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PROBABILISTIC AND STATISTICAL METHODS OF RISK ANALYSIS IN THE INVESTMENTS EFFECTIVENESS EVALUATION AND THEIR APPLICATION IN BUSINESS PRACTICE

The article provides an assessment of the application of probabilistic and statistical methods of risk analysis in evaluating the investments efficiency by companies in Poland. Current economic knowledge concerning the risk assessment of the undertaken investments allows estimating the potential negative consequences of risk and incorporate them in pre-investment plans. The use of quantitative methods' achievements in this field allows minimizing the impact of risks, which can lead to failure of business ventures. However, the research carried out by the authors indicated their little use by enterprises in Poland, even big ones, which confirms the conclusions derived from the studies conducted in other countries in this area. It is alarming that in spite of the development of science and improved techniques applied in risk assessment, their common use is still low.
Keywords: *investments, effectiveness of investments, risk evaluation.*

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ЙМОВІРНІСНО-СТАТИСТИЧНІ МЕТОДИ АНАЛІЗУ РИЗИКУ ПРИ ОЦІНЮВАННІ ЕФЕКТИВНОСТІ ІНВЕСТИЦІЙ ТА ЇХ ВИКОРИСТАННЯ У БІЗНЕСІ

У статті проаналізовано рівень використання ймовірнісно-статистичних методів аналізу ризику при оцінюванні ефективності інвестицій підприємств в Польщі. Актуальні економічні знання, що стосуються оцінювання ризику розпочатих інвестиційних заходів, дозволяють розраховувати можливі негативні наслідки ризику і брати їх до уваги в передінвестиційних проектах. Використання досягнень кількісних методів у цій області надають можливість значно мінімізувати наслідки загроз, які можуть призвести до закриття підприємства. Проведені дослідження показали їх мінімальне використання підприємствами в Польщі, навіть найбільшими, що підтверджує висновки, які випливають з проведених в інших країнах досліджень у цьому напрямку. Насторожує факт, що, незважаючи на розвиток науки і вдосконалення методик, які використовуються при оцінюванні ризику, розповсюдженість їх використання залишається низькою.
Ключові слова: *інвестиції, ефективність інвестицій, оцінка ризику.*
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ВЕРОЯТНОСТНО-СТАТИСТИЧЕСКИЕ МЕТОДЫ АНАЛИЗА РИСКА ПРИ ОЦЕНКЕ ЭФФЕКТИВНОСТИ ИНВЕСТИЦИЙ И ИХ ИСПОЛЬЗОВАНИЕ В БИЗНЕСЕ

В статье проведен анализ уровня использования вероятностно-статистических методов анализа риска при оценке эффективности инвестиций предприятий в Польше. Актуальные экономические знания, касающиеся оценки риска начинаемых инвестиционных мероприятий, позволяют рассчитывать возможные негативные последствия риска и принимать их во внимание в прединвестиционных проектах. Использование достижений количественных методов в этой области дает возможность

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значительно минимизировать последствия угроз, которые могут привести к закрытию предприятия. Проведенные исследования показали их минимальное использование предприятиями в Польше, даже самыми крупными, что подтверждает выводы, исходящие из проведенных в других странах исследований в этом направлении. Настораживает факт, что, несмотря на развитие науки и совершенствование методик, используемых при оценке риска, распространенность их использования остается низкой.
Ключевые слова: *инвестиции, эффективность инвестиций, оценка риска.*

I. Introduction and theoretical issues

Uncertainty and risk are inherent characteristics of each investment. The success of any undertaken investment project depends mainly on identifying and quantifying the potential opportunities and threats. Investment decisions are burdened with a greater or lesser degree of risk. This is due to the fact that:

– making decisions very rarely takes place when the decision-maker knows all the possible outcomes of the action subjected to the choice (i.e. in conditions of certainty);

– while making the decision one often has only a part of the desired information;

– the decisions are largely based on a forecast of future activities;

– each forecast is only an approximation of a future reality.

Together with the growing uncertainty about the outcome of the undertaken investments, a variety of methods to reduce the risk formed. In the literature there are many approaches to risk assessment, as well as many attempts to classify them. It should be noted that in spite of the range and the complexity of construction, one can distinguish two types of methods:

1) the methods which allow to estimate the scale of the risk;

2) the methods only informing about the potential risk aiming at minimizing its effects on investment.

The first group of methods to estimate the risk scale uses the achievements of statistics and probability theory, therefore, a group of these methods can be described as probabilistic and statistical methods, and the second group as non-probabilistic methods.

The probabilistic and statistical methods enable directly to assess the degree of risk borne by the investor. These methods may include in particular:

– effectiveness account with the use of statistical measures (standard deviation, variance, variation coefficient);

– decision trees;

– simulation methods.

