## Paulo Mačãs Nunes ${ }^{1}$ <br> PRICING IN DURABLE GOODS MONOPOLY WITH DISCRETE DEMAND AND CHANGES OF INCOME

This paper shows, considering a model, two periods, discrete demand and two consumers with different levels of income and consequently different willingness to pay, that the Coase conjecture can fail, that is to say, price does not necessarily decrease over time. Maintaining the relationships between the willingness to pay of consumers with high and low incomes in the two periods, if durable goods producing monopoly's option is to discriminate prices over time, the price is more likely to rise, the greater the variation between the high-income consumer's willingness to pay in the future period, due to the perspective of increased income, and his willingness to pay in the present period. In addition, the lower the interest rates, the greater the possibility of the price rising is. If the highincome consumer foresees a considerable reduction in his income in the future period, the lowincome consumer forecasting a comparable increase in income in that period, changing the relationships between consumers' willingness to pay, the price will also increase, since the monopoly manages to absorb the maximum price each consumer is ready to pay in each period.
Keywords: change in income; Coase conjecture; discrete demand; durable goods, willingness to pay. Jel Classifications: D21, D42.

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ЦІНОУТВОРЕННЯ НА ТОВАРИ ТРИВАЛОГО ВЖИТКУ В УМОВАХ МОНОПОЛІЇ ПРИ ДИСКРЕТНОМУ ПОПИТІ І ЗМІНАХ ДОХОДІВ

У статті розглянуто модель двох періодів із дискретним попитом і двома групами споживачів із різним рівнем доходів. Доведено, що в даному випадку припущення Коуза не завжди вірне, тобто зниження ціни з часом не обов'язкове. З урахуванням бажання $i$ можллвостей груп споживачів з низькими і високими доходами платити за товари тривалого вжитку у двох періодах показано, що в умовах монополії ціна на товар може не лише не знизитися, але й піднятися. Це багато в чому залежить від бажання споживачів з високими доходами платити за товар в майбутньому $i$ зараз. Також можливе підвищення цін при зниженні відсоткових ставок у банках. При зміні в платоспроможності споживачів з низькими і високими доходами ціни швидие за все підніматимуться, оскільки в умовах монополії продавець орієнтується на можсливий максимум.

Ключові слова: зміна доходів; припущення Коуза; дискретний попит; товари тривалого вжитку; готовність платити.

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> ЦЕНООБРАЗОВАНИЕ НА ТОВАРЫ ДЛИТЕЛЬНОГО ПОТРЕБЛЕНИЯ В УСЛОВИЯХ МОНОПОЛИИ ПРИ ДИСКРЕТНОМ СПРОСЕ И ИЗМЕНЕНИЯХ ДОХОДОВ

В статье рассмотрена модель двух периодов с дискретным спросом и двумя группами потребителей с разным уровнем доходов. Доказано, ито в данном случае предположение Коуза не всегда верно, то есть снижение цены со временем не обязательно. С учетом желания и возможсностей групп потребителей с низкими и высокими доходами платить за товары длительного потребления в двух периодах показано, что в условиях монополии цена на товар может не только не понизиться, но и подняться. Это во многом зависит

[^0]от желания потребителей с высокими доходами платить за товар в будущем и в настоящем. Также возможно повыиение цен при понижении процентных ставок в банках. При изменении платежеспособности потребителей с низкими и высокими доходами цены скорее всего будут подниматься, поскольку в условиях монополии продавец ориентируется на возможсный максимум.

Ключевые слова: изменение доходов; предположение Коуза; дискретный спрос; товары длительного потребления; готовность платить.

1. Introduction. In a seminal article, Coase (1972) showed that the price of durable goods decreases over time and may reach marginal cost right away in the present period if the distance between periods is quite short. The economic idea implicit in the conclusions of Coase (1972) is that if the time between various possible periods of durable goods purchase by consumers is short, consumers' disutility from not acquiring durable goods in the present period is practically nil, with consumers being prepared to wait to acquire the goods in future periods when the price is lower. In these circumstances, even a monopoly firm can be forced to lower the price in the current period to levels close to the marginal cost.

There is a lack of specific study of the effects of the perspective of variations in income on the demand for and price of durable goods, considering models with discrete demand. On one hand, Macas Nunes (2009) analyzes the effect of the perspective of changes in consumers' income on the Coase conjecture, but considering a model with continuous demand. On the other, Bulow (1982), Gul (1987), Ausubel and Deneckere (1989), Sobel (1991), Waldman (1996), Denicolo and Garella (1999), Mason (2000), Macas Nunes (2006) and Macas Nunes and Serrasqueiro (2008) use discrete demand models with consumers with different willingness to pay, but do not analyze the specific effect of variations in willingness to pay arising from predicted variations in income.

