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LONG STRANGLE STRATEGY USING BARRIER OPTIONS AND ITS APPLICATION IN HEDGING

This paper presents new theoretical results for hedging of an underlying asset price through a static portfolio of European call and put barrier options. We illustrate the theory with numerical examples and discuss the practical implementation. We derive profit functions from secured position in analytical form for two different methods of long strangle strategy formation using barrier options. Based on profit functions, we propose hedging against a price drop of real-traded underlying asset that is SPDR gold shares, analyze the proposed hedging variants and compare the obtained results.

Keywords: *hedging; secured position; long strangle strategy; barrier options; standard options.*

JEL Classification: G11; G15.

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СТРАТЕГІЯ ДОВГОГО СТРЕНГЛА З ВИКОРИСТАННЯМ БАР'ЄРНИХ ОПЦІОНІВ ТА ЇЇ ВИКОРИСТАННЯ В ХЕДЖУВАННІ

У статті представлено нові теоретичні результати хеджування ціни базового активу за допомогою статичного портфеля європейських бар'єрних опціонів пут і колл. Теорію проаналізовано числовими прикладами і обговорено її застосування на практиці. В аналітичній формі виведено функції прибутку з гарантованої позиції для двох методів формування стратегії довгого стренгла з використанням бар'єрних опціонів. Грунтуючись на функціях прибутку, запропоновано хеджування проти зниження вартості реального базового активу – золотих акцій SPDR, проаналізовано запропоновані варіанти хеджування і порівняно отримані результати.

Ключові слова: *хеджування; гарантована позиція; стратегія довгого стренгла; бар'єрні опціони; стандартні опціони.*

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СТРАТЕГИЯ ДЛИННОГО СТРЭНГЛА С ИСПОЛЬЗОВАНИЕМ БАРЬЕРНЫХ ОПЦИОНОВ И ЕЕ ИСПОЛЬЗОВАНИЕ В ХЕДЖИРОВАНИИ

В статье представлены новые теоретические результаты хеджирования цены базового актива посредством статичного портфеля европейских барьерных опционов пут и колл. Проиллюстрировано теорию числовыми примерами и обсуждено ее применение на практике. В аналитической форме выведены функции прибыли из гарантированной позиции для двух методов формирования стратегии длинного стрэнгла с использованием барьерных опционов. Основываясь на функциях прибыли, предлагается хеджирование против снижения стоимости реального базового актива – золотых акций SPDR, анализируются предложенные варианты хеджирования и сравниваются полученные результаты.

Ключевые слова: *хеджирование; гарантированная позиция; стратегия длинного стрэнгла; барьерные опционы; стандартные опционы.*

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1. Introduction. The most important innovation in the financial area of the last decades are options and option strategies that can offer opportunities and advantages to protection or investment from changes in price of various underlying assets (e.g., stocks, bonds, commodities, currencies, indices). Bull, bear, butterfly, condor, spreads along with straddles, strangles, combos, ladders and simple covered or protective call and put are some of the option strategies described in popular books including Cohen (2005), Hull (2005), Chorafas (2008), Mullaney (2009), Smith (2008). Much of trading is concentrated in several popular strategies such as covered call and put, bull and bear spread, strangle and straddle. At present, there are few studies of trading in options strategies, for example, Fahlenbrach et. al. (2010), Chang et. al. (2010), Mugwagwa et. al. (2008), Santa-Clara et. al. (2009), but there is no study of hedging. In this paper we want to demonstrate that option strategies can be a very important risk management tool.

Hedging against a price drop using long strangle strategy with an innovative financial instrument called barrier options is analyzed. Furthermore, obtained hedging results are applied in hedging of SPDR gold shares. Two different methods of long strangle strategy formation using vanilla options are investigated in the paper (Soltes, 2001). Optimal algorithm of this strategy application for practical investment is found as well. The work (Soltes et. al., 2005) is dealing with usage of long strangle strategy using vanilla options in hedging. Hedging against unfavourable price movement by option strategies using vanilla options is described in many other papers, for example Amaitiek (2009), Amaitiek et. al. (2010), Soltes et. al. (2010a), Soltes et. al. (2010b). Analysis of long strangle strategy using barrier option cannot be found in any contemporary literature.

Single call/put barrier options are options with second strike price, named barrier, trigger or outstrike. A knock-out option is considered expired when the barrier is crossed. A knock-in comes into existence when the barrier level is hit. For up and knock-in/out options we have the barrier above the actual spot price of an underlying asset. For down and knock-in/out we have the barrier below the actual spot price. Based on this, 4 basic types of barrier options are:

- up and knock-in (UI) call/put option – is activated if an underlying price during the life of an option increases above upper barrier U;
- down and knock-in (DI) call/put option – is activated if an underlying price during the life of an option decreases below lower barrier D;
- up and knock-out (UO) call/put option – is deactivated if an underlying price during the life of an option increases above upper barrier;
- down and knock-out (DO) call/put option – is deactivated if an underlying price during the life of an option decreases below lower barrier.

More information about barrier options can be found in: Briys et. al. (1998), Kolb (2007), Taleb (1997), Weert (2008). The paper (Ye, 2009) presents boundary analysis of barrier options on which this work is based.

The rest of the paper is organized as follows. Section 2 gives the methodology and the data used in this paper, followed by hedging analysis by long strangle strategy using barrier options in section 3. Section 4 contains practical application in SPDR gold shares. The last section concludes the paper.

