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FINANCIAL AND DEMOGRAPHIC RISK IMPACT ON PRIVATE PAYG PENSION SYSTEM: THE ITALIAN CASE

The aim of this paper is to analyze private pension systems financed by pay-as-you-go, with a focus on the pension funds of the Italian Professional Orders. The research centres on the financial and demographic risks and on their impact on the future evolution of the fund. It presents a model to investigate the dynamics of the types of pension funds which operate according to the pay-as-you-go rule: the two stochastic variables global asset return and new entrants variation rate are modelled by autoregressive processes. The numerical applications are carried out using the data provided by the Italian Chartered Accountants' pension fund.

Keywords: *Pension Funds; PAYG system; demographic risk; stochastic new entrants; extinction risk.*

JEL classification. G23; H55; J11.

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ВПЛИВ ФІНАНСОВИХ ТА ДЕМОГРАФІЧНИХ РИЗИКІВ НА ПРИВАТНУ РОЗПОДІЛЬНУ ПЕНСІЙНУ СИСТЕМУ: НА ПРИКЛАДІ ІТАЛІЇ

У статті проаналізовано приватні пенсійні системи на базі розподільних виплат (PAYG), зосереджуючися на пенсійних фондах італійських профспілок. Дослідження базується на фінансових і демографічних ризиках та їхньому впливі на подальший розвиток фонду. Представлено модель дослідження динаміки таких типів пенсійних фондів, які оперують за принципом розподільних виплат: оборот загальних активів за двома стохастичними змінними та варіативність кількості нових членів змодельовано за авторегресивними процесами. Обчислення здійснено з використанням даних, наданих пенсійним фондом дипломованих бухгалтерів Італії.

Ключові слова: *пенсійні фонди; система PAYG; демографічний ризик; ймовірні нові члени; ризик погашення.*

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ВЛИЯНИЕ ФИНАНСОВЫХ И ДЕМОГРАФИЧЕСКИХ РИСКОВ НА ЧАСТНУЮ РАСПРЕДЕЛИТЕЛЬНУЮ ПЕНСИОННУЮ СИСТЕМУ: НА ПРИМЕРЕ ИТАЛИИ

В статье проанализировано частные пенсионные системы на базе распределительных выплат (PAYG), сосредоточившись на пенсионных фондах итальянских профсоюзов. Исследование основано на финансовых и демографических рисках и их влиянии на последующее развитие фонда. Представлена модель исследования динамики таких типов пенсионных фондов, которые оперируют по принципу распределительных выплат: оборот общих активов по двум стохастическим переменным и вариативность количества новых членов смоделированы в соответствии с авторегрессивными процессами. Вычисления осуществлены с использованием данных, предоставленных пенсионным фондом дипломированных бухгалтеров Италии.

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Ключевые слова: *пенсионные фонды; система PAYG; демографический риск; вероятные новые члены; риск погашения.*

1. Introduction. The function of a pension scheme is to redistribute financial cash flows paid in the work period (contributions) and returned in the retirement time (benefits). Pension schemes are usually classified by the rules used to calculate the benefits (or the contributions) and by financing system. They can be divided into either a defined contribution scheme or a defined benefit scheme. In a defined contribution scheme, the contributions are made to a pension fund without specifying the benefits, which depend on the fund portfolio, whereas, in a defined benefit scheme, a sponsor fixes the benefits in advance and contributions are adapted to maintain the fund in balance.

From the financial point of view, these schemes can be classified as either a funded system or a pay-as-you-go (PAYG) system. In a funded system, contributions are used to purchase assets, which are kept to pay for future benefits; the PAYG system is based on the immediate use of contributions to pay current pensions. In a PAYG system an "intergenerational pact" is established, in which the retired generation is sustained by the active one.

Funded schemes are above all exposed to financial risks. For example, during the financial crisis of 2008-2009, it is estimated that in Europe the pension funds have reduced their wealth accumulation by 15.8 % (OECD, 2009). As highlighted by Borsch-Supam (2010) financial crises have different impact from demographic ones. A financial crisis is a short-term shock of a few years, while demographic change is a long term process lasting several decades.