This paper considers a model with discrete demand, two periods and two consumers with different willingness to pay as a consequence of their different levels of income. Its contribution is to show that if consumers foresee variations in income in future periods with consequent changes in their willingness to pay, the Coase conjecture can fail in various situations.

Maintaining the relationship between the two consumers' readiness to pay, as a consequence of expecting a great increase in income in the future period, if the highincome consumer's expectation is for a significant increase in his willingness to pay in that period, compared to his willingness in the current period, then the price of durable goods can rise over time and not necessarily decrease, as forecast by Coase (1972). Low interest rates may also contribute to a rising price.

Besides, if the high-income consumer foresees a significant fall in income in the future period, his willingness to pay decreases considerably in that period, and the low-income consumer foresees a significant increase in his income in the future period and consequently a considerable increase in his willingness to pay, in such a way that in the future period the income of the low-income consumer in the current period is greater than that of the high-income consumer in the current period, the price may also rise.

The paper is structured as follows, after this introduction: section 2 presents the model, incorporating the assumptions, the optimization process and finally calibration of the model. Section 3 presents the conclusions of the paper.

## 2. The Model

### 2.1 Assumptions

The assumptions of the model are now presented. Just as Bulow (1982), Gul (1987), Ausubel and Deneckere (1989), Sobel (1991), Waldman (1996), Denicolo and Garella (1999), Mason (2000) and Macas Nunes and Serrasqueiro (2008), we consider a model with discrete demand, in which utility is the difference between what consumers are prepared to pay and what they effectively pay for durable goods, i.e., $U T_{t}=\left(V_{t}-P_{t}\right) \delta^{t}$.

We consider two periods, the present period (period 0 ) and the future period (period 1). We assume that the useful life of durable goods is two periods and that they are produced in the present period. Therefore, if consumers acquire durable goods in the current period, they are prepared to pay for their use in two periods, whereas if they postpone purchase until the future period, they are only prepared to pay for use in one period.

Just like Macas Nunes (2006), we consider demand is made up of two consumers with different willingness to pay, the high-income consumer (consumer $h$ ) is prepared to pay $V_{h}$ for durable goods, and the low-income consumer (consumer $l$ ) is prepared to pay $V_{l}$ for durable goods, with $V_{h}>V_{l}$. If the high and low-income consumers acquire durable goods in the present period, they are prepared to pay $V_{h}+V_{h} \delta$ and $V_{l^{+}} V_{l} \delta$ respectively. If they acquire durable goods in the future period, high and lowincome consumers are willing to pay $V_{h}+\Delta V_{h}$ and $V_{l}+\Delta V_{l}$ respectively, with $\Delta$ being the variation of income foreseen by consumers in the future period.

Just like Macas Nunes and Serrasqueiro (2008), we consider that the monopoly's marginal costs are constant, in this paper the marginal cost is k . We also consider that the monopoly produces the units of durable good in sell period. In the case of consumers' utility being the same in the present and future periods, they always choose to acquire durable goods in the present period, that is, if the utility is identical in the present and future periods, consumers prefer to acquire durable goods in the present period rather than postpone purchase until the future period.

### 2.2. Optimization

Initially, we will consider that high and low-income consumers foresee increased income in the future, maintaining the relationship between their willingness to pay. Therefore, we have:

$$
\begin{equation*}
V_{h}+\Delta V_{h}>V_{l}+\Delta V_{l}>V_{h}+V_{h} \delta>V_{l}+V_{l} \delta \tag{1}
\end{equation*}
$$

Lemma 1: If the most advantageous option for the monopoly is to discriminate prices over time, the price may rise, i.e., the price in the future period can be higher than the price in the present period.

Proof:
The monopoly has 5 different options when selling durable goods:
A. Discriminate prices over time, selling one unit of durable goods, at a price between $V_{h}+V_{h} \delta$ and $V_{l}+\Delta V_{l}$, to the high-level consumer in the current period, and
one unit of durable goods, at a price of $V_{l}+\Delta V_{l}$, to the low-income consumer in the future period.
B. Sell one unit of durable goods, at a price of $V_{h}+V_{h} \delta$, to the high-income consumer in the current period.
C. Sell two units of durable goods, at a price of $V_{l^{+}} V_{l} \delta$, to the high and lowincome consumers in the current period.
D. Sell one unit of durable goods to the high-income consumer in the future period at a price of $V_{h}+V_{h} \delta$.
E. Sell two units of durable goods, at a price of $V_{l}+\Delta V_{l}$, to the high and lowincome consumers in the future period.

