Caglar Yurtseven[1]

NETWORK MODEL OF MOBILE COMMUNICATION PRICING

This paper develops a model of competition at mobile communication markets. People's choices are investigated in their social environments with differing utilities for different calls. People get higher utilities from talking to people who are closer to them in the social environment. This paper explains price discrimination with customer necessities stemming from social network structures, without cost differences.

Keywords: *network; social interaction; demand theory; mobile communication; duopoly.*

Чаглар Юртсевен

СОЦІАЛЬНА МОДЕЛЬ ВИЗНАЧЕННЯ ВАРТОСТІ МОБІЛЬНОГО ЗВ'ЯЗКУ

У статті побудовано модель конкуренції на ринку мобільного зв'язку. Досліджено причини вибору послуг у соціальному середовищі з різною цінністю для різних викликів. Розмови з людьми, які знаходяться ближче в соціальному середовищі, являють собою більшу цінність для клієнта. Пояснено цінову дискримінацію за допомогою необхідностей, які випливають зі структури соціальної мережі, без відмінності у вартості.

Ключові слова: *мережа; соціальна взаємодія; теорія попиту; мобільний зв'язок; дуополія.*

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СОЦИАЛЬНАЯ МОДЕЛЬ ОПРЕДЕЛЕНИЯ СТОИМОСТИ МОБИЛЬНОЙ СВЯЗИ

В статье построена модель конкуренции на рынке мобильной связи. Исследованы причины выбора услуг в социальной среде с различной ценностью для различных вызовов. Разговоры с людьми, которые находятся ближе в социальной среде, представляют собой большую ценность для клиента. Объяснена ценовая дискриминация посредством необходимостей, вытекающих из структуры социальной сети, без различия в стоимости.

Ключевые слова: *сеть; социальное взаимодействие; теория спроса; мобильная связь; дуополия.*

1. Introduction. The mobile communication sector is evolving rapidly, and understanding the demand and pricing decisions at this market is getting more important every day. When the related literature is examined, it is seen that the social side of these decisions is not well incorporated into the models. To understand if the incorporation of the social side helps us to understand better the pricing patterns and shares of firms at this market, we are going to establish utilities of people on the "social distance" concept. The introduction of this social concept is the main contribution of the paper and the designed model is aimed to represent the real market decisions in a better way. In addition, the social distance concept allows us explain price discrimination applied to on-net and off-net calls with customer necessities, and without cost differences. This becomes more important now because, with improved technologies, extra costs for off-net calls are becoming negligible. However, price discrimination based on on-net and off-net calls is still seen. By using the social distance concept, we are also able to explain, why at some markets prices charged for

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on-net calls are higher than the prices for off-net. This explanation is also an addition to the literature for understanding of this newly emerging price pattern at the mobile communication markets in Turkey and Greece.

In the leading paper on the topic Laffont, Rey and Tirole (1998) presented an equilibrium in a two-firm wireless market. They use a variety of the well-known hotelling concept for differentiation of the firms. Consumers are located uniformly on a line segment [0,1]. Two networks are located at the two extremities of the segment, namely x_1=0 and x_2=0. In their paper, the product differentiation takes the form of a hotelling. However, for the utility a person derives from a phone call, we believe, distance to a firm is not the only point that matters. We believe social distance used in this paper makes the model more realistic by differentiating between two talks, duration of which are equal but the utility derived is not.

In this paper, in the first scenario we study, there are two firms at the market and they compete on a per unit time charge basis. Consumers choose the firm which maximizes their total utility. We also hypothesize that there is a marginal consumer at the market, who gets the same utility from choosing either firm. The same kind of marginal consumer reasoning, for modeling firm choice, can be seen in the work of Bental and Spiegel (1995). For per unit charge with two firms and differing costs, we found an equilibrium with two firms. However, in the case where the cost difference between two firms is big enough, there might be an equilibrium in which the market goes to a monopoly structure. And if the market has two equal cost firms, the result reached is not surprising in the sense that both firms get equal share from the market. In per unit pricing, we are also able to explain why in some markets prices charged for on-net calls are higher than the prices charged for off-net.

In its second part, the paper focuses on "two-part tariffs" pricing. For this scheme again a duopoly structure is assumed. We allowed the prices for on-net and off-net talks vary. It is shown that there is an equilibrium in which the low-cost firm gets higher share with a higher fixed fee and lower prices. It is observed that firms are able to charge mark-up rates for off-net talks. Firms make profits mainly by their per-minute charges if the person you call is a customer of the other firm.

2. The Model.

2.1 Consumers. The first assumption of the model is that consumers get utility by calling other people. We assume "calling party pays" regime and for the sake of simplicity, only the calling party gets utility. All the people in the model are placed in a circle according to their social features (Figure 1)[2]. A consumer get higher utility from talking to a person whose distance to him is shorter (socially closer people).