2. Methodology and data. The main theoretical aim of this work is to analyze long strangle strategy and suggest its application in hedging against a price drop. A key difference from the previous studies is that in this paper we are concentrated on an analytical expression of profit functions. This approach allows us to derive profit functions from secured position for all possible hedging variants created by long strangle strategy using barrier options. Basing on achieved theoretical results, the practical application in hedging SPDR gold shares is designed. Variants for hedging of these shares are suggested and compared with hedging by long strangle using standard options. Hedging results are considered the main practical contribution of this work.

We use SPDR gold shares option data with expiration time 21 September 2012 (source: http://finance.yahoo.com) for our practical analysis. SPDR gold shares offer investors an innovative way to access the gold market. On 20 October 2011, the shares of SPDR gold shares were traded at the NYSE at approximately USD 158 per share. Real-market vanilla call and put prices on SPDR gold shares is used in barrier option data calculating performed in the statistical program "R". European single barrier options on equities without dividends are evaluated analytically using a model introduced by Reiner and Rubinstein (1991) and later by Haugh (1998). The model is based on the following parameters: type of option (DI/DO/UI/UO CALL/PUT), actual underlying spot price, strike price, barrier level, time to maturity, rebate = 0, risk-free interest rate = cost of carry rate, implied volatility of underlying asset. We use 12-months U.S. Treasury rate as risk-free rate, which was 0.11% on 20 October 2011 (source: Bloomberg). All the data used in our analysis can be provided upon request.

3. Hedging using long strangle strategy formed by barrier options. Let us suppose that we own a portfolio consisting of n pieces of risky underlying asset and at time t. In the future we want to sell them, but we are afraid of price drop.

Profit function of unsecured position in the portfolio at time t is:

$$P(S_t) = nS_t, \tag{1}$$

where S_t is a spot price of underlying asset at time t. The lower S_t, the lower our profit from selling the asset will be.

Therefore, we have decided to hedge against an underlying price drop at time t and we have chosen long strangle strategy using barrier options. Long strangle strategy is formed by combination of long put and long call position, i.e. by buying n put options and at the same time by buying n call options considering European options for the same underlying asset and the same expiration time, but with different strike prices[3]. Related to the amount of strike prices, there are two methods of long strangle strategy formation:

I. By buying n put option with lower strike price X_1 and at the same time by buying n call options with higher strike price X_2.

II. By buying n call option with lower strike price X_1 and at the same time by buying n put options with higher strike price X_2.

By analysing all the possibilities of long strangle formation we can conclude:

[3] If the strike prices are the same, then it is a special case of this strategy formation known as long straddle.

- Hedging possibilities constructed using purchasing down and knock-in put option ensure a minimum selling price at the amount of corresponding barrier level D in the case of any price movement.

- Otherwise (possibilities constructed using purchasing down and knock-out, up and knock-in or up and knock-out put option) we have unsecured scenarios where a price of an underlying asset is not hedge against a price drop.

- Therefore we recommend 8 possibilities of long strangle strategy formation, i.e. constructed by down and knock-in put options in hedging.

I. Let buy n down and knock-in put options with strike price X_1, premium p^0_{1BDI} per options[4], barrier level D in the form of marginal underlying price (an option is activated, if D is reached during the option life) and at the same time we buy n up and knock-in call options with higher strike price X_2, premium c^0_{2SUI} per option, barrier level U. Expiration time of both options is t. We assume that $D < X_1$ and $U > X_2$, because otherwise DI put and UI call option is equivalent to a vanilla put and call. For UI/UO options, we have $U > S_0$, for DI/DO option, we have $D < S_0$, where S_0 is a spot price at time 0.

Profit function from buying n down and knock-in put option at time t is:

$$P(S_t)=\begin{cases}-np_{1BDI} & \text{if } \min\limits_{0\le t\le T}(S_t)>D\wedge S_t<X_1,\\ -n(S_t-X_1+np_{1BDI}) & \text{if } \min\limits_{0\le t\le T}(S_t)\le D\wedge S_t<X_1,\\ -np_{1BDI} & \text{if } S_t\ge X_1,\end{cases} \quad (2)$$

and profit function from buying n up and knock-in call options is:

$$P(S_t)=\begin{cases}-nc_{2BUI} & \text{if } S_t\ge X_2,\\ n(S_t-X_2-nc_{2BUI}) & \text{if } \max\limits_{0\le t\le T}(S_t)\ge U\wedge S_t<X_2,\\ -nc_{2BUI} & \text{if } \max\limits_{0\le t\le T}(S_t)<U\wedge S_t<X_2.\end{cases} \quad (3)$$

Profit function for long strangle strategy is the sum of (2) and (3):

$$P(S_t)=\begin{cases}-n(p_{1BDI}+c_{2BUI}) & \text{if } \min\limits_{0\le t\le T}(S_t)>D\wedge S_t<X_1,\\ -n(S_t-X_1+p_{1BDI}+c_{2BUI}) & \text{if } \min\limits_{0\le t\le T}(S_t)\le D\wedge S_t<X_1,\\ -n(p_{1BDI}+c_{2BUI}) & \text{if } X_1\le S_t<X_2,\\ n(S_t-X_2-p_{1BDI}-c_{2BUI}) & \text{if } \max\limits_{0\le t\le T}(S_t)\ge U\wedge S_t\ge X_2,\\ -n(p_{1BDI}+c_{2BUI}) & \text{if } \max\limits_{0\le t\le T}(S_t)<U\wedge S_t\ge X_2.\end{cases} \quad (4)$$

Profit function from secured position formed by this possibility of long strangle strategy formation at time t can be obtained by adding the profit function from unsecured position to function (4) and has the following form:

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[4] The premium is calculating using the formula for simple or compound interest, i.e. $np_{1BDI} = p^0_{1BDI}(1+rt)$, or $np_{1BDI} = p^0_{1BDI}(1+r)^t$ where r is nominal interest rate. In other profit function of this work we will be using the option premium adjusted by the time value of the money according to the given formulas.

$$ZP_l(S_t)=\begin{cases} n(S_t - p_{1BDI} - c_{2BUI}) & \text{if } \min\limits_{0\le t\le T}(S_t) > D \wedge S_t < X_1, \\ n(X_1 - p_{1BDI} - c_{2BUI}) & \text{if } \min\limits_{0\le t\le T}(S_t) \le D \wedge S_t < X_1, \\ n(S_t - p_{1BDI} - c_{2BUI}) & \text{if } X_1 \le S_t < X_2, \\ n(2S_t - X_2 - p_{1BDI} - c_{2BUI}) & \text{if } \max\limits_{0\le t\le T}(S_t) \ge U \wedge S_t \ge X_2, \\ n(S_t - p_{1BDI} - c_{2BUI}) & \text{if } \max\limits_{0\le t\le T}(S_t) < U \wedge S_t \ge X_2. \end{cases} \quad (5)$$

By analyzing the profit function from unsecured position (1) and the profit function from unsecured position (4) we have the statements:

- If $S_t < X_1 - p_{1BDI} - c_{2BUI}$ and an underlying price during time to maturity decreases below lower barrier, then our profit will be higher than without hedging.
- If $S_t < X_1 - p_{1BDI} - c_{2BUI}$ and an underlying price during time to maturity does not decrease below lower barrier, then we have the constant loss $-n(p_{1BDI} + c_{2BUI})$.
- If $X_1 - p_{1BDI} - c_{2BUI} \le S_t < X_1$ and an underlying price during time to maturity decreases below lower barrier, then our profit will be lower than without hedging but no more than $-n(p_{1BDI} + c_{2BUI})$.
- If $X_1 - p_{1BDI} - c_{2BUI} \le St < X1$ and an underlying price during time to maturity does not decrease below lower barrier, then we have the constant loss $-n(p_{1BDI} + c_{2BUI})$.
- If $X_1 \le S_t < X_2$, then we have the constant loss $-n(p_{1BDI} + c_{2BUI})$.
- If $X_2 \le S_t < X_2 + p_{1BDI} + c_{2BUI}$ and an underlying price during time to maturity increases above higher barrier, then our profit will be lower than without hedging but no more than $-n(p_{1BDI} + c_{2BUI})$.
- If $X_2 \le S_t < X_2 + p_{1BDI} + c_{2BUI}$ and an underlying price during time to maturity does not increase above higher barrier, then we have the constant loss $-n(p_{1BDI} + c_{2BUI})$.
- If $S_t \ge X_2 + p_{1BDI} + c_{2BUI}$ and an underlying price during time to maturity increases above higher barrier, then our profit will be higher than without hedging.
- If $S_t \ge X_2 + p_{1BDI} + c_{2BUI}$ and an underlying price during time to maturity does not increase above higher barrier then we have the constant loss $-n(p_{1BDI} + c_{2BUI})$.

The advantage of hedging using this strategy is ensuring a constant price in case of a price drop below lower barrier and greater profit in case of significant price growth.

Universal profit function from secured position by buying n down and knock-in put options and n call barrier option (up-in, up-out, down-in, down-out) is:

$$ZP_l(S_t)=\begin{cases} n(S_t - p_{1BDI} - c_{2B}) & \text{if } \min\limits_{0\le t\le T}(S_t) > D \wedge S_t < X_1, \\ n(X_1 - p_{1BDI} - c_{2B}) & \text{if } \min\limits_{0\le t\le T}(S_t) \le D \wedge S_t < X_1, \\ n(S_t - p_{1BDI} - c_{2B}) & \text{if } X_1 \le S_t < X_2, \\ n(2S_t - X_2 - p_{1BDI} - c_{2B}) & \text{if condition 1 is fulfilled} \wedge S_t \ge X_2, \\ n(S_t - p_{1BDI} - c_{2B}) & \text{if condition 2 is fulfilled} \wedge S_t \ge X_2. \end{cases} \quad (6)$$

Barrier conditions for particular call barrier options are in Table 1. Substituting corresponding barrier conditions in universal profit function we get the profit function of the selected possibility of long strangle formation.

Table 1. **Barrier conditions for particular call barrier options**

	Type of call barrier option	Barrier condition 1	Barrier condition 2
1.	UI call option	$\max\limits_{0\le t\le T}(S_t)\ge U$	$\max\limits_{0\le t\le T}(S_t)< U$
2.	UO call option	$\max\limits_{0\le t\le T}(S_t)< U$	$\max\limits_{0\le t\le T}(S_t)\ge U$
3.	DI call option	$\min\limits_{0\le t\le T}(S_t)\le D$	$\min\limits_{0\le t\le T}(S_t)> D$
4.	DO call option	$\min\limits_{0\le t\le T}(S_t)> D$	$\min\limits_{0\le t\le T}(S_t)\le D$

Source: Own calculations.

DI/DO call options can have the barrier level below or under the strike price, UO call options only under the strike price.

II. By analogy we analyzed the second method of long strangle formation, i.e. created by buying *n* call options with lower strike price X_1, premium c_{1B}, particular barrier conditions according to Table 1 and at the same time by buying *n* down and knock-in put options with higher strike price X_2, premium p_{2BDI}, barrier level *D*.