If the monopoly chooses to discriminate prices over time, to determine the balance between consumers' expectations and the process of maximizing the monopoly's profits over time, we must resort to the backward induction process.

Consumer $h$ (high income) correctly anticipates the price the durable goods monopoly will set in the future period. Opting to discriminate prices over time, the monopoly sets in the future period a price equal to the low-income consumer's willingness to pay, that is, $\mathrm{P}_{1}=\mathrm{V}_{\mathrm{j}}+\Delta \mathrm{V}_{\mathrm{j}}$.

If consumer $h$ acquires the durable goods in the present period, he will have a utility given by:
$U T_{0}=\left(V_{h}+V_{h} \delta-P_{0}\right)$.
If consumer $h$ opts to acquire durable goods only in the future period, his utility will be given by:

$$
\begin{equation*}
U T_{1}=\left(V_{h}+\Delta V_{h}-P_{1}\right) \delta . \tag{3}
\end{equation*}
$$

Consumer $h$ will be willing to acquire durable goods in the present period if the utility from acquisition in the present period is at least equal to the utility of acquiring it in the future period. Therefore, if the monopoly wishes to discriminate prices over time, it sets $\mathrm{P}_{0}$ so that the utility of consumer $h$ in the present and future periods is equal, therefore:
$U T_{0}=U T_{1} \Leftrightarrow\left(V_{h}+V_{h} \delta-P_{0}\right)=\left(V_{h}+\Delta V_{h}-P_{1}\right) \delta$.
Consumer $h$ knows that in the future period the monopoly sets $\Delta \mathrm{P}_{1}=\mathrm{V}_{\mathrm{j}}+\Delta \mathrm{V}_{\mathrm{j}}$, and so: $\quad\left(V_{h}+V_{h} \delta-P_{0}\right)=\left(V_{h}+\Delta V_{h}-V_{l}-\Delta V_{l}\right) \delta$

For consumer $h$ to acquire durable goods in the present period, the monopoly will have to set a price in that period equal to:

$$
\begin{equation*}
P_{0}=V_{h}+\left(V_{l}+\Delta V_{l}\right) \delta-\Delta V_{h} \delta \tag{6}
\end{equation*}
$$

As mentioned above, in the future period the monopoly producing durable goods sets:

$$
\begin{equation*}
P_{1}=V_{l}+\Delta V_{l} . \tag{7}
\end{equation*}
$$

The monopoly's profit will be given by:
$\pi_{A}=P_{0} \times q_{0}+P_{1} \times q_{1} \delta-k \times(1+\delta)$,
and so we have:
$\left.\pi_{A}=\left(V_{h}+\left(V_{l}+\Delta V_{l}\right) \delta-\Delta V_{h} \delta\right) \times 1+\left(V_{l}+\Delta V_{l}\right) \delta\right) \times 1-k(1+\delta)$,
giving finally:
$\pi_{A}=V_{h}+2\left(V_{l}+\Delta V_{l}\right) \delta-\Delta V_{h} \delta-k(1+\delta)$.
If the monopoly chooses to sell one unit of durable goods in the present period,
since only the high-income consumer buys it, the profit will be:
$\pi_{B}=P_{0} \times q_{0}-k \times 1$,
and so we have:
$\pi_{B}=\left(V_{h}+V_{h} \delta\right) \times 1-k \times 1=V_{h}+V_{h} \delta-k$.
If the monopoly chooses to sell two units of durable goods in the present period, since both consumers (high- and low-income) acquire it, the profit will be:
$\pi_{C}=P_{0} \times q_{0}-k \times 2$,
giving:
$\pi_{C}=\left(V_{l}+V_{l} \delta\right) \times 2-2 k=2\left(V_{l}+V_{l} \delta\right)-2 k$.
If the monopoly chooses to sell one unit of durable goods to the high-income consumer in the future period, we have:

$$
\begin{equation*}
\pi_{D}=P_{1} \times q_{1} \delta-k \times \delta \times 1, \tag{15}
\end{equation*}
$$

and so we have:

$$
\begin{equation*}
\pi_{D}=\left(V_{h}+\Delta V_{h}\right) \delta \times 1-k \delta=\left(V_{h}+\Delta V_{h}\right) \delta-k \delta . \tag{16}
\end{equation*}
$$

