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FINANCIAL MELTDOWN AND BOND PRICES: AN EMPIRICAL ANALYSIS

Yields, over the past few years, have been volatile due primarily to a combination of two factors; the reaction of Federal Reserve (FED) to the events by changing the interest rates and the repricing of risks by investors. These factors have a key impact on determination of bond pricing; this study uses several measurements of bond pricing estimations to illustrate the effect of yield volatility on the pricing of a 10-year US Treasury note issued on February 15, 2007, to 26 February 2010. Even though the analytical evidence shows the existence of volatility affecting the estimated pricing of the observed 10-year US Treasury note, the empirical evidence indicate a possible random walk model.

Keywords: *Federal Reserve; US Treasury; bonds; price nobility; interest rate; GARCH models.*

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ФІНАНСОВА "ВІДЛИГА" ТА ЦІНИ НА ОБЛІГАЦІЇ: ЕМПІРИЧНИЙ АНАЛІЗ

У статті доведено, що в останні кілька років прибутки були нестабільними з двох причин: реакція Федерального резерву США на різні події шляхом зміни відсоткових ставок та переоцінка ризиків інвесторами. Ці чинники також є ключовими при формуванні цін на облігації. Кількома способами дано оцінку формуванню цін на облігації, а волатильність прибутків продемонстровано на прикладі 10-річної облігації Казначейства США, датованої 15 лютого 2007 р., до 26 лютого 2010 р. Аналіз вказує на існування волатильності, яка впливає на вартість облігації, що досліджується. Дані спостережень демонструють, що ціни на облігації слідують моделі випадкових блукань.

Ключові слова: *Федеральний резерв США; Казначейство США; облігації; волатильність цін; відсоткові ставки; GARCH-моделі.*

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ФИНАНСОВАЯ "ОТТЕПЕЛЬ" И ЦЕНЫ НА ОБЛИГАЦИИ: ЭМПИРИЧЕСКИЙ АНАЛИЗ

В статье доказано, что в последние несколько лет прибыли были нестабильны по двум причинам: реакция Федерального резерва США на различные события путем изменения процентных ставок и переоценка рисков инвесторами. Данные факторы являются также ключевыми при ценообразовании облигаций. Несколькими способами оценено ценообразование облигаций, чтобы продемонстрировать волатильность прибылей на примере 10-летней облигации Казначейства США, выпущенной 15 февраля 2007 г., до 26 февраля 2010 г. Анализ указывает на существование волатильности, которая влияет на стоимость исследуемой облигации. Данные наблюдений демонстрируют, что цены на облигации следуют модели случайных блужданий.

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Ключевые слова: *Федеральный резерв США; Казначейство США; облигации; волатильность цен; процентные ставки; GARCH-модели.*

I. Motivation and Background. Recently, we have witnessed a growing volume of research on the current credit crisis and following financial meltdown such as Pariente et al. (2011), Masood et al. (2010), Feldstein (2009), Brunnermeier (2008). While, in times of economic depression, there are in reality two possible options available for FED to raise much needed funds: increasing money supply or debt. In fact, FED usually relies on debt to raise funds to pay for their fiscal stimulus policies as demonstrated by the rise in the US public debt from $8.68 trillion in January 2007 to $12.77 trillion in March 2010, a rise of 47% within just 27 months, according to data obtained from the Bureau of Public Debt as of April 2010. This is a huge increase in the supply of US Treasuries over a short period of time. However, during any period of uncertainties in financial markets, such as the recent global financial crisis, there tends to be an increase in demand for US Treasuries influenced by investors repricing of risks. With the basic rate of interest at or approaching zero, the FED turned to increasing monetary bases in terms of the use of quantitative easing (I-II) which further amplified demand for these securities. As with any situation when demand for an asset is hiked, there always tend to be upward pressures on the equilibrium price. In addition, the yield/price relationship dictates when yields are low prices tend to be high and since one of the factors influencing the yields is the basic rate of interest; hence the FED reacting to the events by lowering the basic rate of interest further increasing the upwards pressures on US Treasuries equilibrium prices. As Piazzesi (2005) hinted, bond yields response to policy decisions of FED and the policy decisions of FED are influenced by bond yields. These factors when combined with the downturn in the economy and financial markets seem to be the main reasons behind the volatile bond yields and prices in recent years. As we attempt to demonstrate that, this volatility has an effect on bond price estimations models.

