Mihaela Bratu

FORECASTS ACCURACY COMPARISONS FOR VAR, AR AND VARMA MODELS

Although the scalar components methodology used to build VARMA models is rather difficult, the VAR models are easier to be used in practice, the forecasts based on the first models have a higher degree of accuracy. This statement is demonstrated for the variables like the 3-month treasury bill rate and the spread between the 10-year government bond yield and the 3-month treasury bill rate, the data are from the U.S. economy. We use a better measure of accuracy than those used in literature before, the generalized forecast error second moment, then adapted to measure the relative accuracy.

Keywords: *VARMA models; accuracy; scalar components methodology; full information maximum likelihood; canonical correlation.*

JEL classification: C32, C51, C53, C82.

Міхаела Брату

ПОРІВНЯННЯ ТОЧНОСТІ ПРОГНОЗІВ МОДЕЛЕЙ VAR, AR ТА VARMA

У статті показано, що хоча методологія скалярних компонентів, яка використовується для моделей VARMA, є доволі складною, а моделі VAR є простішими для застосування на практиці, прогнози за першими точніші, ніж за другими. Дане ствердження підкріплено тестом з такими змінними, як 3-місячний курс казначейських векселів, спред між 10-річним прибутком за облігаціями уряду та даним курсом векселів. Для моделювання використано дані щодо економіки США. Представлено використання нової міри точності, що не була раніше описана в літературі, — узагальнений прогноз помилки другого порядку. Дана міра точності застосована для оцінювання відносної точності.

Ключові слова: *VARMA моделі; точність; методологія скалярних компонентів; максимальна правдоподібність з повною інформацією; канонична кореляція.*

Форм. 6. Табл. 3. Літ. 7.

Михаэла Брату

СРАВНЕНИЯ ТОЧНОСТИ ПРОГНОЗОВ МОДЕЛЕЙ VAR, AR И VARMA

В статье показано, что хотя методология скалярных компонентов, используемая для моделей VARMA, довольно сложна, а модели VAR проще примененить на практике, прогнозы с использованием первых точнее вторых. Данное утверждение подкреплено тестом с такими переменными, как 3-месячный курс казначейских векселей, спред между 10-летним доходом по облигациям правительства и данным курсом векселей. Для моделирования использованы данные экономики США. Представлено использование новой меры точности, не описанной ранее в литературе, — обобщенный прогноз ошибки второго порядка. Данная мера точности применена для оценки относительно й точности.

Ключевые слова: *VARMA модели; точность; методология скалярных компонентов; максимальное правдоподобие с полной информацией; каноническая корреляция.*

[1] PhD Student, Institute for Doctoral Studies, Academy of Economic Studies, Bucharest, Romania.

VARMA models in literature. Vector autoregressive moving average models (VARMA) and the VAR ones are used in econometrics, particularly in time series analysis, to reveal the cross-correlations between series, exceeding the isolated analysis of the data series.

Amid the success of univariate ARMA models in forecasting, it was made the passing to VARMA models for multivariate context. The purpose of their introduction is consistent with the Granger definition of causality and is related to the improvement of forecast accuracy by using a model with interrelated variables. For the first time these models were used by Quenouille (1957). Since then, they have been the subject for research by Tiao and Box (1981), Tiao and Tsay (1983, 1989), Tsay (1989), Wallis (1977), Zellner and Palm (1974). In all these cases, the number of variables was small, no more than 3. Another problem was the inability to identify VARMA representations. These problems were analyzed by Hannan (1970, 1976, 1979, 1981), Dunsmuir and Hannan (1976) and Akaike (1974). Hannan and Deistler (1988) were the first to provide the theoretical presentation of VARMA models. Lautkepohl (1991, 2002) and Reinsel (1993) analyze the forecasts based on these models. ARIMA models, although widely used, fail to describe the dynamics of all the relationships between selected variables.

VARMA models are the result of Wold decomposition theorem for multivariate stationary series, as shown by G. Athanasopoulos and F. Vahid (2007). C. Kascha and C. Trenkler (2011) state that there are very few studies on the performance evaluation of forecasts based on VARMA or cointegrated VARMA models. Poskitt (2003), Athanasopoulos and Vahid (2008) and C. Kascha and C. Trenkler (2011) evaluate the accuracy of forecasts made using VARMA models and they obtain a good performance, exceeding the one of VAR models.