In Europe, a PAYG system is generally used for public pension schemes, instead private schemes (relative to first and second pillar) are managed as funded systems. According to OECD (2009) in most countries of Europe the share of unfunded pension income in total retirement income is more than 90%; among these Italy, France, Belgium, Spain; while in some northern countries it is smaller: the Netherlands (53,5%), UK (56.2%), Ireland (63.9%).

To manage the crisis due to the aging population some countries adopted parametric reforms, rising retirement age and contribution rates, others, like Sweden, Italy, Latvia and Poland introduced notional defined contribution (NDC) system, where pension is linked to contributions paid during the working life.

Until 1995, in Italy, all the workers' pensions were administrated by the state: all social security was publicly managed and the financial mechanism used was PAYG (with a defined benefit). In 1995, a reform of the National Pension Plan was enforced in Italy (L. 335/1995 – the so-called "Dini Reform") establishing the transition from a defined benefit to a defined contribution scheme. Another relevant step of the Reform was privatization of the retirement funds of Italian Professional Orders. In Italy, each professional group recognized by a Board (such as lawyers, doctors, accountants, engineers etc.) administers its own retirement fund. After privatization these funds continued to operate according to a PAYG financing mechanism.

In this paper, we propose a model to describe the evolution of these "closed" pension funds, according to the PAYG rule. The fund is defined closed because it has a restriction on membership: only those who have a specific job or profession have a

right to become the members of the fund. These types of funds show an anomaly as, in general, PAYG systems are applied to an open group. However, this specific PAYG system has a barrier to entry, as only those who pass a public examination can exercise a profession and consequently become members of a fund.

There is a broad literature emphasizing financial and demographic risks associated with private pension funds operating in a funded scheme. Regarding financial risks, the investment risk is linked to the randomness of the return rate; this derives from changes at financial markets where a plan invests and occurs in deviations of the real return rate from its expected values. This issue was discussed in numerous papers. Among these Dufresne (1988), Haberman (1994), Cairn and Parker (1997) analyze investment risk for a defined-benefit pension fund. Blake, Cairns and Dowd (2001, 2003) propose a model for a defined-contribution pension fund, with stochastic wages and returns, in a discrete and continuous time respectively.

Regarding demographic risks, it is well known that the longevity risk, which derives from improvements in the mortality trend and determines systematic deviations in the number of deaths from its expected value. This kind of risk must be faced by all types of pension plans and life insurance products. On this topic an exhaustive literary review can be found in Pitacco (2004).

In particular for a PAYG scheme, financial sustainability is related to the balance between active and retired members. There is a further demographic risk source to take into account: the risk relates to future monetary cash flows necessary to ensure payments of future pensions. This risk is related to the demographic variable "new entrants" and to their future contribution capacity. This is a relevant component because a balance between the number of contributors and the number of pensioners is necessary to sustainability of a fund. In this sense we have identified a further risk related to the demographic variable "new entrants", that we called "extinction risk". We refer to as the possibility that in the short-term changes, either in the job market or regulatory actions in the world of professions produce a more or less sudden reduction in enrolments to the professional order, causing a financial disequilibrium on the fund cash flows, and consequently to the related retirement fund sustainability.

Over the past few years, a vast literature regarding demographic risks in public PAYG system was developed. In this paper we do not examine public PAYG systems, only private ones.

Actuarial literature on pension population is mostly based on deterministic models (Bower et al, 1976, Winklevoss, 1993) which consider the population as stationary. There are some contributions which take into account the fluctuation of the number of new entrants in a pension fund. Mandle and Mazurova (1996) use spectral decomposition of stationary random sequences to investigate a pension scheme under randomly fluctuating rates of return and number of entrants. Iyer (2003) derives algebraic expressions for the variances of, and covariance among important aggregates that characterize the development of a pension scheme taking into account the stochastic variation of the new entrants. Colombo and Haberman (2005) analyze the impact of the stochastic evolution of active membership population on the mismatch between assets and liabilities of a (funded) defined benefit pension scheme. Menoncin (2005) studies the allocation problem for a pension fund, which behaves according to a

PAYG rule considering the total number of workers and pensioners as random variables. Melis and Trudda (2010) propose a model for evolution of a PAYG pension fund, with stochastic new entrants and global asset return, they adopt risk indicators to monitor the solvency of a fund.