In terms of risk management of bonds, there are no market indicators more relevant than duration measures and convexity. Each of these models is in itself a derivative of the yield-price relationship which is at the heart of bond analysis. News directly linked to a bond market has the potential of making yields and hence prices volatile. Underpinning both modified duration and convexity which is the measurement of the "essence of the time element of a loan" as stated by Macaulay (1938, p. 4). Moreover, although as noted by Cox et al. (1979), Macaulay was primarily concerned with the risk properties of duration, yet as stated by Yawitz (1977), duration in its purest form is just a measure of a bond's time dimension. Modified duration is the more relevant model to this research as it measures the bond price "elasticity with respect to a discount ratio", as stated by Hicks (1939, p. 186). The other relevant model is convexity that is the measure of sensitivity of duration to yield movement according to Kritzman (1982). Both modified duration and convexity can be used to estimate bond prices and all these models are important bond market indicators, especially in fixed income portfolio management.

Yawitz (1977) proved mathematically and empirically that bond price volatility is influenced by yield volatility and duration and states when concerning information about price volatility, duration is obviously superior to maturity. However, as Cox et al. (1979) pointed, the problem is a bond with long duration may not proportionally be affected more than a bond with short duration by changes in the spot rate. Therefore, they concluded that basis risk can only be measured by Macaulay duration under conditions, which are both theoretically and empirically unrealistic in that any small change in the interest rate will cause an approximately equal shift in the yield curve across all maturities. Therefore, leading to a new measure of duration, stochastic duration, defined as the sensitivity of a price of a bond (or portfolio of bonds) with respect to any change of the term structure consistent with the model. The consensus of past empirical evidences and theoretical models support the view that credit risk could shorten effective duration. However, Babbel et al. (1999) stated that basic risk is associated with uncertain movement in the yield spread between a financial instrument and the reference interest rate over time. Hence, in addition to default risk, the differences between Macaulay duration and effective duration stem from different behaviour of various reference interest rates used. The use of a common reference interest rate would allow for greater confidence in measurement of duration within a portfolio of bonds subjected to varying credit risks. Crack & Nawalkha (2001) argued that it is dollar duration that measures the absolute dollar change in the bond price for a given change in yield. Thus, the changing slope in the price/yield relationship curve does not illustrates changing Macaulay duration or modified duration; rather it illustrates changing dollar duration. Similarly, curvature in the price/yield relationship curve illustrate dollar convexity and differs substantially from the convexity. Both dollar duration and convexity decrease rapidly with increasing yields regardless coupon rates and others factors being equal increase with increasing coupon levels.

Each bond is affected by yield volatility over time and reacts differently regarding the term structure. In the periods of consistently high volatility like the recent crisis and subsequent financial meltdown leading to the sovereign debt crisis, the best times series model is known as GARCH model. However, technically this is a very restricted view of the GARCH model, in essence that GARCH is a model of time varying volatility in financial or economical time series noted by numerous papers such as Nelson & Cao (1992). A key factor in financial observations is that on occasions the distribution can be skewed as noted by Franses & Djik (1996). Some GARCH models cannot cope with such skewness; hence, this would result in some forecasts being bias for skewed time series. However, there are models such as the quadratic GARCH model and the GJR model that explicitly takes into account skewed distribution. As noted by Engel (2001), in general in financial analysis there are occasions when periods of risks are greater than others and these periods are not scattered randomly; hence meaning there is a degree of autocorrelation in risks. To a certain degree bond prices are affected by uniformed periods of risks hinting at a degree of autocorrelation of riskiness. This is mainly due to the effect of outside factors on the yield which has a knock on effect on bond prices. The main benefit of the GARCH model has been observed to capture sudden movement in volatility and recover faster.