As a result of "diminishing marginal utility of talk," we assume each person talks one minute with others. As in Laffont, Rey and Tirole (1998), firms are placed on two extremes of the curve, points 0 and 1. This placement is exogenous. Firms appeal to different consumer types with their services. Consumers want a match between their preferences and firms' services. However, a perfect match is unlikely. Therefore, by

[2] The distances are calculated as if the consumers are located on a straight line. The circumference of the circle is 2 units. Points that are shown as 0 and 1 are placed on the two extremes of the semicircles and they don't represent the locations of the points on the x-axis.

choosing either firm 0 or firm 1, a consumer wants to minimize the disutility instead[3]. Utility function of a person "x" on the circle can be stated as:

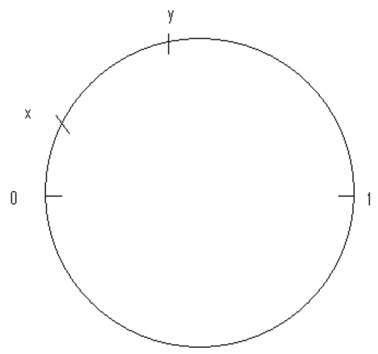

Figure 1. **The Social Circle**

That is, a person located at the point x and call all the y' s on a set Φ get the total

$$U_X(\Phi,t) = \int_{Y\in\Phi} \frac{1}{|y-x|+1} dy - ln(\tau+1) \qquad \tau \in \{(x),(1-x)\}$$

utility above from cell phone talking. Observe that according to firm choice, part of the total utility function that is coming from firm match, appears either as -ln(x+1) or -ln((1-x)+1). The term 1 in the denominator of $\frac{1}{|y-x|+1}$ and in ln(t+1) is added for mathematical purposes[4].

On the social circle, it is hypothesized that marginal consumers x^*' s exist and their place is normalized so that they are symmetrically located around the points 0 and 1 (Figure 2).

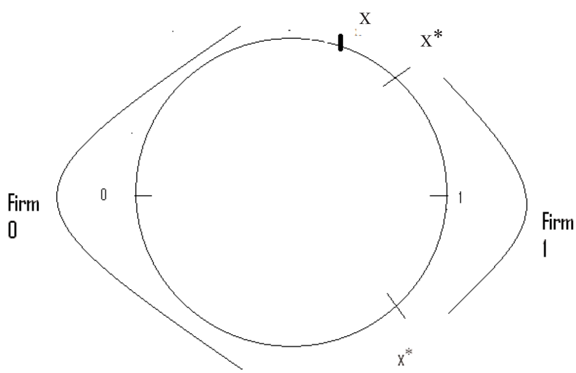

Figure 2. **Marginal Consumers and Shares**

Marginal consumers are the consumers indifferent in choosing a firm. The utility they can get from any firm would be equal considering prices, firms' and their own

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[3]In the consumers' utility function, the disutility from firm choice is designed big enough to make a person who is right next to a firm choose his/her neighbor firm.

[4]Note that $y\in\Phi$ $\int_{y\in\Phi} \frac{1}{|y-x|+1}dy$ also becomes an *ln* expression after the integration.

locations on the social circle. There are two firms at the market and since we are working on a (0,1) space, place of the marginal consumers tells us the shares of firms. (From *0* to x^* firm 0's customers and from x^* to 1 firm 1 customers) The total utility of the person *x* who is the customer of firm 0 (x_0) can be written as[5]:

$$maxU_{x_0}(x_{00},x_{01}) = ln(x_{00} - x^* + x + 1) + ln(x_{01} + x^* - x + 1) - ln(x+1),$$

where:

x_{00} : total on-net talk of x_0 (between *x* firm 0 customers.)

x_{01} : off-net talk of x_0 (between firm 0 and firm 1 customers)

—ln(x+1) is the utility, consumer located at *x* get, from choosing firm 0.

As a result of the normalization done about the place of marginal consumer, we are able to solve the model through solving only one semicircle between points 0 and 1. Everything is completely symmetric.

Everybody has *m* dollars to spend on wireless communication. This result comes up as the result of the form of the Cobb-Douglas demand function assumed. Therefore, a firm 0 customer is trying to maximize his utility as:

$$maxU_{x_0}(x_{00},x_{01}) = ln(x_{00} - x^* + x + 1) + ln(x_{01} + x^* - x + 1) - ln(x+1)$$

subject to

$$x_{00}p_{00} + x_{01}p_{01} = m$$

where

p_{00}: on-net per unit price of firm 0.

p_{01}: is off-net per unit price of firm 0.

When the maximization problem is solved:

if a customer of firm 0:

$$x_{00} = \frac{m + p_{01}(x^* - x + 1) + p_{00}(x^* - x - 1)}{2p_{00}} \quad \text{(demand for on-net calls)}$$

$$x_{01} = \frac{m - p_{00}(x^* - x - 1) - p_{01}(x^* - x + 1)}{2p_{01}} \quad \text{(demand for off-net calls)}$$

and symmetrically for firm 1 customers.

2.2. Firms. Before solving firms' profit maximization problem, we have to differentiate between consumers who talk only on-net and consumers who talk on-net and off-net.

Proposition 1. For both firms there exists consumer r_i who acts as a borderline between only on-net talking consumers and both on-net and off-net talking consumers. In the following part, we show how to find the consumer r_i on the social circle.