In a pure PAYG pension system revenues equal outlays each year (Towerbridge, 1952). Here we analyze a spurious PAYG scheme, yet being in a growth phase (contributors greater than pensioners) where there is the accumulation of partial reserves. The active population evolves according to the process of new entrants, which can be different for each fund. Therefore, the main problem is to analyze the flow of new entrants into the fund, to find out if the number of future taxpayers is sufficient to maintain a system in balance, and thus to ensure its solvency.

We study the impact of the stochastic component "new entrants" analyzing how its variation rate influences future cash flows of a fund. The goal is to measure and to compare the influence of the stochastic variable "new entrants" and the stochastic variable global asset return, with respect to the evolution of the overall fund.

A numerical application to the data of Cassa Nazionale di Previdenza e Assistenza a favore dei Dottori Commercialisti (CNPADC – the pension fund managing the pension system of Italian chartered accountants professional order) is developed. Some comparisons between the influence of the financial variable "global asset return" and the demographic variable "new entrants" are carried out to show the most persuasive one by analyzing the impact of the single stochastic variable.

The work is organized as follows: Section 2 illustrates the single components of the cash flows of the fund; in Section 3 a model to measure the fund evolution is proposed; Section 4 presents a numerical application of the CNPADC data; finally, in Section 5 conclusions and proposals are drawn from the empirical investigation.

2. Fund Evolution. As already stated, the purpose of the analysis is to study the impact of the evolution of new entrants in a pension fund financed with PAYG. There are different approaches to analyze future flows of new entrants for these types of professions. Here we propose a model for evolution of new entrants based on the analysis of the variable "new entrants variation rate".

In this section we study the fund evolution through the dynamical analysis of its single components.

The amount of the total assets belonging to the pension scheme at a specific time t represents the fund value. Excluding the fixed cost of management the evolution of the fund can be represented as follows:

$$f(t+1)=[f(t)+C(t)-B(t)][1+r(t,t+1)] \quad (1)$$

where $C(t)$ and $B(t)$ represent respectively the annual contribution income and pension benefits paid at the beginning of the year t and $r(t,t+1)$ indicates the global asset return related to the period between time t and time $(t+1)$.

The evolution of the annual contributions $C(t)$ is analyzed starting in a generic time zero and adopting a recursive dynamic year by year. The assumptions are as follows:

- α is the only entry age to the scheme;
- τ is the retirement age; in this way $\tau - \alpha$ is the length of the contribution period;
- ω is the extreme age; $\omega - \tau$ is the maximum length of the retirement period;

- the scheme provides pensions only upon reaching the age of retirement (disability or survivors' pensions are not considered);

- administrative costs are not included;

- in the theoretical framework for notation simplicity we consider the same contributory seniority for the members of the same age.

The analysis is simply extendible releasing to these hypotheses (in the applications real seniority and administrative costs are considered).

In this way it is possible to study the amount of contributions $C(t)$, paid at time t, decomposing the total quantity by the age of contributors.

In a generic time t the total contributions are calculated by the summation for all the ages x between α and t of the number of contributors aged x at time t multiplied by the average wage at time t and the contribution rate.

If we separate the original population (members already in the scheme at time 0) from new entrants (members entered in the scheme after time 0), we have the sum of two summations to calculate the contributions, one for the contributions paid by the original population, and the other one – by new entrants.

The active population evolves according to the force of mortality.

As stated in the introduction a PAYG system needs equilibrium between revenues and outlays in each year: it is necessary that active members inside a fund pay a sufficient amount of contributions to cover benefit payments of the retired members. Active members evolve according to the process of new entrants. It is assumed that the random number of new entrants in the period k depends on the number of new entrants in the previous time multiplied by $(1 + \lambda_k)$ being λ_k the new entrants variation rate at time k.