Dungey et al. (2009) used the GARCH model to analyse flight to quality in respect to the US Treasuries market. Using the asymmetric GARCH model TGARCH (or TARCH), they explained the positive sign asymmetries found in most flights to quality. During any period of uncertainty such as the recent banking crisis, increasingly risk averse investors tend to sell high risk assets and buy low risk assets. As noted by Dungey et al. (2009), this leads to low risk asset markets, such as the US Treasuries, exhibiting positive sign asymmetries, i.e., "a positive price shock in the low risk asset may generate a disproportionately large volatility response". While the high risk asset will suffer from negative asymmetries, Jones et al. (1998) used the GARCH model among other models to analyse the persistence of volatility on several treasury bonds through macroeconomic news announcement shocks. However, Bollerslev et al. (2000) researched on 5 min intraday return volatility in US Treasury bond futures over the period 1994-1997 using the fractionally integrate GARCH (FIGARCH) model seem to indicate long-memory volatility in certain macroeconomic announcements.

The main goal of this paper is to establish the effect of volatile yields during the uncertain period of the last few years on bond price estimations. For this purpose, the paper is structured into 3 sections including theoretical and empirical parts. Following the motivation and background, the second section researches the theories and models underpinning bond analysis: duration, modified duration and convexity together with the GARCH models and evaluates the bond pricing estimation models during the current crisis of 2007-2009 and financial meltdown. Then, it focuses on the analysis of the yields and estimated bond prices using some GARCH models. Finally, the last section is assigned to concluding remarks.

II. Effect of Volatile Yields on Bond Pricing Estimation Models and Empirical Evidence. The US Treasuries yield data used in this research is obtained from the Federal Reserve Bank of St. Louis on 2nd March 2010. The sample range from 16th February 2007 to 26th February 2010, total the observations are 791. The sample data is interpolated by setting observations with NA to the last known data. It is essential to note that unlike the auction where the price is gained from the yield bid, at the secondary market the yield is gained from the price. As noted by the US Treasury's website as of April 2010, the daily yields are obtained and derived from the close of market bid yield on actively traded on the run US Treasury securities by New York FED using a quasicubic hermite spline algorithm.

In the absence of access to data services such as Bloomberg and DataStream, it is decided to calculate the prices using the yields obtained from the US Treasury website. Although this would seem to be rather odd using yields obtained from prices to calculate the prices through the use of the present value equation, yet in the absence of daily market prices this is the optimal way to get the prices. Therefore, prices may vary from the officially listed prices for the US Treasury 10-year note issued 15th February 2010 but since as mentioned above the yield is obtained from the market prices, therefore the prices are an accurate approximation based on the yield at any given time. Hence, for the purpose of this article this estimated price using the present value equation is regarded as the listed bid price.

Graph 1 shows the trend of the yields during the period of observation on a daily basis. The most important aspect of the shape is, as clearly shown by the trend line, the downwards sloping curve which is usually a sign of high demand. Graph 2 shows the trend in bond prices over the period on a daily basis. Given the trend of the yield is downwards, the prices are upwards trending. The striking feature of both graphs is around the spring/summer 2007 and the autumn of 2008, both highlight the low and high yields/prices of this US Treasury note. Analysing Graph 3 would hint at uncertainty in the economy and increasing risk aversion during 2007. However, the Federal Reserve intervention thru aggressive interest rates cuts throughout the first part of 2008 and the introduction of quantitative easing at the height of the banking crisis means an inverted yield curve is averted. In bond analysis, probably the most relevant factors are the price/yield and term/yield, more commonly known as the yield curve relationships. Under the normal conditions, both relationships are "opposite attract" meaning that in general the term/yield has a positive correlation while the price/yield has a negative correlation. The bond analysis is based on the following equations as illustrated by Llano-Ferro (2009):

$$Dur_{Mac} = \frac{\sum_{t=1}^{T} \frac{tC_t}{(1+y)^t}}{P}$$

$$MDur = \frac{Mac_{Dur}}{1+\left(\frac{y}{2}\right)}$$

$$CV = \frac{\frac{1}{P(1+Y)^2} \sum_{t=1}^{T} \frac{C_t}{(1+y)^2} t(t+1)}{f^2}$$

Graph 4 shows that all 3 models are reliant on the term structure. Notice the regular shifts in the downwards trending curves, these shifts are steeply upwards in duration and modified duration and steeply downwards in convexity. The importance of these shifts is that they coincide with the semi-annual payments therefore hinting at the frequency of payments having an effect on the overall structure of these models.