With the normalization of the place of the marginal consumer x^*, we hypothesize to get a social distribution of the market as shown in Figure 3. In the figure, consumers until x^* are the customers of firm 0 and consumers from x^* to 1 are the customers of firm 1. (One can find the place of marginal consumer by using the fact that he should get the same utility by choosing either firm.)

[5] For the total utility of person *x* rather than writing the direct expression we get from this integration, another form of the expression which makes use of the "1 minute" assumption will be stated. That form makes the utility function easier to follow.

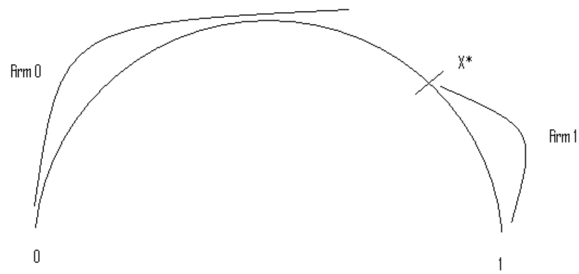

Figure 3. **Shares of firms on the simplified semicircle**

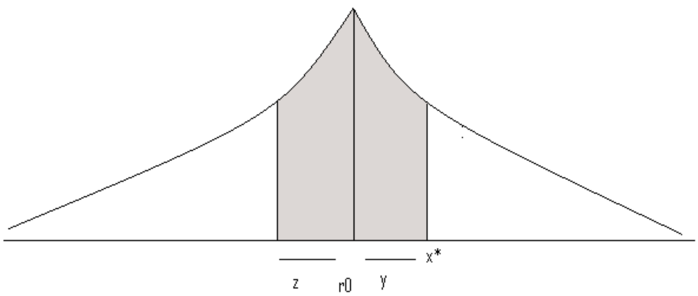

Figure 4. **For firm 0, consumers up to r0 speak only on-line**

With the budget restriction and the "duration of each call is 1 minute" assumption in mind, we can understand that to maximize utility, customers try to call as much people as possible, starting with the closest people on the right and left of themselves. It is also clear that consumers up to x^*-y use network 0 only. (Figures 5 and 6.) For consumer r_0 the marginal utility for talking to the person at y-z and the marginal utility for talking to marginal consumer x^* should be equal. Then,

$$\frac{p_{01}}{p_{00}} = \frac{f_x(x^*)}{f_x(x^* - y - z)} \qquad \frac{p_{01}}{p_{00}} = \frac{\frac{1}{y+1}}{\frac{1}{z+1}} \; (1)$$

and the budget constraint of the people r_0 (the person on the boundary who talks only on-line)

$$\frac{m}{p_{00}} = z + y \; (2)$$

Using some algebra, we can show that for the customers of firm 0, people from 0 to x^* —

$$x^* - \frac{m + p_{00} - p_{01}}{p_{01} + p_{00}}$$

speak only on-net and people from x^* — $x^* - \frac{m + p_{00} - p_{01}}{p_{01} + p_{00}}$ to x^* speak both on-net and off-net. Similarly for firm 1, customers; people from x^* to $x^* + \frac{m + p_{11} - p_{10}}{p_{11} + p_{10}}$ speak both on-net and off-net and people from $x^* + \frac{m + p_{11} - p_{10}}{p_{11} + p_{10}}$ to 1 speak only on-net (Figures 5 and 6).

Relevant distinctions being noted, the total demand for firm 0 can be stated according to types of consumers as (the total demand for firm 1 is symmetric, and it will be represented with *J*, *K* and *L* instead of *A*, *B* and *D*).

$$A = \int_{0}^{x^*-m+p_{00}-p_{01}/p_{00}+p_{01}} \frac{m+p_{01}(x^*-x+1)+p_{00}(x^*-x-1)}{2p_{00}}dx$$

$$B = \int_{x^*-m+p_{00}-p_{01}/p_{00}+p_{01}}^{x^*} \frac{m-p_{00}(x^*-x-1)-p_{01}(x^*-x+1)}{2p_{01}}dx$$

$$D = \int_{x^*-m+p_{00}-p_{01}/p_{00}+p_{01}}^{x^*} \frac{m+p_{01}(x^*-x+1)+p_{00}(x^*-x-1)}{2p_{00}}dx$$

where *A* represents total demand of only on-net calling customers and *B*+*D* represent total demand of both on-net and off-net calling customers.

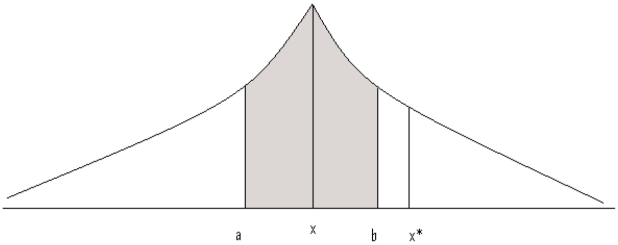

Figure 5. **A firm 0 customer who talks only on-line**

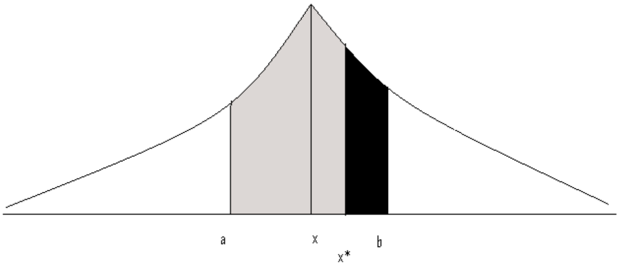

Figure 6. **A firm 0 customer who talks both on-net and off-line**

3. Pricing Structures

3.1 Per unit pricing. In this part we look for an equilibrium at the market with two firms, when they charge only for the minutes, their consumers are using. Firm 0 has a marginal cost of c_1 and firm 1 has a marginal cost of c_0 It is assumed there is a cost incurred only to the calling party's firm. There is no connection charge. First, firms announce their prices, then consumers make their provider choices. We look specifically for the existence of a two firm equilibrium in which the social distance curve is separated into two pieces.