In a generic time t the total contributions are calculated as follows:

$$C(t) = \gamma \left\{ \sum_{x=\alpha+t}^{\tau-1} A_{x,t} w_{x,t} + \sum_{x=\alpha}^{\alpha+t-1} A'_{x,t} w_{x,t} \right\} =$$
$$= \gamma \left\{ \sum_{x=\alpha+t}^{\tau-1} A_{x-t,0}\ {}_t p_{x-t} w_{x,t} + \sum_{x=\alpha}^{\alpha+t-1} {}_{(x-\alpha)}p_{\alpha} A'_{\alpha,0} \left[\prod_{i=1}^{t-(x-\alpha)} (1+\lambda_i) \right] w_{x,t} \right\}, \quad (2)$$

where γ is the fixed contribution rate, $A(x, t)$ indicates the number of the original members aged x at time t, $A'(x, t)$ indicates the number of new members aged x at time t, ${}_{(x-\alpha)}p_{\alpha}$ is the probability of a member aged α remains in the scheme for x-α years and $w_{x,t}$ indicates the average income for a member aged x at time t.

As we can see the total amount of contributions is directly influenced by the random variable "new entrants" (as well as the pension benefits when future active members become pensioners), depending by the value of the parameters.

The same approach is used here for the total pensions $B(t)$. The pension benefit is obtained considering the number of retired members and the average amount of benefit. The benefit depends on the amount of the accumulated contributions and the transformation coefficient based on the age, that is the annuitization coefficient used for the conversion into annuity of the notional contribution amount accumulated by each worker. In the scheme all the agents retire at the same age τ; furthermore, at τ they have paid the same average amount of contribution and consequently they receive the same amount of periodic benefit $B_{x,t}$, which represents the average pen-

sion of people aged x at time t. The total amount of the pension is expressed as follows:

$$B(t) = \sum_{x=\tau}^{\omega} P_{x,t} \overline{B}_{x,t}, \quad (3)$$

where $P_{x,t}$ represents the number of pensioners aged x in the generic time t. The total amount of the pension benefits is influenced by the random variable "new entrants" in the long run, when active members become pensioners and when new entrants reach the age $x \geq \tau$. In other words, when the projection time is higher than the contribution period $(t \geq \tau - \alpha)$, the survivor members that entered at time 1 become pensioners: i.e., the new entrants at time $1\ A'_{\alpha,1}$ become pensioners $P_{\tau,d}$ in the period $t = d$ and so on in the following years.

3. The model. The model includes two stochastic variables: the demographic variable "new entrants variation rate" and the financial variable "global asset return".

There are different approaches to analyze future flows of new entrants for these types of funds. An approach consists of studying variables related to demographic evolution of population, the development of education and the attractiveness of the profession, through the analysis of the transition probabilities from states of the population (university students, graduates, employment rates, active workers, members of the pension fund). This method is useful for short-term forecass (5-10 years). As the aim is to study the fund dynamics in the long run, here we propose a model for the evolution of new entrants based on the analysis of the variable "new entrants variation rate". This indicates the variation that occurs in the number of new entrants from one year to the next.

The rate of variation in the number of new entrants can be represented as an autoregressive and moving average process of order (p, q). Indeed through an accurate analysis of real data for different retirement funds, we notice that for the analyzed funds an ARMA(1,1) is well suited to describe the new entrants variation rate, splitting the population into males and females. Therefore, we assume that new entrants variation rate follows an ARMA (1,1) model, defined by linear difference equations with constant coefficients, written as follows:

$$\lambda_t = \varphi_1 \lambda_{t-1} + Z_t - \vartheta_1 Z_{t-1}, \quad (4)$$

where φ_1 and ϑ_1 are respectively the autoregressive and the moving average parameters, and Z_t are normal mutually independent random variables with mean 0 and variance σ^2_Z.

