	0.531	0.425	0.60851115
0.705	0.189	0.530	0.426	0.56784073
0.704	0.19	0.529	0.427	0.52765412
0.703	0.191	0.528	0.428	0.48794169
0.702	0.192	0.527	0.429	0.44869405
0.701	0.193	0.526	0.430	0.40990205
0.700	0.194	0.525	0.431	0.37155678
0.699	0.195	0.524	0.432	0.33364952
0.698	0.196	0.523	0.433	0.29617180
0.697	0.197	0.522	0.434	0.25911533
0.696	0.198	0.521	0.435	0.22247205
0.695	0.199	0.520	0.436	0.18623406
0.694	0.200	0.519	0.437	0.15039367
0.693	0.201	0.518	0.438	0.11494339
0.692	0.202	0.517	0.439	0.07987587
0.691	0.203	0.516	0.440	0.04518398
0.690	0.204	0.515	0.441	0.01086072

Table 3. **θ_1, θ_2, λ_A, λ_B of equation (32)**

θ_1	θ_2	λ_A	λ_B	Equation (32)
0.290	0.6700	0.296	0.710	0.99984853
0.291	0.6701	0.297	0.709	0.97721393
0.292	0.6702	0.298	0.708	0.95483617
0.293	0.6703	0.299	0.707	0.93271173
0.294	0.6704	0.300	0.706	0.91083717
0.295	0.6705	0.301	0.705	0.88920908
0.296	0.6706	0.302	0.704	0.86782414
0.297	0.6707	0.303	0.703	0.84667907
0.298	0.6708	0.304	0.702	0.82577064
0.299	0.6709	0.305	0.701	0.80509570
0.300	0.6710	0.306	0.700	0.78465112
0.301	0.6711	0.307	0.699	0.76443386
0.302	0.6712	0.308	0.698	0.74444089
0.303	0.6713	0.309	0.697	0.72466926
0.304	0.6714	0.310	0.696	0.70511606
0.305	0.6715	0.311	0.695	0.68577843
0.306	0.6716	0.312	0.694	0.66665355
0.307	0.6717	0.313	0.693	0.64773865
0.308	0.6718	0.314	0.692	0.62903103
0.309	0.6719	0.315	0.691	0.60152799
0.310	0.6720	0.316	0.690	0.59222690
0.311	0.6721	0.317	0.689	0.57412517
0.312	0.6722	0.318	0.688	0.55622026
0.313	0.6723	0.319	0.687	0.53850966
0.314	0.6724	0.32	0.686	0.52099089
0.315	0.6725	0.321	0.685	0.50366154
0.316	0.6726	0.322	0.684	0.48651921
0.317	0.6727	0.323	0.683	0.46956155
0.318	0.6728	0.324	0.682	0.45278625
0.319	0.6729	0.325	0.681	0.43619104
0.320	0.6730	0.326	0.680	0.41977367

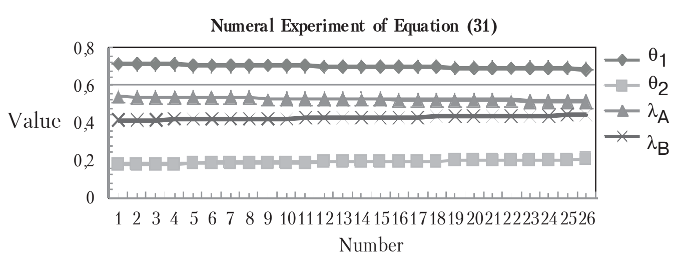

Figure 6. **θ_1, θ_2, λ_A, λ_B**

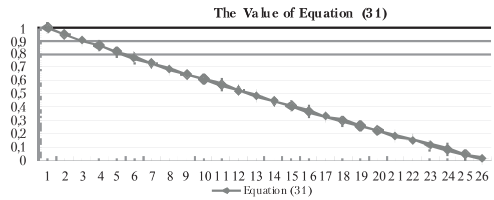

Figure 7. $0 < \partial n_A / \partial \lambda_A - \partial n_B / \partial \lambda_A < 1$