Proposition 2. At the market constructed, there is only one marginal consumer on the semisocial distance curve[6].

[6]One, in each half.

Existence of marginal consumers is quite obvious. With the distance of a consumer to his firm part in the utility function we make sure a consumer who is next to a firm is better off by choosing his "neighbour" firm. Therefore, on the social curve there have to be marginal consumers[7]. However, how can we make sure there is only one?

Suppose there are more than one marginal consumer: x_1* and x_2*. There are two consumers who are indifferent between choosing either firm. Then the equilibrium would appear as in Figure 7 with 3 pieces. However, if the consumer x_1* is indifferent between choosing either firm with given prices and its place according to firms, the consumer x_2* has to be better off by choosing firm 1. Existence of the utility one can get from firm choice rules out the possibility of many marginal consumers on the semicircle (Proof can be found in Appendix 2).

For the model, we also assume costs of firms are greater than zero and different from each other. In addition, as the meaningful cost range, we are interested in the cost interval of (0,1). This restriction is necessary due to the size of the social distance curve. And it is also assumed a firms' both type of prices are higher than their own marginal costs. Given the demands above, firms try to maximize their profits as follows:

$$\pi_1 = J(p_{11} - c_1) + K(p_{11} - c_1) + L(p_{10} - c_1)$$

and

$$\pi_1 = J(p_{11} - c_1) + K(p_{11} - c_1) + L(p_{10} - c_1)$$

marginal cost of firm 0 is predetermined as c_0, so firm 0 tries to maximize its profit with respect to p_{00} and p_{01}.

So we have two first order conditions for firm 0:

$$u_0 = \frac{\partial \pi_0}{\partial p_{00}} \qquad v_0 = \frac{\partial \pi_0}{\partial p_{01}}$$

and the first order conditions for firm 1:

$$u_1 = \frac{\partial \pi_1}{\partial p_{11}} \qquad v_1 = \frac{\partial \pi_1}{\partial p_{10}}$$

We have 4 equations for our 4 unknowns. Instead of not interpretable open form solutions, we provide the proof for firm 0's equilibrium prices' uniqueness.

Proposition 3. Under the assumptions of per unit pricing, for marginal costs greater than 0 and different for each firm, and each firm's both type of prices are higher than their own marginal costs, there exists a two-firm equilibrium in which the lower cost firm gets a higher share with its lower prices.

Proof 1: For the existence of the single equilibrium, we need to show successively that no consumer is better off by switching between firms and no firm has a reason to change its pricing scheme in the range we are looking for: (0,1).

In our model consumers have no reason to change their providers. Marginal consumers get the same utility by choosing either firm and so all the consumers other than marginals are better off by being a customer of a particular firm according to their place on the social curve.

[7] One also can easily see by the symmetry of utility functions for firm 0 and firm 1 customers and by the strong firm-consumer relationship that the derivative of the difference between the utility of consumer x from buying from firm 0 and utility of x from buying from firm 1 is non-increasing.

It is proposed that in equilibrium, small cost firm has lower prices and higher share. Intuitively, small cost firm uses its advantage in costs to ask prices low enough to attract more consumers and increase its share, and high enough to have a profit margin to get higher profit from a customer. Big cost firm is still able to get share from the market mainly because it offers high utility from firm choice to the close consumers. We need to show that stated prices which are found by maximizing firms' profits are the only stable equilibrium prices. That is we need to show both firms' profit functions have only one local maximum for given costs and phone call budget. In other words, we need to prove profit functions are concave, according to their two choice variables; on-net (p_{00},p_{11}) and off-net (p_{01},p_{10}) prices.

Lemma 1: Let *f* be a function of many variables with continuous partial derivatives of the first and the second order on the convex open set *S*. Denote the Hessian of *f* at the point *x* by *H*(*x*). If *H*(*x*) is negative definite for all *x S* then *f* is strictly concave.

Lemma 2: Second partial derivative test: For a two variable function, if the determinant of the Hessian matrix is positive and sign of the leading principal minors is negative, then *H(x)* is negative definite.