The global asset return can be represented in different ways according to the asset composition of the fund's investment portfolio and the consequent risk associated. The funds analyzed in this paper deal with first pillar pension schemes and therefore they usually present prudential portfolios. The global asset returns associated usually show limited variation around their historical trends. For this reason to represent the interest rate dynamics we use the following model (see Orlando and Trudda, 2004):

$$r(t, t+1) = \hat{r}_{t+1} + X_r(t+1), \quad (5)$$

where the interest rate is the sum of a deterministic component r_{t+1} and a stochastic one $X_r(t+1)$, described by an autoregressive process of first order $(AR1)$, expressed by the following non-homogeneous equation:

$$X_r(t+1)=\varphi_r X_r(t)+\sigma_\alpha a_{t+1} \tag{6}$$

which expresses the autoregressive dependence of order one, where φ_r and σ_α are the parameters of the process and a_t are normal mutually independent random variables (with mean 0 and variance 1).

This model is a discrete representation of the Vasicek model and it is suitable to represent the global asset return on a risky asset portfolio, since the return can reach a negative value, as there can be losses of capital. The choice of a mean reverting stochastic process is due to the fact that the analyzed funds are characterized by prudential portfolios composed with low-risk assets (the heritage is in large part composed of real estate and liquidity and only in limited part of stock funds).

Taking the above into consideration, the general function of the fund (1) can be represented as follows:

$$f(t+1)=\left\{f(t)+\gamma\left[\sum_{x=\alpha+t}^{\tau-1}A_{x-t,0}\ {}_tp_{x-t}w_{x,t}+\sum_{x=\alpha}^{\alpha+t-1}{}_{(x-\alpha)}p_\alpha A'_{\alpha,0}\prod_{i=1}^{t-(x-\alpha)}(1+\varphi_1\lambda_{i-1}+Z_i-\theta_1 Z_{i-1})w_{x,t}\right]+\right.$$
$$\left.\sum_{x=\tau}^{\omega}\overline{B}_{x,t}P_{x,t}\right]\ \left[1+\hat{r}_{t+1}+\varphi_r X_r(t)+\sigma_\alpha a_{t+1}\right] \tag{7}$$

4. Numerical application. The application is carried out on the data provided by the CNPADC, the pension fund of Italian Chartered Accountants, which is a defined contribution pension fund, financed by a PAYG system. Data are available from 1976 to 2006. It is one of the Italian Professional Order retirement funds privatized by Legislative Decree (D.Lgs) 509/1994. Until 1995 these organizations, managing the social security of given categories of self-employed professionals, were administrated by the State that would step in, in case of insolvency. Since 1995 they have managed the security of a growing number of self-employed without being sponsored by the state. New funds, built by D.Lgs 103/1996, follow a fully funded financial scheme.

The pension funds of Professional Orders are self-managed and they continue to operate according to a PAYG financing mechanism, although they were privatized. As already highlighted in the introduction this is an anomaly because private closed schemes are usually funded. In this particular system financial self-sufficiency is certainly guaranteed only at the initial phase, because there are many contributors and absence of pensioners. In the medium run the performance depends on the accumulated capitals. In the long run it is necessary for financial sustainability of the pension plan that the number of pensioners remains proportional to the number of workers. If the ratio active/retired decreases, the increase in the financial burden can entail a situation of financial disequilibrium. This is most relevant for the retirement funds of each specific professional order for which, unlike in a public system, there is indeed no intergroup compensation.

The social security of professional orders is not marginal within the Italian pension system; about 1.3 mln. workers, equal to 5.6% of total employment, are registered as contributors to the private pension system of Italian Professional Orders. The subscribers share has increased notably for some funds (engineers and architects,

lawyers and chartered accountants), others remain stationary (notaries), while some have reduced (accountants). Table 1 and Figure 1 show the number of contributors and pensions of Italian Professional Orders Pension Funds in the last 3 years. The last two columns of the table indicate respectively the financing mechanism of the fund (PAYG or fully funded) and the type of benefit (defined benefit or defined contribution).