As can be seen in Table 1, the variance and standard deviations are large enough to have impact on all 3 models. The correlation with yields for all 3 models would suggest there is a positive relationship meaning that when yield moves the models move in the same direction. However, the covariance between yields and 3 models is insignificantly low. There is evidence of a negative correlation with the price which hints at an opposite movement. There seem to be strong evidence of a negative covariance with price which suggests that when price moves there is a similar movement in the opposite direction.

Table 1. **Statistical evidence on 3 bond analysis models**

	Duration	Modified Duration	Convexity
Mean	7.061	6.928	59.085
Variance	0.340	0.311	82.491
Standard Deviation	0.583	0.557	9.082
Covariance with Yields	0.003	0.002	0.039
Correlation with Yields	0.649	0.625	0.646
Covariance with Price	-1.665	-1.521	-25.525
Correlation with Price	-0.587	-0.561	-0.577

Even though at the first sight evidence seems to suggest there are no signs of effect on 3 models of analysis from the crisis and the following recession. However, looking at the curves closer, we notice how towards the end of 2008 there is a noticeable hike on all 3 curves. At that time, the crisis was at the peak and the FED decided to introduce quantitative easing by lowering the basic rate of interest to approximately zero and increasing monetary bases to hold the overnight interest rate at this rate, hence high demand and low interest rates were increasing prices and decreasing yields which may have affected the bond analysis at the time.

There are two price estimations methods, which use modified duration or convexity techniques to estimate the bond prices. Given that modified duration is the first derivative and convexity is the second derivative of the price about the price/yield relationship; hence by differentiating:

$$MDur = \frac{-Dur_{Mac}}{(1+y)}$$

We should get

$$\Delta P \cong -MDur\Delta y$$

Using Taylor series expansion the convexity model becomes

$$\Delta P \cong -MDur\Delta y + \frac{CV(\Delta y)^2}{2}$$

Since price can be estimated by the following equation:

$$P \cong P_{-1} + \Delta P$$

Therefore, substituting equations 4 or 5 for ΔP in equation 6 would give us price estimation using either modified duration or convexity. However, there is a difference in two price estimations as illustrated by Llano-Ferro (2009).

In the concept of this research, the relevancy of the GARCH model using student-t is that it picks up changes in the volatility of the time series. The equations of the estimation models for yield and the estimated prices are given by:

$$Yield_t = \beta_1 + \beta_2 Yield_{t-1} + \beta_3 Yield_{t-2}$$

$$\mathrm{Pr}ice_t = \beta_1 + \beta_2 \mathrm{Pr}ice_{t-1} + \beta_3 \mathrm{Pr}ice_{t-2}$$

Graph 5 shows the distribution of the US Treasury 10-year note's yields. The distribution of the yield has a negative skew of -0.074290 and a positive kurtosis of 2.757866 which is often referred to as leptokurtic. This means that the distribution is not normal and the deviation from normality.

Table 2. **Distribution Statistics of Price Estimations Models**

	Yields	Present Value	Modified Duration	Convexity
$Resid^2_{-1}$	0.039520	0.044784	0.045939	0.045929
$GARCH_{-1}$	0.957542	0.952370	0.946623	0.946880

There is a similar distribution in all three estimation models but there are noticeable differences between the PV model and the other two as shown in Graphs 6-8 and Table 2.

Table 3. **GARCH Key Variables Output**

	Yields	Present Value	Modified Duration	Convexity
$Resid^2_{-1}$	0.039520	0.044784	0.045939	0.045929
$GARCH_{-1}$	0.957542	0.952370	0.946623	0.946880

The GARCH model for conditional variance is given by Bollerslev, (1986). $\text{var}_t = \delta + \alpha_1 e^2_{t-1} + \beta_1 \text{var}_{t-1}$. Given the US Treasury 10-year yields and prices, these two models with Student's t distribution would produce the results in Table 3. Theoretically, one of the rules for GARCH is that $\alpha_1 + \beta_1$ is between 0 and 1, however if $\alpha_1 + \beta_1 \geq 1$ it is assumed to be an IGARCH model (Engel & Bollerslev, 1986). As Table 4 rows 1 and 2 hint the sums of these two coefficients for all the variables are approaching 1; they are on the verge of IGARCH but still within the range of GARCH.