That is we need to show both firms profit functions' in the given pricing scheme are negative (semi) definite. Simplified Hessian of firm 0's profit function can be shown as (open form of the Hessian is very long, massy and difficult to follow, that form is available upon request):

$$\begin{bmatrix} \frac{-a_0(-c_0+p_{00})(A_0^2)-b_0(B_0^2)}{C_0^2} & \pi_{p_{00},p_{01}} \\ \pi_{p_{01},p_{00}} & \frac{-w_0(-c_0+p_{01})(D_0^2)-z_0(E_0^2)}{F_0^2} \end{bmatrix}$$

Both of the principal minors ($\pi_{p_{00},p_{00}}$, $\pi_{p_{01},p_{01}}$) are negative. ($a_0>0$, $b_0>0$, $w_0>0$, $z_0>0$ and note the assumption that p_{00} and p_{01} should be greater than marginal cost c_0).

And the determinant of the Hessian can be shown in a simplified form as:

$$\frac{(-c_0+p_{00})G_0^2*H_0^2+I_0^2*J_0^2+K_0^2}{L_0^4}$$

which is positive. Again, note the assumption that $p_{00}>c_0$. This shows that firm 0's profit function matrix is negative definite and so it is concave. Proof for the other firm's profit function's concavity is completely similar.

Q.E.D

3.1.1 Comparative Statistics and Numerical Examples. As the costs of firms increase, prices of firms rise as well.

$$\frac{\partial p_{00}}{c_0}>0 \quad \frac{\partial p_{01}}{c_0}>0 \quad \frac{\partial p_{11}}{c1}>0 \quad \frac{\partial p_{10}}{c1}>0$$

If the cost of a firm increases, the original firm increases its own prices and in response other firm raises its prices as well, by increasing its profit margins.

When we compare different cost pairs for firms, it is seen that prices of both firms are close to each other in the equilibrium. It means that lower cost firm charges a bigger profit margin. (As an example: for m=4, c_0=0.1, c_1=0.2 ➤ p_{00}=0.45, p_{01}=0.38, p_{11}=0.47, p_{10}=0.41, share of firm 0: 52%, share of firm 1: 47%). For moderate changes in cost differences one can claim that shares of firms do not change radically; instead profit margins change and again prices of firms can be considered close to each other. (For m=4, c_0=0.1, c_1=0.5 ➤ p_{00}=0.59, p_{01}=0.56, p_{11}=0.72, p_{10}=0.67, share of firm 0: 69%, share of firm 1: 30%). This causes price elasticities of small-cost firm to be smaller than big-cost firm[8]. Since small-cost firm has high profit margins it is able to adjust its margin and keep its share rather than changing the price directly. Big-cost firm has no more chance than reflecting the change in cost onto its prices, since its profit margins are already low. Trends for prices, profits and shares of two firms are provided in Figures 8-10, where the cost of firm 0 is fixed at 0.1 and cost of firm 1 is getting bigger. (0,2 to 1)

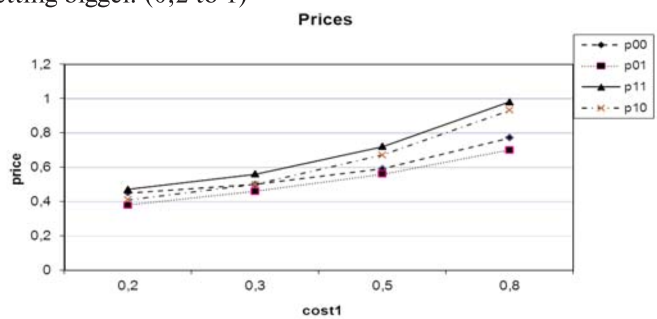

Figure 8. **Per unit charging prices (firm 0's cost: 0.1. firm 1's cost: X-axis)**

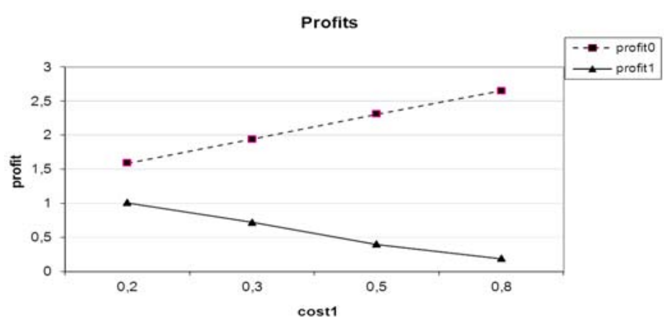

Figure 9. **Per unit charging profits (firm 0's cost: 0.1. firm 1's cost: X-axis)**

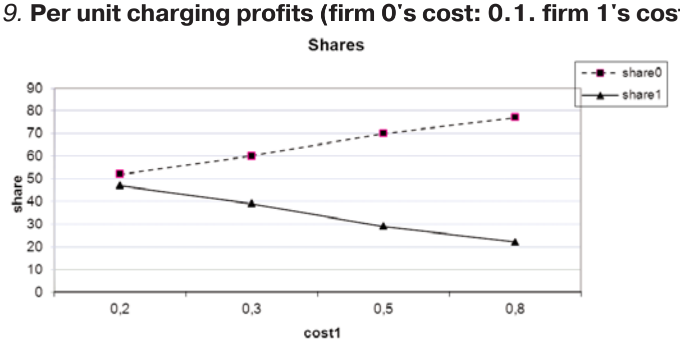

Figure 10. **Per unit charging shares (firm 0's cost: 0.1. firm 1's cost: X-axis)**

[8] Price elasticities with respect to costs.