Universal profit function from secured position for this method is:

$$ZP_{II}(S_t)=\begin{cases} n(S_t-c_{1B}-p_{2BDI}) & \text{if } \min\limits_{0\le t\le T}(S_t)>D\wedge S_t<X_1,\\ n(X_2-c_{1B}-p_{2BDI}) & \text{if } \min\limits_{0\le t\le T}(S_t)\le D\wedge S_t<X_1,\\ n(2S_t-X_1-c_{1B}-p_{2BDI}) & \text{if condition 1 is fulfilled} \wedge \min\limits_{0\le t\le T}(S_t)>D\wedge X_1\le S_t<X_2,\\ n(S_t+X_2-X_1-c_{1B}-p_{2BDI}) & \text{if condition 1 is fulfilled} \wedge \min\limits_{0\le t\le T}(S_t)\le D\wedge X_1\le S_t<X_2,\\ n(S_t-c_{1B}-p_{2BDI}) & \text{if condition 2 is fulfilled} \wedge \min\limits_{0\le t\le T}(S_t)>D\wedge X_1\le S_t<X_2,\\ n(X_2-c_{1B}-p_{2BDI}) & \text{if condition 2 is fulfilled} \wedge \min\limits_{0\le t\le T}(S_t)\le D\wedge X_1\le S_t<X_2,\\ n(2S_t-X_1-c_{1B}-p_{2BDI}) & \text{if condition 1 is fulfilled} \wedge S_t\ge X_2,\\ n(S_t-c_{1B}-p_{2BDI}) & \text{if condition 2 is fulfilled} \wedge S_t\ge X_2. \end{cases} \quad (7)$$

4. Application in SPDR gold shares. Let us assume that now (October 2011) we own a portfolio consisting of 100 SPDR gold shares and we are afraid of a price drop in the future (September 2012). We have decided to hedge using long strangle strategy.

4.1. Hedging of SPDR gold shares using I method of long strangle formation. Let us form long strangle strategy by buying 100 DI put options with the strike price 145, barrier level 135, premium 10.99 per option and at the same time:

a) by buying 100 UI call options with the strike price 175, barrier level 185, premium 11.77 per option;

b) by buying 100 UO call options with the strike price 175, barrier level 185, and premium 0.02 per option.

The profit function from secured position in case a) is:

$$ZP_{Ia}(S_t)=\begin{cases}100S_t-2276 & if \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t<145,\\ 12224 & if \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t<145,\\ 100S_t-2276 & if\ 145\le S_t<175,\\ 200S_t-19776 & if \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge S_t\ge 175,\\ 100S_t-2276 & if \max\limits_{0\le t\le T}(S_t)<185 \wedge S_t\ge 175.\end{cases} \quad (8)$$

The profit function from secured position in case b) is:

$$ZP_{Ib}(S_t)=\begin{cases}100S_t-1101 & if \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t<145,\\ 13399 & if \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t<145,\\ 100S_t-1101 & if\ 145\le S_t<175,\\ 200S_t-18601 & if \max\limits_{0\le t\le T}(S_t)<185 \wedge S_t\ge 175,\\ 100S_t-1101 & if \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge S_t\ge 175.\end{cases} \quad (9)$$

The hedging variant a) ensures a constant price 145 per a share in case of a price drop below 135 or a minimum price 135 in case of a price movement above 135. In a case of significant price growth we hedge a greater profit than without hedging.

It is obvious that hedging case b) is not appropriate if we expect a significant price increase. It is balanced by a lower option premium paid for buying UO call options.

Let us buy 100 DI put options with the strike price 145, barrier level 135, premium 10.99 per option and at the same time:

c) buy 100 DI call options with the strike price 175, barrier level 135, premium 1.52 per option;

d) buy 100 DO call options with the strike price 175, barrier level 135, premium 10.13 per option.

$$ZP_{Ic}(S_t)=\begin{cases}100S_t-1251 & if \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t<145,\\ 13249 & if \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t<145,\\ 100S_t-1251 & if\ 145\le S_t<175,\\ 200S_t-18751 & if \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t\ge 175,\\ 100S_t-1251 & if \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t\ge 175.\end{cases} \quad (10)$$

$$ZP_{Id}(S_t)=\begin{cases}100S_t-2112 & if \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t<145,\\ 12388 & if \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t<145,\\ 100S_t-2112 & if\ 145\le S_t<175,\\ 200S_t-19612 & if \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t\ge 175,\\ 100S_t-2112 & if \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t\ge 175.\end{cases} \quad (11)$$

Hedging case c) is especially interesting for investors, because in case of a significant price movement (below barrier 135), it protects investors from bursting of a bubble and allows them profit more in possible further bubble increasing than without hedging.

The case d) is also interesting for investors, because in case of a small decrease (above barrier 135), we are hedged and we can profit in possible price growth.

4.2. Hedging of SPDR gold shares using II method of long strangle formation.

Let us form long strangle strategy:

a) by buying 100 UI call options with the strike price 145, barrier level 185, premium 22.69 per option;

b) by buying 100 UO call options with the strike price 145, barrier level 185, premium 1.59 per option;

and at the same time by buying 100 DI put options with the strike price 175, barrier level 135, premium 25.73 per option.