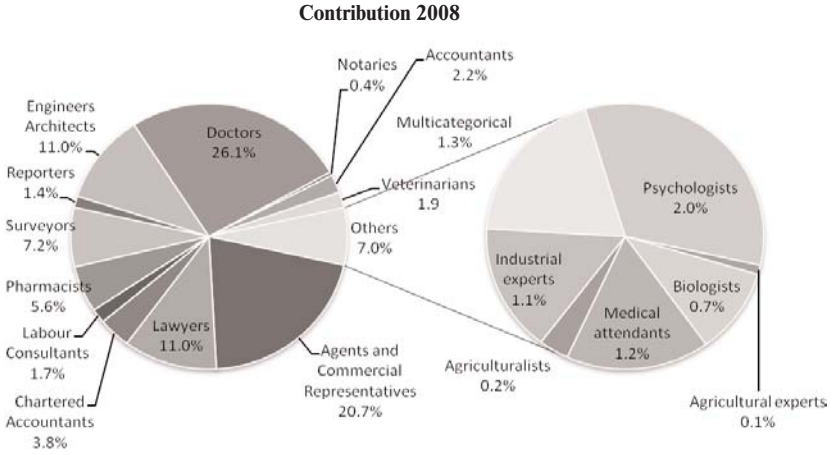

Figure 1. **Contributors of Italian Professional Orders Pension Funds classified by profession (2008)**

Table 1. **Contributors and pensions of Italian Professional Orders Pension Funds (2006-2008)**

Fund		2006	2007	2008		
Agents and Commercial Representatives	Contributors	n.r.	271,093	271,002	PAYG	DB
	Pensions	n.r.	112,167	111,658		
Lawyers	Contributors	129,359	136,818	144,070	PAYG	DB
	Pensions	22,997	23,697	24,358		
Chartered Accountants	Contributors	45,353	47,322	49,759	PAYG	DC
	Pensions	4,619	4,833	4,946		
Labour Consultants	Contributors	21,684	22,225	22,897	PAYG	DB
	Pensions	5,951	6,282	6,782		
Pharmacists	Contributors	69,572	71,373	73,728	PAYG	DB
	Pensions	27,060	27,298	27,431		
Surveyors	Contributors	92,779	93,487	94,486	PAYG	DB
	Pensions	22,219	23,786	24,774		
Reporters	Contributors	17,344	17,681	18,163	PAYG	DB
	Pensions	5,794	6,002	6,230		
Engineers, Architects	Contributors	131,095	138,124	143,851	PAYG	DB
	Pensions	11,756	12,086	12,706		
Doctors	Contributors	332,834	337,798	342,260	PAYG	DB
	Pensions	80,770	81,390	82,501		
Notaries	Contributors	5,312	5,312	5,312	PAYG	DB
	Pensions	2,362	2,380	2,409		
Accountants	Contributors	29,690	29,297	28,659	PAYG	DC
	Pensions	5,431	5,751	6,268		

The End of *Table 1*

Fund		2006	2007	2008		
Veterinarians	Contributors	24,123	24,902	25,478	PAYG	DB
	Pensions	5,996	5,980	5,963		
Agricultural experts	Contributors	1,096	1,121	1,048	FF	DC
	Pensions	0	0	2		
Biologists	Contributors	8,874	9,155	9,477	FF	DC
	Pensions	105	153	219		
Medical attendants	Contributors	12,183	14,275	15,286	FF	DC
	Pensions	141	214	300		
Agriculturalists	Contributors	3,234	3,184	3,203	FF	DC
	Pensions	221	247	277		
Industrial experts	Contributors	13,639	13,828	14,093	FF	DC
	Pensions	759	938	1187		
Multicategorical	Contributors	17,101	17,556	17,628	FF	DC
	Pensions	452	522	680		
Psychologists	Contributors	25,876	27,911	30,101	FF	DC
	Pensions	577	719	885		
Total contributors		981,148	1,282,462	1,310,501		
Total pensions		197,210	314,445	319,576		
Total members		1,178,358	1,596,907	1,630,077		

The following assumptions are used:

- all the new members join the fund at the same average age $\alpha = 30$ and they retire at the same age $\tau = 65$;
- for the actual members real entry age and contributory seniority is considered;
- a subjective contribution rate γ is equal to 10,7% of annual professional income[3];
- the evolution of the population is based on IPS55 male and female mortality tables[4];
- the pension benefit value is obtained by multiplying the accumulated contribution and transformation coefficients based on the age[5];
- administrative costs A_t are considered resulting from 2005 balance sheet, appreciated at the 3% annual rate. Thus the general equation of the fund becomes:

$$f(t+1) = [f(t) + C(t) - B(t) - A(t)][1 + r(t, t+1)]$$

- professional incomes are appreciated at rate of inflation;
- the inflation rate is fixed at 2%.