For unrealistically big cost differences the market may go to a monopoly structure. However, these cases are out of the scope of our paper. Also, when the budget for phone calls increases, prices have a tendency to rise as well.

$$(\frac{\partial p_{00}}{m} > 0 \quad \frac{\partial p_{01}}{m} > 0 \quad \frac{\partial p_{11}}{m} > 0 \quad \frac{\partial p_{10}}{m} > 0)$$

3.1.2. Comparison with the results of the baseline model and a note on price discrimination. If one tries to solve the model of the paper without the social distance concept, there would be no opportunity for firms to price discriminate. Laffont, Rey and Tirole (1998) justified different prices for on-net and off-net calls with different cost structures for on-net and off-net calls. This explains price discrimination with customer necessities and without cost differences. As a result of the construction of the model in this paper, firms charge higher prices for on-net calls. People around you are possibly in the same network company with you due to the similar tastes you have. Therefore, it makes sense that you and your friends feel closer to the same firm. Therefore, firms tend to charge higher prices for on-net calls. It is more difficult, due to high utility, for a consumer not to call his close friends, relatives etc. This is an important strength because it explains how firms can price-discriminate in a highly regulated network market structure with sophisticated technology, where extra costs for off-net calls are becoming negligible. (These market conditions become more realistic everyday and actually observed at Turkish cellular communication market.)

However, in many countries one can still assume positive extra costs for reaching another network's customer. For this extra cost assumption, our model perfectly estimates different prices at many of the markets, in the sense that off-net calls are more expensive than on-net calls. In Figures 11-13 we provide the corresponding graphs to 8-10, in which we assume access cost of 0.1.

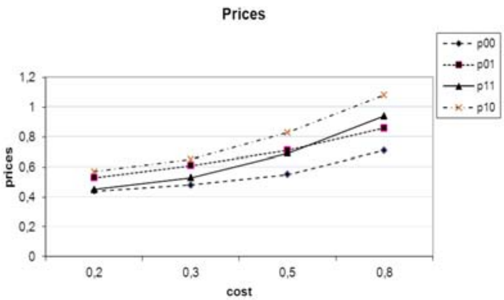

Figure 11. **Per unit charging, with 0.1 access, prices. (firm 0's cost: 0.1. firm 1's cost: X-axis)**

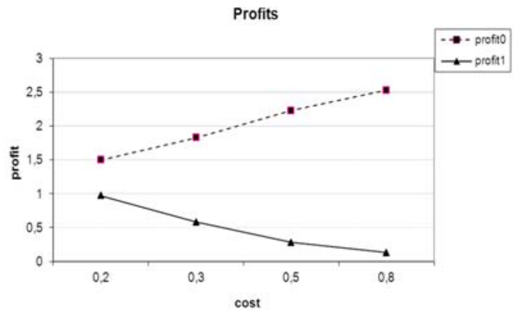

Figure 12. **Per unit charging, with 0.1 access, profits (firm 0's cost: 0.1. firm 1's cost: X-axis)**

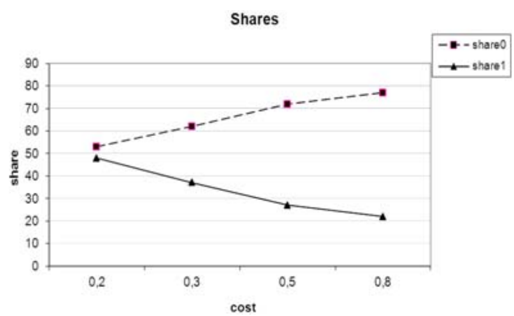

Figure 13. **Per unit charging, with 0.1 access, shares (firm 0's cost: 0.1. firm 1's cost: X-axis)**

3.2 Two-Part Tariffs. In this pricing scheme, firms charge consumers a fixed fee. Firm 0 charges a fixed fee of h_0 and firm 1 charges a fixed fee of h_1 (assume h_0 and h_1 are less than m). From m dollars budget, in this tariff structure there remains $m—h_0$ dollars to spend on phone calls for firm 0 customers ($m—h_0$ dollars for firm 1).

For firms, observing the real market, we assume that firms use marginal cost pricing for on-net calls[9]. They can still make profits from off-net. With these new assumptions, firms' choice variables are fixed fee and off-net pricing. Everything else remains same as in the one part tariff case. Demand for firm 0 can be represented as:

$$A^* = \int_0^{x^*-m-h_0+p_{00}-p_{01}/p_{00}+p_{01}} \frac{m-h_0+p_{01}(x*-x+1)+p_{00}(x*-x-1)}{2p_{00}}dx$$

$$B^* = \int_{x^*-m-h_0+p_{00}-p_{01}/p_{00}+p_{01}}^{x^*} \frac{m-h_0-p_{00}(x^*-x-1)-p_{01}(x^*-x+1)}{2p_{01}}dx$$

$$D^* = \int_{x^*-m-h_0+p_{00}-p_{01}/p_{00}+p_{01}}^{x^*} \frac{m-h_0+p_{01}(x*-x+1)+p_{00}(x*-x-1)}{2p_{00}}dx,$$

[9]In many markets, including the US, firms charge very low prices for on-line. Most of the time on-line price is as low as firms' marginal costs and in some cases, they are equal to zero.

where A^* represents total demand of only on-net talking customers and B^*+D^* represent total demand of both on-net and off-net talking customers (demand for firm 1 is symmetric and represented with J^*, K^* and L^*).