The profit functions from secured position are:

$$ZP_{IIa}(S_t)=\begin{cases} 100S_t-4842 & \text{if } \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t<145, \\ 12658 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t<145, \\ 200S_t-19342 & \text{if } \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge \min\limits_{0\le t\le T}(S_t)>135 \wedge 145\le S_t<175, \\ 100S_t-1842 & \text{if } \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge \min\limits_{0\le t\le T}(S_t)\le 135 \wedge 145\le S_t<175, \\ 100S_t-4842 & \text{if } \max\limits_{0\le t\le T}(S_t)<185 \wedge \min\limits_{0\le t\le T}(S_t)>135 \wedge 145\le S_t<175, \\ 12658 & \text{if } \max\limits_{0\le t\le T}(S_t)<185 \wedge \min\limits_{0\le t\le T}(S_t)\le 135 \wedge 145\le S_t<175, \\ 200S_t-19342 & \text{if } \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge S_t\ge 175, \\ 100S_t-4842 & \text{if } \max\limits_{0\le t\le T}(S_t)<185 \wedge S_t\ge 175,. \end{cases} \qquad (12)$$

$$ZP_{IIb}(S_t)=\begin{cases} 100S_t-2732 & \text{if } \min\limits_{0\le t\le T}(S_t)>135 \wedge S_t<145, \\ 14768 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135 \wedge S_t<145, \\ 200S_t-17323 & \text{if } \max\limits_{0\le t\le T}(S_t)<185 \wedge \min\limits_{0\le t\le T}(S_t)>135 \wedge 145\le S_t<175, \\ 100S_t+268 & \text{if } \max\limits_{0\le t\le T}(S_t)<185 \wedge \min\limits_{0\le t\le T}(S_t)\le 135 \wedge 145\le S_t<175, \\ 100S_t-2732 & \text{if } \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge \min\limits_{0\le t\le T}(S_t)>135 \wedge 145\le S_t<175, \\ 14768 & \text{if } \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge \min\limits_{0\le t\le T}(S_t)\le 135 \wedge 145\le S_t<175, \\ 200S_t-17323 & \text{if } \max\limits_{0\le t\le T}(S_t)<185 \wedge S_t\ge 175, \\ 100S_t-2732 & \text{if } \max\limits_{0\le t\le T}(S_t)\ge 185 \wedge S_t\ge 175,. \end{cases} \qquad (13)$$

Fig. 1 depicts profit function from unsecured position and profit functions from secured position using I and II methods of long strangle strategy formation with the same parameters, i.e. functions (8) and (12).

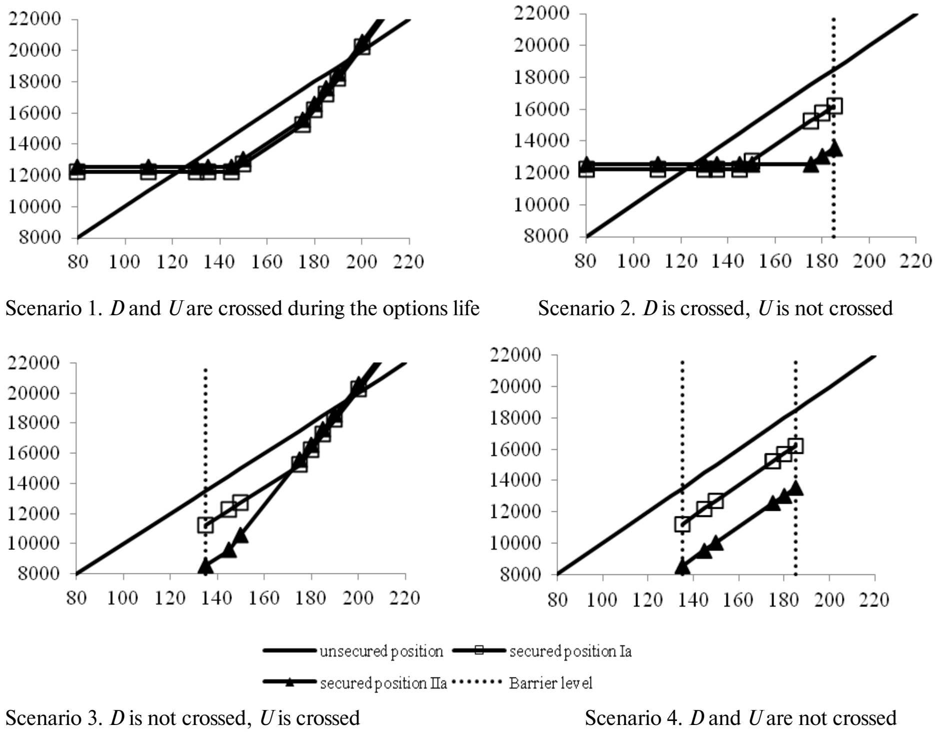


Fig. 1. **Comparison of profit functions of proposed hedging variants Ia and IIa**

Source: Own constructions.

The results of the analysis:

- If the price develops according to the following scenarios: scenario 1, scenario 2 interval $S_t < X_1$, scenario 3 interval $S_t \geq X_1$, and the condition $X_1 - X_2 - p_{1BDI} - c_{2BUI} + c_{1BUI} + p_{2BDI} > 0$, is fulfilled, then I method of long strangle strategy formation is better, otherwise II method. In our case, 145 - 175 - 10.99 - 11.77 + 22.69 + 25.73 = -4.34 < 0, therefore II method is better for these scenarios.

- If the price develops according to the scenarios: scenario 2 interval $S_t \geq X_2$, scenario 3 interval $S_t < X_1$, scenario 4 and the condition $c_{1BUI} + p_{2BDI} - p_{1BDI} - c_{2BUI} > 0$, is fulfilled, then I method is better, otherwise II method. In our case, 22.69 + 25.73 - 10.99 - 11.77 = 25.66 > 0, therefore I method is better for the given scenarios.