Figure 2 shows the demographic structure of the CNPADC fund on January, 1st, 2006. There is a high component of young members: the main class of age is represented by 35-45 category. The fund is in a strong growth phase and thus the cash flows of contributions $C(t)$ are much higher than payments of pensions $B(t)$. The analysis, carried out on a time horizon of 40 years, demonstrates how the variable "new entrants" affects the probability of default of the fund. The "hump" of 35-45 year old actual members will retire after 20-30 years (2027-2037): the application shows how

[3] Subjective contribution is the contribution paid by a member of the CNPADC pension fund. It is calculated applying to the professional annual income a contribution rate which varies electively between 10% and 17%. In 2005 the average rate was 10.71% (Source CNPADC).

[4] IPS55 are projected life tables for Italian males and females, cohort 1955.

[5] The transformation coefficients of Italian Law have been employed: in particular the value corresponding to age 65 is 6.13%.

in the long run the fund could go to zero in the case that the future flows of new entrants were not sufficiently high. The demographic scheme is very similar to that of other most important analysed funds (architects, engineers and doctors).

From this demographical situation a dynamic fund evolution has been developed.

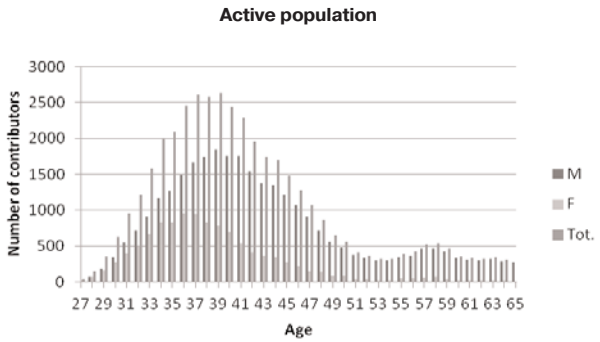

Figure 2. **Actual contributors classified by age and gender**

10 ths. Monte Carlo simulations have been run with both stochastic components (interest rates and new entrants).

Figure 3 shows the percentiles of frequency distribution and the expected value (value is expressed in million euros).

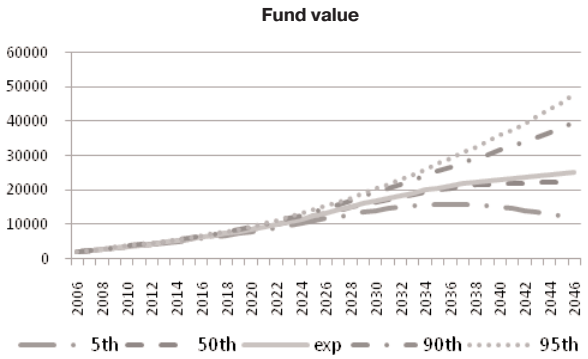

Figure 3. **Fund dynamics with stochastic interest rates and new entrants (percentiles and expected value)**

The chart indicates there is a probability for the lower percentiles that the fund could reach its peak in 2035, then taking a downward trend until reaching the default. Developing the projections in a long-term horizon (40 years) we can observe that, if there is not a sufficient number of new entrants and therefore a sufficient flow of contributions, the fund value will tend to decrease rapidly, as the contributions and the investment performance are not sufficient to cover pensions and administrative costs and the fund will tend to achieve zero and so default.

In order to analyze the single influence of two processes, two further simulations have been considered using only one stochastic variable and fixing the other one on the expected value. The results are illustrated in Figures 4 and 5. In the former case (fixed new entrants and stochastic interest rates) the fund reaches a peak after about 30 years and then begins to decrease for all the percentiles. In the latter (fixed interest rates and stochastic new entrants) the fund continues to increase, and begins to decrease in about 2035, only at lower percentiles. The comparison shows how in our application the new entrants variation rate has a stronger influence on the fund value than the global asset return.