The use of statistical measures in investment efficiency account results in an analysis of the key measures of the dispersion of a random variable around its expected value, i.e.:

– standard deviation;

– variance;

– coefficient of variation (Jozwiak, Podgorski 1994).

The expected value is an average value for the random variable. For the discrete random variable in case of economic account one can specify the following formula:

$$E(X)=\sum_{i=1}^{n}x_i\times p_i \tag{1}$$

where p_i – the probability function of a random variable X taking values $x_1 (i=1,2,...n)$.

Thus, the expected value of NPV can be described as:

$$E(NPV)=\sum_{i=1}^{n}NPV_i\times p_i \tag{2}$$

For the same values of efficiency (e.g. NPV) the different level of dispersion indicates a more risky project. The basic measure of dispersion is the variance. The variance of X is defined as the weighted average of the deviations squares of each variable from its expected value, where the weights are the probabilities of the events.

$$D^2(X)=E[X-E(X)]^2=\sum_{i=1}^{n}[x_i-E(X)]^2\times p_i \tag{3}$$

The variance has always non-negative values, where as the value equals to 0 only if all possible results are the same. In this case, there is uncertainty about the future outcome, so the decision is not subjected to risks. In other words, the risk is equal to zero. The higher the variance, the greater the risk of making a particular decision (Zelias, 1998).

Due to the fact that in case of variation the deviations from the expected value are raised to a square, the most convenient category from the point of view of interpretation is the standard deviation.

The standard deviation is the square root of the variance and is defined by the formula:

$$D(X)=\sqrt{E[X-E(X)]^2}=\sqrt{\sum_{i=1}^{n}[x_i-E(X)]^2\times p_i} \tag{4}$$

Just as the variance, standard deviation has non-negative values. When the standard deviation equals 0, it means there is no risk. This risk increases with the standard deviation.

The fact of squaring deviations from the expected value causes that in case of large individual variations may be an overstate of the level of the surveyed investment risk (Smith, Pielichaty, Quilt, 1999). In such cases it may be appropriate to use the average deviation category:

$$R(X)=\sum_{i=1}^{n}p_i\times|x_i-E(X)| \tag{5}$$

The average deviation is a weighted average of absolute values of deviations of the random variable x_1 from the expected value. Like in the previous category, the level of risk incurred increases with increasing rate.

In the analysis of the above ratios both the positive and negative deviations from the expected value are treated in the same way. However, the negative deviation indicates an undesirable situation for the investor, and the positive deviation of the win-win situation. Thus, it is appropriate not only to examine the degree of dispersion, but also its asymmetry. Left-handed asymmetry of the distribution will inform about the advantage of the possible results higher than the expected value, while the right-handed asymmetry signals about the lower results. To determine the degree of asymmetry understood as the degree of deviation of the distribution from the symmetric distribution, the third order central moment is used. The asymmetry coefficient is defined by:

$$\gamma = \frac{\mu_3}{D^3(X)}, \tag{6}$$

where μ_3 — third order central moment.

If the asymmetry index is positive, the right-handed asymmetry occurs and when it has a value less than 0 — its an example of the left-handed asymmetry.

The third order central moment is the expected value of the function $g(x)=[X-E(X)]^3$ of this variable (Jozwiak, Podgorski, 1994). For the discrete variable it takes the following form:

$$\mu_3 = \sum_{i=1}^{n} [x_i - E(X)]^3 \times p_i \tag{7}$$

An alternative to the asymmetry coefficient may be the use of semi-variance and standard semi-deviation, which take into account only the negative deviations from the expected value (Zelias, 1998). Semi-variance is defined as:

$$D_s^2(X) = \sum_{i=1}^{n} p_i \times d_i^2, \tag{8}$$

where d_i — negative deviation from the expected value

$$d_i = \begin{cases} 0, \ when \ x_i \geq E(X) \\ x_i - E(X), \ when \ x_i < E(X) \end{cases}$$

Standard semi-deviation that will inform how much on average the results would deviate from the expected value in case on unfavorable economic condition is presented as:

$$D_s(X) = \sqrt{\sum_{i=1}^{n} p_i \times d_i^2} \tag{9}$$

The standard deviation and variances show the absolute values of differences, but do not demonstrate the relationship between the dispersion and the expected value. The measure which allows for such a comparison is the coefficient of variation (Nowak, Pielichaty, Poszwa, 1999). For projects which vastly differ in scale, the use of this index seems to be much more appropriate. Taking into account the standard deviation, the index takes the form:

$$V(X) = \frac{D(X)}{E(X)}, \tag{10}$$

while we will consider the average deviation:

$$V(X) = \frac{R(X)}{D(X)}. \tag{11}$$

These ratios are measures of a relative risk. As other measures of dispersion they take non-negative values. Along with their increase the risk goes up as well.