- If the price develops according to the scenario 2 interval $X_1 < S_t < X_2$ and the condition $S_t > X_2 + p_{1BDI} + c_{2BUI} - c_{1BUI} - p_{2BDI}$, is fulfilled then I method of long strangle strategy is better, otherwise II method. In our case, S_t = 175 + 10.99 + 11.77 - 22.69 - 25.73 = 149.34, therefore if S_t <149.34, then II method is better and if S_t >149.34, then I method is better.

- If the price develops according to the scenario 3 interval $X_1 < S_t < X_2$ and the condition $S_t < X_1 + c_{1BUI} + p_{2BDI} - p_{1BDI} - c_{2BUI}$ is fulfilled, then I method is better, otherwise II method. In our case, S_t = 145 + 22.69 + 25.73 - 10.99 - 11.77 = 170.66, therefore if S_t <170.66, then I method is better and if S_t >170.66, then II method is better.

As we mentioned earlier, this hedging variant is suitable for an investor who expects a significant drop or growth, i.e. scenario 1 is the most expected. In this particular case we recommend II method of this strategy formation to use in hedging for investors the awaited price development in accordance with scenario 1. The optimal algorithm of long strangle strategy using barrier options cannot be found. The selection of appropriate method must be made by an investor depending on his expectations.

Let us form long strangle strategy using II method:

c) by buying 100 DI call options with the strike price 145, barrier level 135, premium 4.96 per option;

d) by buying 100 DO call options with the strike price 145, barrier level 135, premium 19.32 per option

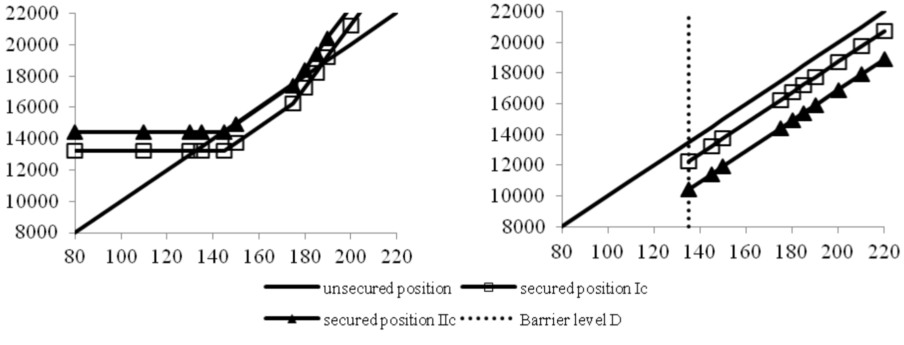
and at the same time by buying 100 DI put options with the strike price 175, barrier level 135, premium 25.73 per option.

The profit functions from secured position are:

$$ZP_{IIc}(S_t)=\begin{cases}100S_t-3069 & \text{if } \min\limits_{0\le t\le T}(S_t)>135\wedge S_t<145,\\ 14431 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135\wedge S_t<145,\\ 100S_t-69 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135\wedge 145\le S_t<175,\\ 100S_t-3069 & \text{if } \min\limits_{0\le t\le T}(S_t)>135\wedge 145\le S_t<175,\\ 200S_t-17569 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135\wedge S_t\ge 175,\\ 100S_t-3069 & \text{if } \min\limits_{0\le t\le T}(S_t)>135\wedge S_t\ge 175.\end{cases} \quad (14)$$

$$ZP_{IId}(S_t)=\begin{cases}100S_t-4505 & \text{if } \min\limits_{0\le t\le T}(S_t)>135\wedge S_t<145,\\ 12995 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135\wedge S_t<145,\\ 200S_t-19005 & \text{if } \min\limits_{0\le t\le T}(S_t)>135\wedge 145\le S_t<175,\\ 12995 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135\wedge 145\le S_t<175,\\ 200S_t-19005 & \text{if } \min\limits_{0\le t\le T}(S_t)>135\wedge S_t\ge 175,\\ 100S_t-4505 & \text{if } \min\limits_{0\le t\le T}(S_t)\le 135\wedge S_t\ge 175.\end{cases} \quad (15)$$

By analogy, Fig. 2 depicts profit function from unsecured position and functions from secured position Ic and IIc, i.e. (10) and (14).



Scenario 1. *D* is crossed during the options life

Scenario 2. *D* is not crossed

Fig. 2. **Comparison of profit functions of proposed hedging variants Ic and IIc**

Source: Own constructions.

We can deduce the following conclusions:

- If the price develops according to the scenario 1 and $X_1 - X_2 - p_{1BDI} - c_{2BDI} + c_{1BDI} + p_{2BDI} > 0$, then I method of long strangle strategy formation is better, otherwise II method. In our case, 145 - 175 - 10.99 - 1.52 + 4.96 + 25.73 = -11.82 < 0, therefore, II method is better for this scenario.

- If the price develops according to the 2 scenario and $c_{1BDI} + p_{2BDI} - p_{1BDI} - c_{2BDI} > 0$, then I method is better, otherwise II method. In our case, 4.96 + 25.73 - 10.99 - 1.52 = 18.18 > 0, therefore I method is better for the given scenarios.

The optimal algorithm for usage of long strangle strategy cannot be found.

In the next section we compare hedging variants Ic and IIc with hedging variant formed by standard options.

4.3. Comparison with long strangle hedging results using standard options. As we mentioned, long strangle strategy can be formed by buying n put options with a strike price X_1 and at the same time by buying n call options with a strike price X^2. We assume options with the same underlying asset and time to maturity. The profit functions from secured position for both methods of formation is derived in paper (Soltes et. al., 2005), where the optimal algorithm of long strangle strategy usage for practical investment can be found as well.