Finally, if the monopoly opts to sell two units of durable goods to high and lowincome consumers in the future period, we have:

$$
\begin{equation*}
\pi_{E}=P_{1} \times q_{1} \delta-k \times \delta \times 2, \tag{17}
\end{equation*}
$$

giving:
$\pi_{E}=\left(V_{l}+\Delta V_{l}\right) \delta \times 2-2 k=2\left(V_{l}+\Delta V_{l}\right) \delta-2 k \delta$.
The monopoly chooses to discriminate prices over time, selling one unit of durable goods to the high-income consumer in the present period and one unit to the low-income consumer in the future period, if:

$$
\begin{align*}
& \pi_{A}>\pi_{B} \Leftrightarrow 2\left(V_{l}+\Delta V_{l}\right)>V_{h}+\Delta V_{h}+k \delta,  \tag{19}\\
& \pi_{A}>\pi_{C} \Leftrightarrow V_{h}+2 \Delta V_{l} \delta+k(1-\delta)>\Delta V_{h} \delta+2 V_{l},  \tag{20}\\
& \pi_{A}>\pi_{D} \Leftrightarrow V_{h}(1-\delta)+2 \delta\left(V_{l}+\Delta V_{l}\right)>2 \Delta V_{h} \delta+k,  \tag{21}\\
& \pi_{A}>\pi_{E} \Leftrightarrow V_{h}>\Delta V_{h} \delta+k(1-\delta) . \tag{22}
\end{align*}
$$

Given the situations presented above, the monopoly chooses to discriminate prices over time. The price will tend to increase if:

$$
\begin{equation*}
P_{1} \delta>P_{0} \tag{23}
\end{equation*}
$$

that is,
$\left(V_{l}+\Delta V_{l}\right) \delta>V_{h}+\left(V_{l}+\Delta V_{l}\right) \delta-\Delta V_{h} \delta$,
and finally:
$\Delta V_{h} \delta>V_{h}$.
We find that the greater the forecast of increased income for the high-income consumer in the future period, and the lower the interest rate, the greater the possibility of the price increasing, that is, the greater the possibility of the price in the future period to be higher than the price in the present period. In these circumstances, a fall in the price of durable goods over time may not be found, contrary to the stated by Coase (1972).

Next we will assume the possibility of the high-income consumer forecasting a sharp fall in his income in the future period, and the low-income consumer expecting an equivalent increase in his income in that period.

Lemma 2: If the high-income consumer forecasts a sharp fall in his income in the future period, and the low-income consumer forecasts a significant increase in his income
in the future period, the price can also show a rising tendency, i.e., the price in the future period can also be higher than that in the current period.

Proof:
If the high-income consumer in the current period expects a sharp fall in his income in the future period, his willingness to pay diminishes considerably in that period, and the low-income consumer in the current period expects his income to increase greatly in that period, and his willingness to pay increases considerably in that period, the following relationship may be found between the consumers' willingness to pay:

$$
\begin{equation*}
V_{1}+\Delta V_{l}>V_{n}+V_{h} \delta>V_{l}+V_{l} \delta>V_{h}+\Delta V_{h} \tag{26}
\end{equation*}
$$

In the circumstances described above, in the current period, the low-income consumer cannot acquire durable goods, since $V_{h}+V_{h} \delta>V_{l}+V_{l} \delta$. Therefore, in the present period the monopoly sets $P_{o}=V_{h}+V_{h} \delta$.

In the future period, taking advantage of the low-income consumer's increased income, the monopoly sets $P_{l}=V_{l}+D V_{l}$, the high-income consumer being unable to buy durable goods because $V_{l}+\mathrm{D} V_{l}>V_{h}+\mathrm{D} V_{h}$. So the monopoly manages to absorb all the consumer excess, since it sets a price equal to the consumers' maximum willingness to pay in each period. Therefore, the price is shown to rise since $P_{1} d>P_{0}$, if

$$
\left(V_{l}+\Delta V_{l}\right) \delta>V_{h}+V_{h} \delta
$$

3. Conclusion. Considering a model with discrete demand, two periods and two consumers with different willingness to pay, this article's contribution is to show that the tendency for the price of durable goods to fall may not be found, in this way contradicting the conjecture of Coase (1972).

On one hand, if both consumers expect increased income in the future period, maintaining the relationship between the two consumers' willingness to pay, the possibility of the price increasing will be greater, the greater the increase in the highincome consumer's willingness to pay in the future period compared to his willingness to pay in the current period, and the lower the market interest rate.

On the other hand, if the high-income consumer foresees a considerable decrease in his income in the future period, with a consequently significant reduction in his willingness to pay in that period, and the low-income consumer expects a considerable increase in his income in the future period, and consequently a significant increase in his willingness to pay, the price may rise, since the monopoly can fix a price equal to consumers' maximum willingness to pay in each period.

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