As a result of the marginal cost pricing assumption, firms do not make profits from on-line calls. Firm 0 gets a revenue of

$$\varphi_0 = \int_0^{x^*} h_0 dx$$

from fixed fees. The revenue firm 1 gets from fixed fees is:

$$\varphi_1 = \int_{x^*}^{1} h_1 dx$$

Total profits of firms 0 and 1; — π_0^* and π_1^* can be represented as follow:

$$\pi_0^* = \varphi_0 + D^*(p_{01} - c_0)$$

and

$$\pi_1^* = \varphi_1 + L^*(p_{10} - c_1)$$

Firms try to maximize their profits with respect to fixed fees and off-net prices. So we have two first order conditions for firm 0:

$$\mu_0 = \frac{\partial \pi_0^*}{\partial p_{01}} \qquad \lambda_0 = \frac{\partial \pi_1^*}{\partial h_0}$$

and the first order conditions for firm 1:

$$\mu_1 = \frac{\partial \pi_1^*}{\partial p_{10}} \qquad \lambda_1 \frac{\partial \pi_1^*}{\partial h_1}$$

Proposition 4. Under the assumption of two-part tariffs, for marginal costs greater than 0 and different from each other, and each firms off-net prices higher than their own on-net prices, there exists a two-firm equilibrium where higher cost firm charges higher prices and lower fixed fee whereas the lower cost firm charges lower prices and higher fixed fee.

In this pricing scheme low-cost firm offers lower on-net prices than the high-cost firm to get use of its cost advantage. Due to the same reason and marginal pricing assumption its off-net prices are lower as well. Therefore, low-cost firm is still able to get a high share from the market by asking higher fees than the high-cost firm. Intuitively, it asks fixed fees high enough to get more profit from a customer and low enough to attract people and get higher share. By asking lower fixed fees and having a natural advantage on the closer consumers, high-cost firm is able to maintain its place in the market with the stated prices and fees. The proof for the uniqueness of the equilibrium is very similar to Proof 1 and available upon request.

3.2.1 Comparative Statistics and Numerical examples. As the cost of a firm increases, off-net price of a firm tends to increase, however, its fixed fee tends to decrease. Intuitively it wants to keep its market share by a decrease in fixed fee in response to a increase in its prices. There is no significant relationship between the magnitudes of price elasticities this time. The direction of changes in prices with respect to changes in costs are:

$$\frac{\partial p_{01}}{c_0} > 0 \quad \frac{\partial h_0}{c_0} < 0 \quad \frac{\partial p_{10}}{c1} > 0 \quad \frac{\partial h_1}{c1} < 0$$

When different cost pairs for firms are compared, one can observe that again fees and prices of firms are close to each other. (As an example: for m=4, c_0=0.1, c_1=0.2 ➔ p_{00}=0.1, p_{01}=0.32, p_{11}=0.2, p_{10}=0.37, h_0=3, h_1=2.5 share of firm 0:57%, share of firm 1:42%). While cost differences are getting larger, the gap between firms' fixed fees gets larger as well. (For m=4, c_0=0.1, c_1=0.5 ➔ p_{00}=0.1, p_{01}=0.43, p_{11}=0.5, p_{10}=0.53, h_0=3.2, h_1=2.0, share of firm 0:70%, share of firm 1:29%). Unrealistically high cost differences may lead to a monopoly structure at the market which is out of the scope of this paper. In addition, when the budget for mobile calls increases, prices for off-net calls and fixed fees tend to increase as well.

$$(\frac{\partial h_0}{m}>0 \quad \frac{\partial p_{01}}{m}>0 \quad \frac{\partial h_1}{m}>0 \quad \frac{\partial p_{10}}{m}>0)$$

See Figures 14-16, where the cost of firm 0 is fixed at 0.1 and cost of firm 1 is getting bigger.

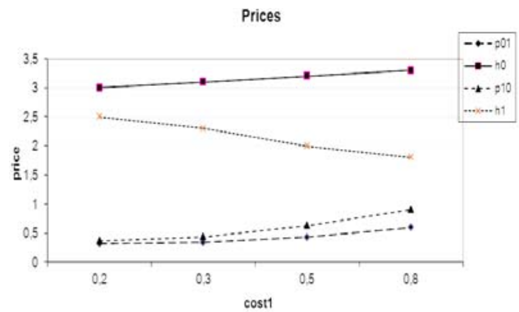

Figure 14. **Two-part tariff prices (firm 0's cost: 0.1. firm 1's cost: X-axis)**

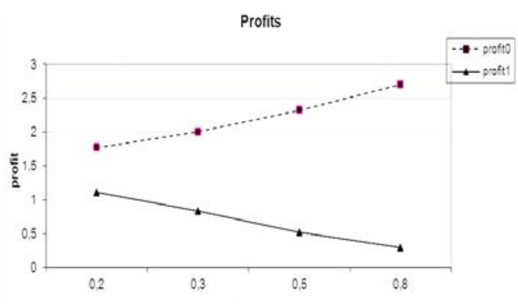

Figure 15. **Two-part tariff profits (firm 0's cost: 0.1. firm 1's cost: X-axis)**