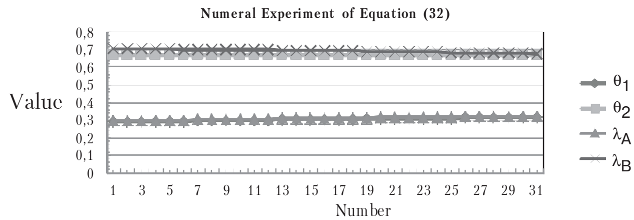

Figure 8. $\theta_1, \theta_2, \lambda_A, \lambda_B$

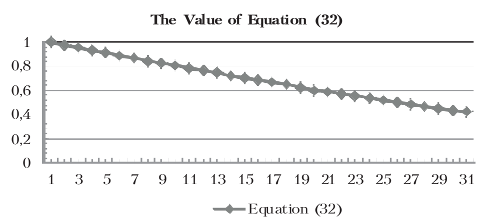

Figure 9. $0 < \partial n_A / \partial \lambda_B - \partial n_B / \partial \lambda_A < 1$

As in Figures 6 and 7, the values of $0 < \partial n_A / \partial \lambda_A - \partial n_B / \partial \lambda_A < 1$ are plotted and calculated under the assumed values on Table 2. When assumed values exceed the given range, there are two results that are $0 < \partial n_A / \partial \lambda_A - \partial n_B / \partial \lambda_A < 0$. Similarly, as in Figures 8 and 9, the values of $0 < \partial n_A / \partial \lambda_B - \partial n_B / \partial \lambda_A < 1$ are plotted and calculated under the assumed values on Table 2. When assumed values exceed the given range, there are two results that are $\partial n_A / \partial \lambda_B - \partial n_B / \partial \lambda_B < 0$ and $\partial n_A / \partial \lambda_B - \partial n_B / \partial \lambda_B > 1$.

We evidenced firm A and firm B correlations between partial compatibility of routing profiles, degree of compatibility and subsidy by precisely numeral analysis. Furthermore, the value range of equation (31) is wider than equation (32). There are two meanings. One is that public bus firm has more empowerment to influence private bus firm. The other one is that bus firms recognize how to set these values when public and the private bus firms cooperate.

Conclusions. Bus firms consider how to formulate more applicable routing profiles of DRTS under rational pricing so that impaired and elderly people could pay the minimum price to use the Vanpool taxi or Fu-Kang bus; the government must be pondering what to benefit to bus firms and riders. Besides, they also have to manage their budgets more efficiently and effectively. Because of these motivations, we establish a model with subsidy differentiation, routing profiles, network effects and the degree of the routing partial compatibility at a duopoly transport market. Traditionally, both bus firms own their vehicles and routing profiles to serve riders. As a result, they become competitors. Unfortunately, the transport market scale is too narrow to enter this market that it may result in abandonment of bus transport service.

In this paper, we attempt to answer the question "Is it a possible kind of transport operating method to become a partner?" The answer is "Yes". We use the concept of economic networks to assume that both bus firms own their routing profiles that can be of partial compatibility so that they can obtain differentiated subsidy. Furthermore, consider the price that a rider pays least for routing profiles and organize them into this model. Finally, use the subgame perfect method to find the equilibrium solutions.

We solve Nash equilibriums that are $P^*_A=Y(1/3+\theta_2/3\theta_1)$, $P^*_B=Y(1/3+\theta_1/3\theta_2)$, and calculate that $P^*_A=P^*_B=0.67$ is the equilibrium value when both bus firm obtain the equal subsidy. Our results are similar to the discount rate which is from 50% to 66.7% when a rider uses a Fu-Kang bus. We also find out the relationship between subsidy and partial compatibility of routing profiles. Furthermore, we find that partial compatibility of routing profiles influences degree of compatibility under the subsidy ranges that include and $1/2\ \theta_2<\theta_1<2\theta_2$ and $1/2\ \theta_2<2\theta_2<\theta_1$. Finally, we use the numeral experiments to search the values of adaptive subsidy and degree of compatibility.

According to our findings, the following suggestions are recommended. Under the limited subsidy range and the degree of partially routing profiles both bus firms might know how to adjust policy to benefit riders. Because of the constricted-budget bus operating, we attempt to solve this model based on the sub-game perfect equilibrium; i.e., the competitive method, and to find out their relation, so that decision makers have more information to consider for a paratransit policy to be more appropriate for riders. Furthermore, decision makers should consider economical efficiency of bus operating and sustainability of elderly and impaired people transporting, more precise subsidy mechanism must be constructed.

Nowadays, most of subsidy types are a kind of discount from transport fares. According to this paper, we can combine subsidies, routing profiles supporting services and partial compatibility to match riders and bus firm behavior. Due to differentiated goals, the authority must have power to integrate bus firms so that the latter final mission can be achieved: transport elderly and impaired people sustainably.

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