VARMA models have been used by many researchers as Quenouille (1957), Hannan (1969), Tunnicliffe-Wilson (1973), Hillmer and Tiao (1979), Tiao and Box (1981), Tiao and Tsay (1989), Tsay (1991), Poskitt (1992), Lutkepohl (1993), Lutkepohl and Poskitt (1996), Reinsel (1997), Tiao (2001), G. Athanasopoulos and F. Vahid (2004, 2005, 2006, 2007, 2008). However, finite order VAR models are preferred to VARMA, since in literature there is no question about their alternative use and the identification of VAR is easier, a lot of software allows the development of these models. Economic theory is not in accordance with the process modeling using VAR, the moving average terms couldn't be excluded. Cooley and Dwyer (1998) argue that macroeconomic time series modeling using VAR models is not consistent with the economic theory. But the difficulty with VARMA methodology imposed the selection of VAR models, whose results are quite good. Likelihood function is based on the normality assumption and it can be recursively determined, the first p observations being set, and the next being zero. Starting from the state space form of the model this likelihood function can be exactly calculated. The determination of VARMA(p, q) model orders is quite difficult, given the fact that the parameters must follow certain restrictions. Kascha and Mertens (2009) carry out a comparative analysis of the identification of structural form for VAR and VARMA models and for representation in the state space form.

Feunou (2009) uses a VARMA model to represent the yield curve, eliminating the restrictions on cointegration. Dufour and Pelletier (2005) propose a modified information criterion to determine VARMA orders, this being only a generalization of the Hannan and Rissanen (1982) criterion. Mainassara (2010) brings a change in AIC criterion used in VARMA models selection by Tsay and Hurvich (1989), resulting the AICc criterion, which is an almost unbiased estimator of the Kullback-Leibler divergence. If d-data sets are analyzed, the number of parameters to be estimated is: Choosing a too small VARMA (p, q) order implies inconsistent estimators and a too large order brings a decrease in forecast accuracy, as shown by Mainassara (2010).

Procedures for specifying and estimating cointegrated VARMA models have been developed by Yap and Reinsel (1995), Lutkepohl and Claessen (1997), Poskitt (2003, 2006, 2009). All these procedures are based on the "echelon form". This form is a set of restrictions for parameters to ensure the rest of parameters are obtained using likelihood function. Kascha C. and C. Trenkler (2011) extend the representations of Dufour and Pelletier (2008) that were valid for the non-stationary series.

Kascha C. and C. Trenkler (2011) start from the last significant results related to the VARMA models, proposing a strategy for specification and estimation for cointegrated series. The authors made predictions based on these models for the U.S. interest rate.

Athanasopoulos G. and Vahid F. (2007) showed that the forecasts based on VARMA models are better than the ones based on VAR.

In literature there are several methods to identify the VARMA. Athanasopoulos G. and Vahid F. (2008a) identified two methodologies that can be applied to obtain a unique identification of VARMA. The authors made comparisons of the forecasts made on VARMA models. The first methodology is an extension of Tiao's and Tsay's one (1989). The second methodology, the echelon form one, involves the estimation of the Kronecker indices, calculated as the maximum rank of each row from each equation of the model, and the specification the canonical echelon form. Kronecker indices are estimated using the least squares method applied to regressions. The innovation estimates with lag are derived from the first stage of a VAR presented by Hannan and Rissanen (1982). Kronecker indices are determined at the second stage using a model selection criterion, as shown by Hannan and Diestler (1988) and Lutkepohl and Poskitt (1996). This methodology is very simple, being used by Akaike (1974, 1976), Kailath (1980) and Kavalieris Hannan (1984), Solo (1986), Hannan and Deistler (1988), Tsay (1991), LAutkepohl (1993), Nsiri and Roy (1992.1996), Poskitt (1992), LAutkepohl and Poskitt (1996). Using Monte Carlo simulations, Athanasopoulos and Vahid (2006) evaluated the ability of two methodologies to identify VARMA models. Based on the real data, the authors compared the performance of VARMA models that used these two methodologies.