Table 4. **T-GARCH Key Variables Output**

	Yields	Present Value	Modified Duration	Convexity
$Resid^2_{-1}$ x $(Resid_{-1})$ Where $Resid_{-1} < 0$	-0.005880	-0.002275	0.010642	0.013199

Another factor to consider is the T-GARCH model of asymmetric effects; this would differentiate between bad news and good news (Zakoian, 1994).

The equation for T-GARCH is $\text{var}_t = \delta + \alpha_1 e^2_{t-1} + \gamma d_{t-1} e^2_{t-1} + \beta_1 \text{var}_{t-1}$

where $d_1 = \begin{cases} 1, e_t \prec 0 (badnews) \\ 0, e_t \geq 0 (goodnews) \end{cases}$.

This means that a T-GARCH would collapse to a simple GARCH model if there is good news. As shown in Table 4, given coefficients of the first two asymmetric terms are negative numbers, then d must have been 1; however, both are not sufficient enough to say there is evidence of asymmetric effects in the variance of yields and prices. Yet, it seems that the last two have collapse to a simple GARCH model due to their positive coefficients, in short, this means there is no evidence of asymmetric effects in the variance of the two estimation models.

Table 5. **GARCH in Mean Key Variables Output**

	Yields	Present Value	Modified Duration	Convexity
GARCH	-288.173	0.248142	0.231191	0.198768
(z-statistics)	-2.021588	1.593647	1.060225	0.921263
T-stat (DOF)	7.914127	8.074627	8.441192	8.493225
$Coefficient_{-1}$	0.962694	0.963651	1.032232	1.033990
$Coefficient_{-2}$	0.032121	0.030370	-0.038009	-0.039724

The third factor to consider is the GARCH-M model (Engel et al., 1987) which determines whether volatility affects yields and prices. This model is given by the equation $y = \beta_0 + \theta \text{var}_t + e_t$. Given that the z-statistics of the GARCH variable in Table 5 is greater than 2 in absolute value, this would suggest there is evidence of volatility affecting the yield but the problem is that volatility must be very small for a coefficient of -286.165. With absolute z-statistic values of less than 2, there seems to be even less and reducing evidence of volatility across all 3 price estimation models. Still analysing the coefficients of all the estimation models' GARCH variable would suggest otherwise. Given that the t-statistics is sufficiently large, it is possible that there is some degree of volatility affecting the price estimation models.

Since β_3 is insufficiently low and β_2 is approximately 1, the resulting equation of is $X_t = \beta_2 X_{t-1} + \beta_1$ the random walks model. Therefore, all the models have signs of a random walk. This is supported by Graphs 9 to 12, which show that the variances seem to follow the random walk model.

The evidence from all the models seems to point to a second order stationary point with a first order non-stationary point. Using the augmented Dickey-Fuller test (Dickey & Fuller, 1979) provides us with another possible reason for inconclusive results given by the GARCH models, the existence of unit roots in all four-time series datasets. The evidence from all the datasets seem to point to a second order stationary point at with a first order non-stationary point. The augmented Dickey-Fuller test confirms this to be the case with a first order test result greater than the 5% level and a second order test result less than the 5% level.

III. Final Remarks. In financial research, the essence of empirical studies is based naturally on econometric models. Since, this study is essentially a time-series analysis of the yields/price relationship under uncertainties during the credit crisis and following recession of the late 2000s. The evidence from the analysis of the 3 estimation models seem that yield volatility does make price estimation harder. This is illustrated by the crisis and recession of the last couple of years. However, the evidence also gives the impressions that the FED's reaction to events is main cause of the volatility. It seems that all the pricing estimation models follow a random walk model, hence the GARCH model of variance over time for both factors are limited. However, there is a case to be made over a longer period and given different issues and maturities of bonds. The fact is that each bond has its own yields and prices, which are affected by coupon rate and term to maturity, so this might be a major factor to consider when undertaking a thorough analysis of the bonds' prices and yields. Lastly, the suspicion is that the restricted observational data which was taken during a period of high volatility is limiting the scope of the study. A more robust study would extend the period over a longer time scale and include various US Treasury bonds with different maturities and coupons rates. In addition, to analyse the estimated prices, there would be a requirement to include the official daily bid-ask prices as well as yields.

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