Profit function if $X_1 < X_2$, has the following form:

$$ZP_I(S_t) = \begin{cases} n(X_1 - p_{1B} - c_{2B}) & \text{if } S_t < X_1, \\ n(S_t - p_{1B} - c_{2B}) & \text{if } X_1 \le S_t < X_2, \\ n(2S_t - X_2 - p_{1B} - c_{2B}) & \text{if } S_t \ge X_2. \end{cases} \tag{16}$$

Profit function if $X_1 > X_2$ is:

$$ZP_{II}(S_t) = \begin{cases} n(X_2 - c_{1B} - p_{2B}) & \text{if } S_t < X_1, \\ n(S_t - X_1 + X_2 - c_{1B} - p_{2B}) & \text{if } X_1 \le S_t < X_2, \\ n(2S_t - X_1 - c_{1B} - p_{2B}) & \text{if } S_t \ge X_2. \end{cases} \tag{17}$$

Let us suppose that we want to hedge the future selling price of 100 SPDR gold shares using long strangle strategy formed by standard option. According to the optimal algorithm of this strategy usage, if $X_2 - X_1 - c_{1B} + p_{1B} + c_{2B} - p_{2B}$ = 175 - 145 - 24.28 + 11.03 + 11.65 - 27.90 = 0.5 > 0, then I method is better.

Let us create this strategy by buying 100 put options with the strike price 145, premium 11.03 per option and at the same time buy 100 call options with the strike price 175, premium 11.65 per option.

The profit function from a secured position is:

$$ZP_{Ie}(S_t) = \begin{cases} 12232 & \text{if } S_t < 145, \\ 100S_t - 2268 & \text{if } 145 \le S_t < 175, \\ 200S_t - 19768 & \text{if } S_t \ge 175. \end{cases} \tag{18}$$

Now we will compare selected proposed secured positions using barrier options (Ic and IIc) and proposed secured position using vanilla options (Ie) with unsecured position, i.e. 100 S_t. If secured position ensures higher profit at particular intervals of spot price at time *t*, then it will be evaluate as profitable (P), on the contrary as lossy (L). In the case of profit/loss we will calculate minimum (min) profit/loss, maximum profit/loss (max) valid for individual intervals. The results of comparative analysis are in Table 2.

Table 2. **A comparison of hedging alternatives Ie, Ic and IIc with unsecured position**

Spot price intervals at time *t*	Hedging alternative Ie			Barrier conditions	Hedging alternative Ic			Hedging alternative IIc		
	P/L	min	max		P/L	min	max	P/L	min	max
$S_t \le 122.32$	P	0	12 232	$\min_{0\le t\le T}(S_t)\le 135$	P	1 012	13 249	P	2 199	14 431
$122.32 \le S_t \le 132.49$	L	0	1 017	$\min_{0\le t\le T}(S_t)\le 135$	P	0	1 012	P	1 182	2 199
$132.49 \le S_t \le 135$	L	1 017	1 268	$\min_{0\le t\le T}(S_t)\le 135$	L	0	251	P	931	1 182
$135 \le S_t \le 144.31$	L	1 268	2 199	$\min_{0\le t\le T}(S_t)\le 135$	L	251	1 182	P	0	931
	L	1 268	2 199	$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	
$144.31 \le S_t \le 145$	L	2 199	2 268	$\min_{0\le t\le T}(S_t)\le 135$	L	1 182	1 251	L	0	69
	L	2 199	2 268	$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	
$145 \le S_t \le 175$	CL	2 268		$\min_{0\le t\le T}(S_t)\le 135$	CL	1 251		CL	69	
	CL	2 268		$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	
$175 \le S_T \le 175.69$	L	2 199	2 268	$\min_{0\le t\le T}(S_t)\le 135$	L	1 182	1 251	L	0	69
	L	2 199	2 268	$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	
$175.69 \le S_t \le 185.17$	L	1 251	2 199	$\min_{0\le t\le T}(S_t)\le 135$	L	234	1 182	P	0	948
	L	1 251	2 199	$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	
$185.17 \le S_t \le 187.51$	L	1 017	1 251	$\min_{0\le t\le T}(S_t)\le 135$	L	0	234	P	948	1 182
	L	1 017	1 251	$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	
$187.51 \le S_t \le 197.68$	L	0	1 017	$\min_{0\le t\le T}(S_t)\le 135$	P	0	1 017	P	1 182	2 199
	L	0	1 017	$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	
$S_t \ge 197.68$	P	0	infinity	$\min_{0\le t\le T}(S_t)\le 135$	P	1 017	infinity	P	2 199	infinity
	P	0	infinity	$\min_{0\le t\le T}(S_t)> 135$	CL	1 251		CL	3 069	

Explanatory note: CP/L = Constant Profit/Loss.

We can see that if our assumption is fulfilled and the price falls below 135, then the hedging variant IIc ensures the highest profit respectively the lowest loss at all intervals of spot price at time *t*.

5. Conclusion. This paper represents two main contributions. The first one is the proposal of long strangle strategy formation using barrier options in hedging against the price drop of an underlying asset. The new approach based on the analytical expression of the functions of profit from secured position is introduced. The other one is the practical application of this strategy in hedging of SPDR gold shares, which can be also used in practice as a priceless aid in deciding which hedging variant is the most suitable. The detailed analysis of both methods of long strangle formation is performed. The comparison with hedging using the long strangle strategy formed by standard option is presented as well.