Tiao and Say (1989) proposed the first method, fairly criticized, later it was improved by G. Athanasopoulos and F. Vahid (2006). Their methodology has 3 stages:

- Identification of scalar components of the model (SCM) by applying canonical correlation tests between different sets of variables;
- Identification of the structural form of a model using the same tests and some certain logical deductions;
- Estimation of a model using the method of full information maximum likelihood (FIML).

Tiao's and Say's (1989) methodology estimates the parameters in two stages, but at the first stage there is a deviation of standard errors results, this error being corrected later in the three-stages version proposed of G. Athanasopoulos and F. Vahid (2006).

In 1989, Tiao and Tsay presented their SCM methodology to the Royal Society of Statistics. The critiques related to their methodology were formulated by Chatfield, Hannan, Reinsel and Tunnicliffe-Wilson, but excluding Tsay's intervention in 1991, the methodology was developed later only by two authors. G. Athanasopoulos and Vahid F. are those who extended the methodology, the results of its application being periodically published, especially in the recent 10 years.

Tiao's and Say's methodology critiques are related to determination of transformation matrix can be summarized as follows:

- the use of transformation matrix does not lead to the most efficient estimators for parameters;
- standard errors can not be calculated for the estimated parameters in matrix A;
- the total number of the estimated parameters in A should be included info the model parameters to reduce the number of degrees of freedom;
- identification of VARMA (p, q) is based on the transformed variables and not on the original ones.

All these problems are solved by G. Athanasopoulos and F. Vahid (2008b), which provide a formula for determining the number of redundant parameters of matrix A. They describe the procedure by which certain parameters are normalized to 1, but they are different from those that should be set to zero. The authors give up to the estimated canonical covariates by choosing an estimate of full information maximum likelihood parameters. However, they keep the way of determining the order of the K scalar components.

The VARMA modeling methodology based on scalar components. In order to identify the VARMA model we examined the presence of simple structures in the process. Scalar component methodology of Tiao and Say (1989) considers a K-dimensional VARMA (p,q) model ($x_t = \Phi_1 x_{t-1} + \ldots + \Phi_p x_{t-p} + \eta_t - \Theta_1\eta_{t-1} - \ldots - \Theta_q\eta_{t-q}$) a non-zero linear combination: $z_t = a'x_t$ follows a SCM (p,q) process if α satisfies the following properties:

$$\alpha'\Phi_{p_1} \neq 0^T, 0 \leq p_1 \leq p$$

$$\alpha'\Phi_l = 0^T, l = p_1+1,\ldots,p$$

$$\alpha'\Theta_{q_1} \neq 0^T, 0 \leq q_1 \leq q$$

$$\alpha'\Theta_{q_l} = 0^T, l = q_1+1,\ldots,q$$

Scalar random variable admits an ARMA representation with orders that vary from p1 to q1, depending on lags from 1 to p1 and its innovations depend on lags from 1 to q1. The identification starts with the SCM (0,0), which is actually a white noise. The basic idea is to find out linearly independent vectors K that achieve a rotation operation of VARMA (p,q) process in a new process with a dynamic structure, but with fewer parameters. The linearly independent vectors form a matrix $A = (\alpha_1, \ldots, \alpha_k)'$. Then, $z_t = Ax_t$.

The VARMA process with transformed variables keeps all the rows of zero restrictions from AR component, the parameters matrix from MA having the form:

$$z_t = \Phi_1^* z_{t-1} + \ldots + \Phi_p^* z_{t-p} + \varepsilon_t - \Theta_1^* \varepsilon_{t-1} - \ldots - \Theta_q^* \varepsilon_{t-q} \qquad (1)$$

where $\Phi_i^* = A\Phi_i A^{-1}, \varepsilon_t = A\eta_t, \Theta_i^* = A\Theta_i A^{-1}$.