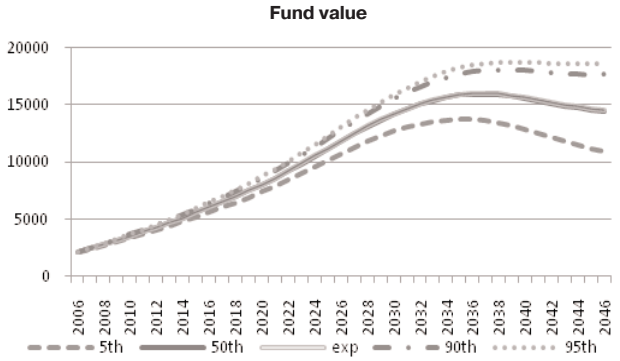

Figure 4. **Evolution of the fund, with deterministic new entrants and stochastic interest rates**

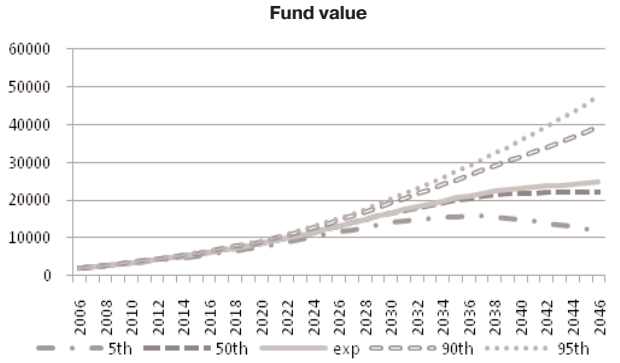

Figure 5. **Evolution of the fund with deterministic interest rates and stochastic new entrants**

To compare the influence of the new entrance on the default probability, the Value at Risk (VaR) at 95% confidence level of the pension fund value for 4 cases is calculated: in the first case, we consider interest rates and new entrants both stochastic; in the other two cases we considered only one variable as stochastic and the other one as fixed on its expected value; finally we considered the case in which there are no new entrants: the results are shown in Figure 6. As we can see, the "new entrants" influence is stronger than the one of financial variable in this case as well.

To evaluate the "extinction risk" of the profession, we considered also the case of total absence of new entrants. In this case we verify that the fund would quickly shortfall.

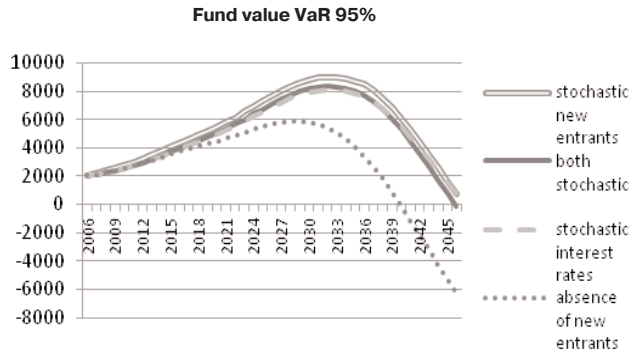

Figure 6. **Fund value VaR at 95%**

5. Conclusions. The paper examines private pension funds financed by PAYG, developing a stochastic model to describe the evolution of the fund. Some applications were carried out on the pension funds of the Italian professional orders.

The most relevant risk sources of these funds were analyzed focusing on the demographic risk related to insufficient number of new members into the fund to ensure the payment of future pensions; this hypothesis has to be considered, given the potential barrier at labour markets associated with these funds. The empirical analysis shows how the new entrants variation rate has a stronger influence on the fund value with respect to the global asset return. This kind of risk, that we called "extinction risk" of the insured professional category, is not an insurable risk, but it can be diversified and shared among the whole system of the different professions' pension funds. In the long run this strategy would reduce this risk for a single professional group, through funding gained from all the professional categories. Using our model it is possible to monitor the financial sustainability in the medium-long run, in function of the random variables considered. In this way the management has at its disposal a control instrument to forecast financial disequilibrium situations and to promptly activate the actions to rebalance the fund with respect to a shock in a determinant variable of the fund.

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