It is not possible to explicitly conclude that one of the described hedging variants is better in every practical situation. It depends on the real spot price of the underlying asset at the options expiration time. The selection of appropriate hedging variant must be made by an investor depending on his preferences and expectations.

Our results indicate that hedging using barrier options expands hedging opportunities, thereby it offers more alternatives for price hedging. It allows securing only the most likely unfavourable future price movement scenarios, i.e. allows adaptation to hedger's specific individual requirements, which reduces additional costs of hedging.

References

Amaitiek, O.,F., S. (2009). Vertical ratio call back spread strategy and its application to hedging. Transactions of the Universities of Kosice, 2009, No. 2, pp. 7-10.

Amaitiek, O.,F., S., Balint, T., Resovsky, M. (2010). The Short Call Ladder strategy and its application in trading and hedging. Acta Montanistica Slovaca, 2010, Vol. 15, No. 3., pp. 171-182.

Andreeva, G.I. (2011). Structure of exchange market for derivatives and principles of its organization. Actual Problems of Economics, 2011, Vol. 119, Issue 5, pp 183-187.

Briys, E. Mai, H. M., Bellalah, M., Varenne, F. (1998). Options, Futures and Exotic Derivatives. John Wiley & Sons, Ltd, 1998, 449 p.

Burdenko, I.M., Makarenko, I.O. (2010). Financial derivatives as a form of fictitious capital. Actual Problems of Economics, 2010, Vol. 113, Issue 11, pp 181-188.

Cohen, G. (2005). The bible of options strategies: the definitive guide for practical trading strategies. Financial Times Prentice Hall, 2005, 356 p.

Fahlenbrach, R., Sandas, P. (2010). Does information drive trading in option strategies? Journal of Banking and Finance, 2010, Vol. 34, pp. 2370-2385.

Glova, J., Sabol, T. (2011). Analyza dlhopisov s vlozenymi opciami. E+M Ekonomie a Management, 2011, Vol. 14, No. 3, pp. 77-86.

Haug, E. (1998). The Complete Guide to Option Pricing Formulas. McGraw-Hill, 1998, 232 p.

Hull, J.C. (2005). Options, Futures and Other Derivatives. (6th ed.). Prentice Hall, 2005, 816 p.

Chang, C.C., Hsieh, P.F., Wang, Y.H. (2010). Information content of options trading volume for future volatility: Evidence from the Taiwan options market. Journal of Banking and Finance, 2010, Vol. 34, pp. 174-183.

Chorafas, D. N. (2008). Introduction to Derivative Financial Instruments: Options, Futures, Forwards, Swaps, and Hedging. New York, USA: McGraw-Hill Professional Publishing, 2008, 384 p.

Kolb, R. W. (2007). Futures, Options, & Swaps. (5rd ed.). Wiley-Blackwell Publishers, 2007, 840 p.

Mugwagwa, T., Ramiah, V., Naughton, T. (2008). The Efficiency of the Buy-Write Strategy: Evidence from Australia. Seminar presentation, 2008. RMIT Melbourne.

Mullaney, M. (2009). The complete guide to option strategies: advanced and basic strategies on stocks, ETFs, indexes, and stock indexes. John Wiley & Sons, Ltd, 2009, 556 p.

Rubinstein, M., Reiner, E. (1991). Breaking Down the Barriers. Journal of Risk. 1991, Vol. 4, No. 8, pp. 28-35.

Santa-Clara, P., Saretto, A. (2009). Option strategies: Good deals and margin calls. Journal of Financial Markets, 2009. Vol. 12, pp. 391-417.

Smith, C. (2008). Option Strategies: Profit-Making Techniques for Stock, Stock Index, and Commodity Option. (3rd ed.). John Wiley and Sons, Inc., 2008. 308 p.

Soltes, M. (2010a). Crude oil trading using turbo certificates. Acta Montanistica Slovaca, 2010, Vol. 15, No. 1 pp. 1-4.

Soltes, M. (2010b): Vztah Spreed certifikatov a inverznej Vertical Ratio Call Back Spread opcnej strategie, E+M Ekonomie a management, 2010, No. 2, pp. 119-123.

Soltes, V. (2001). Analyza strategie Long Strangle a navrh optimalneho algoritmu na jej vyuzitie pri praktickom investovan? Ekonomicke rozhsady, 2001, Vol. 30, No. 2, pp. 188-193.

Soltes, V., Soltes, M. (2005). Hedging pomocou opcnych strategii Long Combo a Long Strangle. AIESA – Budovanie spolocnosti zalozenej na vedomostiach. Bratislava: Vydavatelstvo Ekonom, 2005, 5 p.

Soltes, V., Amaitiek, O., F., S. (2010a). Inverse vertical ratio put spread strategy and its application in hedging against a price drop. Journal of Advanced Studies in Finance, 2010, Vol. 1, No. 1, pp. 100-107.

Soltes, V., Amaitiek, O., F., S. (2010b). The short put ladder strategy and its application in trading and hedging. Theory Methodology Practice, 2010, Vol. 6, No. 2, p. 77-84.

Taleb, N.N. (1997). Dynamic Hedging: Managing Vanilla and Exotic Options. Hardcover: Wiley & Sons, 1997. 528 p.

Ye, G.L. (2009). Exotic options: Boundary analyses. Journal of Derivatives and Hedge Funds. 2009, Vol. 15, No. 2, pp. 149-157.

Weert, D.F. (2008). Exotic Options Trading. John Wiley & Sons, Ltd, 2008. 204 p.

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