If we identify two scalar components:

$$z_{r,t} = SCM(p_r, q_r) \text{ and } z_{s,t} = SCM(p_s, q_s), \text{ where } p_r > p_s, q_r > q_s,$$

the lags of $Z_{s,t}$ from the right part of the dynamic equation for $Z_{r,t}$ can be expressed in terms of variables from right side of $Z_{r,t}$. Lags orders of $Z_{s,t}$ can take values between 1 and the minimum of $\{p_r - p_s, q_r - q_s\}$ The parameters on the right side of the dynamic equation for $Z_{r,t}$ can be determined only if the maximum lag order is set to zero.

Using canonical correlation tests, the form of models with scalar components embedded is identified. The SCM (0,0) combination is a linear one, the canonical correlations between past and present being identified by a simple generalization made by Hotelling (1935) for time series. The squares of these canonical correlations will be noted with $\hat{\lambda}_1 < \hat{\lambda}_2 < \ldots < \hat{\lambda}_K$. The likelihood ratio test is applied when the null hypothesis is that there are at least s scalar components and the alternative one refers to the existence of less than s unpredictable components. The test statistic is:

$$C(s) = -(n-h)\sum_{k=1}^{s} \ln(1-\hat{\lambda}_k) \sim \chi^2_{s\cdot[(h-1)K+s]}$$

Consistent estimates of scalar components are given by the canonical covariances corresponding to insignificant canonical correlations. Generalized method of moments based on the test with the same hypothesis as the above one has the statistics, as shown in Anderson and Vahid (1998):

$$(n-h)\sum_{k=1}^{s} \hat{\lambda}_k$$

Using the squared canonical correlations between $x_{p,t} \equiv (x_t', \ldots, x_{t-p}')'$ and $x_{h.t-1} \equiv (x_{t-1}', \ldots, x_{t-1-h}')', h \geq p$ and a similar test a SCM (p, 0) is determined.

SCM (p, j) are linear combinations of $x_{p,t}$, for which linear predictions can not be made in history before *t-j* moment. By the structure of the weighted matrix obtained applying the generalized method of moments is determined a linear combination, which is a moving average of order *j*. In this context a test of supraidentifying the restrictions is applied. Tiao and Say (1989) proposed a statistics:

$$C(s) = -(n-h-j)\sum_{k=1}^{s} \ln\left(1-\frac{\hat{\lambda}_k}{d_k}\right) \sim \chi^2_{s\cdot[(h-p)K+s]}$$

d_k is a correction factor that arises because canonical variations may be moving average processes of order *j*. Thus: $d_k = 1 + 2\sum_{v=1}^{j} \hat{\rho}_v(\hat{r}_k' x_{p,t})\hat{\rho}_v(\hat{g}_k' x_{h,t-1-j})$ ρ_v (.) being the autocorrelation of order v corresponding to the argument and the terms in brackets are the canonical variances of the k canonical correlation. Higher value orders are identified below by testing the orders. Tiao and Say (1989) pointed out the results of the tests in a table that represents the rules for identifying the orders of SCMs. These authors obtained a consistent estimator of the transformation matrix A, starting from the estimated canonical coefficients. They identify the appropriate null eigen vectors in the applying statistical tests.

Athanasopoulos G. and Vahid F. (2008a) present the following rules to ensure unique identification of the system:

- The model structure does not change if each row of the matrix A is multiplied by a constant. This allows normalization of the parameters on each row by one. Using tests of predictability for subsets of variables, we verify that a parameter set to zero is not normalized by one.

- Any linear combination of SCM (p_1, q_1) and SCM (p_2, q_2) is a SCM (max (p_1, p_2, (q_1, q_2)). When there are only two scalar components in the SCM (p_1, q_1), random multiples could be included without changing the structure. Because in this case the line of matrix A corresponding to SCM (p_1, q_1) is not identified, the parameter in column k corresponding to the line from A is normalized by 1. The parameter from line k that corresponds to SCM (p_2, q_2) is restricted to zero.

- If and a submatrix identity is formed and the previous rule is applied twice. If there is only one SCM with an AR/MA of minimal, the corresponded row from A is uniquely identified.

In the original methodology an estimator for A $(\hat{A})$ was obtained and $z_t = \hat{A} \cdot x_t$ was determined, then $\Phi^*_1,\ldots\Phi^*_p$ and $\Theta^*_1,\ldots\Theta^*_q$ which have many null restrictions. The improved methodology of G. Athanasopoulos and F. Vahid (2008a) rewrited the original system variables and by identification of A restrictions we obtain estimates using as method the full information maximum likelihood.