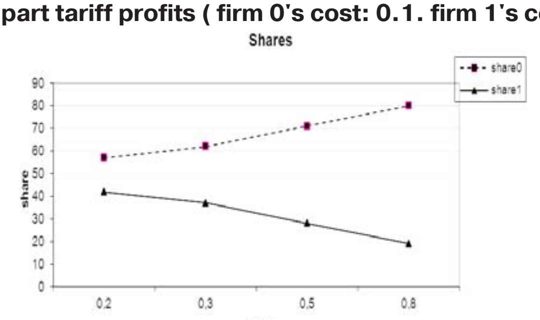

Figure 16. **Two-part tariff shares (firm 0's cost: 0.1. firm 1's cost: X-axis)**

3.2.2 Comparison with the results of the baseline model. In a baseline model, where people get same utility from talking to different people, both on-net and off-net prices are equal to firms' marginal costs. Firms are able to make profits only from fixed fees. This is clearly not the case in the real world. When we look at the market of fixed fee tariffs it can be seen that calling somebody from your own network is most of the times cheaper than talking with somebody from other networks (with lower prices for on-net calls, or completely unlimited calling for on-net calls at high fixed fee tariffs).

4. Concluding Remarks. The article has developed a social interaction model of wireless pricing to study the network competition. As a distinguishing feature of the model, the utility consumers get from cell phone talking is based on a social structure. People get higher utilities from talking to people who are closer to them in the social environment. Now, let us recap the main results, starting with the case of per unit pricing.

In per unit charging, we come up with a two-firm equilibrium. Assuming each firm has different costs which are larger than zero, smaller cost firm gets higher share with lower prices in the equilibrium. Higher on-net prices for zero access charge and higher off-net prices for positive access charge. Elasticities are also stated in the text.

We also investigated two-part tariffs, in which customers pay a fixed fee for a period. Many firms charge very low prices for on-net calls under this tariff. Considering this, we assume that firms charge their marginal costs as their on-net prices. A two-firm equilibrium is reached in which higher cost firm charges higher prices and lower fixed fee whereas the lower cost firm charges lower prices and higher fixed fee.

In both pricing schemes the model is able to explain price discrimination at the market for on-net calls and off-net calls without the use of any assumption of different costs for different types of calls. We believe this is an important addition to the literature in the sense that differences in these costs in today's sophisticated communication technology is becoming negligible everyday.

Although the paper is on the cell-phone market, the model in the paper is a more general two-way network model. Any network structure in which other people socially matter can be investigated under this mechanism. Networking websites where you pay a subscription fee to become a member are good examples of the phenomena examined. People generally choose websites which they feel like to find more people socially closer to them. With modifications to price structure and other necessary changes, the core of our model has a wide application area.

Appendix

$$U_{X_1}(x_{10}, x_{11}) = ln(x_{10} - x^* + x + 1) + ln(x_{11} + x^* - x + 1) - ln(2 - x)$$

Utility of a person x, who is a customer of firm 1 (x_1)

x_{11} is the total on-net call of x_1 (between x who is a firm 1 customer and other people who are firm 1 customers);

x_{10} is the off-net call of x_1 (between x who is a firm 1 customer and people who are firm 0 customers);

$-ln(2-x)$ is the utility consumer located at x gets, from choosing firm 1.

Total Demand for firm 1: Per unit charging

Total demand for firm 1 can be shown as:

$$J = \int_{x^*}^{x^*+m+p_{11}-p_{10}/p_{11}+p_{10}} \frac{m+p_{10}(x^*-x+1)+p_{11}(x^*-x-1)}{2p_{11}}dx$$

$$K = \int_{x^*}^{x^*+m+p_{11}-p_{10}/p_{11}+p_{10}} \frac{m-p_{11}(x^*-x-1)-p_{10}(x^*-x+1)}{2p_{10}}dx$$

$$L = \int_{x^*}^{x^*+m+p_{11}-p_{10}/p_{11}+p_{10}} \frac{m+p_{10}(x^*-x+1)+p_{11}(x^*-x-1)}{2p_{11}}dx,$$

where J represents total demand of only on-net talking customers and $K+L$ represent total demand of both on-net and off-net talking customers.

Proof 2: Possibility of equilibria with more than one marginal consumer for the semi-circle

Due to simplifications, we can use derivative of difference function to show that utility of consumer x^* from buying from firm 0 minus utility of x from buying from firm 1 is strictly decreasing at x^* so that there is only one marginal consumer.

$$f'(ln(x_{00}+1)+ln(x_{01}+1)-ln(x+1)-ln(x_{10}+1)-ln(x_{11}+1)+ln(2-x))$$

$$=-\frac{1}{x^*+1}-\frac{1}{2-x^*}$$

$$=\frac{-3}{(x^*+1)(2-x^*)}$$

given $0<x<1$, this ratio is negative, hence f is strictly decreasing.

Q.E.D.

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