Setting at zero the MA coefficients is equivalent to replacing the MA process variables on the right side of the equation ($z_{t-1},\ldots,z_{t-p}$) with variables $x_{t-1},\ldots,x_{t-p}$, maintaining the system structure. Taking into account the replacement of $z_{t-1},\ldots,z_{t-p}$ with A $\cdot x_{t-1},\ldots,$A $\cdot x_{t-p}$ and the obtain of the system:

$$z_t = \psi_1 z_{t-1} + \ldots + \psi_p z_{t-p} + \varepsilon_t - \Theta_1^* \varepsilon_{t-1} - \ldots - \Theta_q^* \varepsilon_{t-q}, \tag{2}$$

G. Athanasopoulos and F. Vahid (2008b) showed that have the same zero restrictions as $\Phi^*_1,\ldots\Phi^*_p$. This lemma leads to:

$$A \cdot x_t = \psi_1 x_{t-1} + \ldots + \psi_p x_{t-p} + \varepsilon_t - \Theta_1^* \varepsilon_{t-1} - \ldots - \Theta_q^* \varepsilon_{t-q}, \tag{3}$$

in which the parameters satisfy the same restrictions as matrix parameters from the right part of the equation (1). Since not all matrix parameters are free, the system is still unidentified. This situation restricts the matrix A to have a uniquely determined system.

The matrix A is identified if and only if the single matrix H so that $HAx_t = H\psi_1 x_{t-1} + \ldots + H\psi_p x_{t-p} + H\varepsilon_t - H\Theta_1^* \varepsilon_{t-1} - \ldots - H\Theta_q^* \varepsilon_{t-q}$ has the same restrictions as (3) and it is the unit matrix of order K.

It is also assumed that the k row from the system is SCM(p_1, q_1). Null restrictions on the right side of the system show that the row k of matrix H may differ from that of an identity matrix, if there are other SCM (p_1, q_1) models. The row of rank k from matrix is transformed into a row of identity matrix by normalization to 1 of an item in this row.

A canonical representation SCM VARMA has the following characteristics:

i. The orders for SCM are as small as possible;

ii. In order to obtain a unique identification, all the redundant parameters from transformed matrix A are restricted;

iii. Zero restrictions used to determine the number of redundant parameters are set for the corresponding coefficients of MA process.

An empirical example of VARMA modeling methodology. The procedure applied by G. Athanasopoulos and F. Vahid (2007) to ensure that the element that must be set to zero is not normalized to one starts with a SCM of minimum order. One of the variables is excluded and the test of predictability is applied for the rest of the variables. If the test is rejected, then the eliminated variable coefficient is set to 1 and the rest of the coefficients are zero. If the test is accepted (an SCM is formed with the rest of variables) the coefficient corresponding to the eliminated variable is set to zero and the test continues after the elimination of another variable. Tests applied in this case are GMM tests, which are tests of generalized method of moments proposed by Hansen (1982).

1. The identification of scalar components

Tiao's and Say's (1989) methodology for this stage consists of two steps, to which Athanasopoulos G. and F. Vahid (2008b) added a rule of elimination.

The two steps are:

a. Determination of the overall order

All null canonical correlations between $x_{m,t\,c}$and $x_{m,t-1-j}$ are determined, beginning with $m = 0$ and $j = 0$. A table with two parts is built.

Determine all canonical correlations between the null and since $m = 0$ and $j = 0$. It is a table composed of two parts. We start from the top left corner and look the first occurrence of zero eigen values s + K, where s is the number of null eigen values in position (p-1, q-1) of the table. It is considered that (p, q) is the general order of the system. In case we identify several orders of this form, we will select only one using an information criterion.

Table 1. **Criterion table**

		j				
	m	0	1	2	3	4
	0	140.11	90	3.47	2.57	2.55
	1	3.2	0.9	0.98	0.94	0.94
	2	1.23	1.05	1.23	0.9	1.03
	3	1.05	1.08	0.92	1.05	1
	4	0.89	0.92	1.93	0.9	0.98

Table 2. **Root table**

		j				
	m	0	1	2	3	4
	0	1	1	1	1	1
	1	1	4	4	4	4
	2	1	5	9	9	9
	3	3	9	11	15	16
	4	4	10	17	19	24

For 4 macroeconomic variables (inflation rate (π_t), the GDP growth rate (g_t), the 3-month treasury bill rate (r_t), the spread between the 10 year government bond yield and the 3-month treasury bill rate (s_t)) we form the series of quarterly data from the U.S. economy. These variables were used in recent researches in models with observable factors or in the new Keynesian DSGE ones. Federal Reserve Economics Database (FRED) was used to create the data series. Quarterly interest rate data were obtained from monthly data as average. Inflation rate used in VARMA model is different from the one published by FRED, being calculated by multiplying by 400 the difference between the logarithm of consumer price index in the last month of the current quarter and the last month of the previous one. Levin (1999) recommends the first order differentiation of interest rate in monetary policy models. His indication is argued by the fact that it can develop rules for monetary policy less affected by model uncertainty. One of the reasons for which variables as interest rate in first difference are used in this study and the GDP growth rate is related to the need of applying canonical correlation tests with a chi-square asymptotic distribution for stationary series.

We use quarterly data for the period first quarter 1955 – fourth quarter 2000 to build VAR and VARMA models and make predictions based on these models for the horizon first quarter 2001 – second quarter 2011. The identified models are: VARMA (2,1), VAR with 3 lags when the selection criteria is AIC and VAR model with two lags when the selection criteria is BIC, univariate AR (AR(1) for GDP growth rate, AR(3) for inflation rate and AR(2) for other variables.

The VARMA model is:

$$\left\langle\begin{matrix}1 & 0 & 0 & 0\\ -0{,}82 & -0{,}03 & 0{,}15 & 1\end{matrix}\right\rangle\cdot\left\langle\begin{matrix}g_t\\ \pi_t\\ s_t\\ \Delta r_t\end{matrix}\right\rangle=\left\langle\begin{matrix}0{,}72\\ 0{,}5\\ 0{,}23\\ -0{,}64\end{matrix}\right\rangle+\left\langle\begin{matrix}0{,}13 & -0{,}05 & 0{,}41 & 0{,}55\\ -0{,}38 & 0{,}89 & 0{,}3 & 0{,}24\\ -0{,}04 & 0{,}04 & 1 & -0{,}11\\ 0 & 0 & 0 & 0\end{matrix}\right\rangle\cdot\left\langle\begin{matrix}g_{t-1}\\ \pi_{t-1}\\ s_{t-1}\\ \Delta r_{t-1}\end{matrix}\right\rangle+$$

$$\left\langle\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ -0{,}02 & -0{,}07 & -0{,}15 & 0{,}05\\ 0 & 0 & 0 & 0\end{matrix}\right\rangle\cdot\left\langle\begin{matrix}g_{t-2}\\ \pi_{t-2}\\ s_{t-2}\\ \Delta r_{t-2}\end{matrix}\right\rangle+\left\langle\begin{matrix}0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0{,}62 & -0{,}57 & -1{,}23 & 0\\ 0 & 0 & 0 & 0\end{matrix}\right\rangle\cdot\left\langle\begin{matrix}e_{1,t-1}\\ e_{2,t-1}\\ e_{3,t-1}\\ e_{4,t-1}\end{matrix}\right\rangle+\left\langle\begin{matrix}e_{1,t}\\ e_{2,t}\\ e_{3,t}\\ e_{4,t}\end{matrix}\right\rangle$$

b. The determination of scalar components orders

We test the null canonical correlations between $x_{m,t}$ and $x_{m+(q-j)t-1-j}$, where m = 0 ,..., p and j = 0 ,..., q. The SCM (m, j) includes all the scalar components of smaller order, in (m, j) position there are s scalar components will be and scaling components, where $s = min\ \{m_{-p1}+1_{,\,j-1}\}$ for each SCM (p_1, q_1).

2. The place of restrictions of identification

Identification rules are applied to determine the structure of the matrix A.

3. The estimation of the system of unique identified parameters

The parameters estimation method is the full information maximum likelihood presented by Durbin (1963). This method provides estimates for both parameters, as well as standard errors of parameters, including the free ones.

The trace and the determinant of the mean square errors matrix are classical measures of forecast accuracy, used by G. Athanasopoulos and F. Vahid (2007). We use the generalized forecast error second moment as measure of accuracy. This is calculated according to Clements and Hendry (1993) as a determinant of the expected value of the vector forecast errors for future times on the horizon of interest. If we study a number until h quarters, this indicator is calculated as:

$$GFESM = \left| E\begin{pmatrix} e_{t+1} \\ e_{t+2} \\ \dots \\ e_{t+h} \end{pmatrix} \cdot \begin{pmatrix} e_{t+1} \\ e_{t+2} \\ \dots \\ e_{t+h} \end{pmatrix}^T \right|$$

e_{t-h} – N-dimensional forecast error of the model of n variables on the forecasting horizon h

It is considered that GFESM is a better measure of accuracy because it is invariant to elementary operations with variables, unlike the trace of MSFE, and also it is a measure invariant to elementary operations for the same variables on different forecasting horizons, unlike the trace or the determinant of MSFE.

We propose as a measure to compare the accuracy of forecasts basing on these models a new indicator: the ratio GFESM relative to the VARMA model.

We calculate the ratio GFESM relative to the VARMA model one quarter ahead for the two models ($h = 1$). For multivariate models we also calculate GFESM separately.

Table 3. **Ratio of the GFESM relative to the VARMA model for one step ahead forecasts**

Model	GDP growth rate	Inflation	s_t	Δr_t	| GFESM |
VAR (AIC)	0.93	1.01	1.22 **	1.59 *	1.69
VAR (BIC)	0.91	1.06	1.24 *	2.06 *	2.11
AR	0.82	1.03	1.09	1.22 *	-
Naive model	0.81	1.35	6.82 *	1.45 *	-

* For the significance level of 5% the ratio differs significantly from 1.
** For the significance level of 10% the ratio differs significantly from 1.

Analyzing Table 3, the VARMA model provides forecasts with a higher degree of accuracy than VAR models for variables S_t and Δr_t. Knowing that these variables are not affected by structural shocks it is likely that forecasts based on VARMA models are better than those based on VAR models.

Conclusions. Scalar components methodology used in building VARMA models is quite difficult to apply on practice, but for small time horizons, the forecasts based on these models are better than others for the variables unaffected by structural shocks. This conclusion has been also reached by G. Athanasopoulos and F. Vahid, but indicators used to measure the accuracy were the classical ones: the trace and the determinant of MSFE. In this study, the accuracy is evaluated using as indicator the generalized forecast error second moment. We introduce a new measure for evaluating the relative accuracy in order to make comparisons between forecasts: the ratio of the GFESMs relative to the VARMA model.

Bibliography:

1. *Athanasopoulos, G., Poskitt, D.S., Vahid, F.* (2007). Two canonical VARMA forms: Scalar component models vis-a-vis the Echelon form, Working paper 10/07. Department of Econometrics and Business Statistics, Monash University.

2. *Athanasopoulos, G., Vahid, F.* (2008a). A complete VARMA modeling methodology based on Scalar Components, Journal of Time Series Analysis 29(3), p. 533-554.

3. *Athanasopoulos, G., Vahid, F.* (2008b). VARMA versus VAR for macroeconomic forecastin, Journal of Business and Economic Statistics 26(2), 237-252.

4. *Clements, M.P., Hendry, D.F.* (1993). On the limitations of comparing mean squared forecast errors (with discussions), Journal of Forecasting, 12, p. 617-637.

5. *Mainassara, Y.B.* (2010). Selection of weak VARMA models by Akaike's information criteria, MPRA Paper No. 23412, http://mpra.ub.uni-muenchen.de/23412/

6. *Tiao, G., Tsay, R.* (1989). Model specification in multivariate time series (with discussions), Journal of the Royal Statistical Society B 51, p. 157-213

7. Federal Reserve Bank of St. Louis. Economic information. Available at http://research.stlouisfed.org/fred2/ , accessed July 31, 2011.

Стаття надійшла до редакції 08.09.2011.