The obvious difficulty is the use of statistical measures is probability estimation (Bednarski, Wasniewski, 1996). However, due to the knowledge of the market, using the experience of the past, it is possible to build a probability distribution of the expected results of the project.

Here it may be useful to apply the decision trees and dendrites of risk. They are mapping a set of possible states of the real and the interrelated ones in such a way that the existence of a particular state is conditioned by what had happened before, and allows for the existence of a particular set of successive states. States, which are represented by the branches of the tree correspond to different values of the random variable representing an economic effect. In case of a decision tree the branches represent alternative decision-making, whereas in case of the dendrite risk, the states are just random events (Starczyk, Glowacz, 1997). In case of a decision tree, the decision is indicated by means of a square, from which the extended branches represent alternative measures. However, with the help of the circle, which extend the branches representing the possible outcomes, random events are defined. Assigning to each event a certain probability to occur, it is possible to quantify the risk by calculating the probability of each of the situations represented by each terminal branch.

The main disadvantage of this method is that a very large expansion of trees in case of complex investments, and hence the need for a large number of assumptions, many of which are subjective in nature (Marcinek, 2001). However, the most important advantage is the transparency and clarity of the analysis.

A more advanced technique is represented by simulation methods. These methods rely on constructing models that describe the structure and functioning of the researched system (e.g. investment project) in a mathematical way and then reproduce with these models the successive states of the analyzed system in order to determine its properties. In case of an investment project it is possible to examine the impact of several variables and to get the data determining the probable pattern of behavior of the considered investment project (Marcinek, 2001). The most commonly used simulation method in the analysis of investment risk is the Monte Carlo method. It involves the repetition of the procedure for calculating the value of the investment performance indices (e.g. NPV) according to the model for randomly generated uncertain variables affecting the value. In this analysis, in contrast to the non-probabilistic methods, uncertain variables are considered together, taking into account the relationship between them. The simulation cycle consists of 5 main stages (Ostrowska, 2002):

– model construction of the investment project, which extracts the most relevant variables and relationships between them;

– establishing a hypothetical distribution of probable values for each variable burdened with uncertainty;

– random selection of the vales from the hypothetical distribution of an uncertain variable and the setting the baseline;

– conducting a specific series of simulation experiments to obtain different values of the output variable;

– determining the empirical distribution of the output variable value obtained from a series of simulation experiments and estimation of this distribution.

Simulation methods are one of the most effective tools due to the simultaneous consideration of several variables, but their widespread use is limited with the high degree of complexity and thus the high cost of this type of analyses.

In spite of the variety of methods which may affect the assessment of the risks of the investments undertaken their use is limited. This is not a problem only for Poland but in the whole world. The studies conducted in the UK among 236 companies demonstrated that 65% of companies did not use any methods of risk assessment (Graham, Harvey, 2001). Prior to the transformation of the economy in a centralized market system and the initial phase of transformation in Poland, such studies were not necessary. Currently taking into account regional economic integration they are useful on the micro-, regional and national levels. There is a need for theoretical and practical research to develop similar activities in Poland. The main goal of this paper is to diagnose the degree of application of these methods to assess the risk involved in business ventures by the Polish companies.

II. Research methodology

In order to assess the degree of use of the above methods in economic analysis a research among 500 companies was conducted. The study focused on five sectors with the greatest capital expenditures. Such narrowing of the field of studies was predetermined, on the one hand by a multitude of businesses that could be potential objects of research, and on the other one, such a choice was related with the purpose of the paper for which the observation of objects undertaking the development projects was necessary. The following selected five sectors were:

- industrial manufacturing,
- real estate and business, science
- trade and service,
- transport, storage and communication,
- construction.

Companies which are the object of studies were divided by means of cluster analysis (k-means method) into three groups:

- small enterprises,
- big enterprises,
- middle enterprises.

Clustering was performed in four dimensions. As these dimensions were adopted:

- fixed assets,
- current assests,
- number of employees,
- revenue from the sale.

Companies with the lowest values of the tested variables were in the second cluster. This cluster was identified as a 217-element group of the smallest companies in the sample. In this group, the smallest deviations from the average has the number of employees ≈0.38, and the greatest the fixed assets ≈0.58.

The largest values of the tested variables were characterized by a 93-element cluster of three, identified as the largest group of companies in the sample. In this group, the average of the tested variables is more than 2.5 times higher than the average in the group of small enterprises. The largest deviation from the expected value in this group have revenues from the sale of about 1.46, and the smallest the number of employees ≈0.67.

The middle group between the above is the first 190-element concentration, which can be described as medium-sized enterprises. Here the largest standard deviation is characterized by the number of employees of approximately 0.89, and the smallest current assets are about 0.53.

The respondents, on a 5-position scale from 0 to 4 (where 0 – never, 4 – always) had the opportunity to indicate the frequency of using these methods. When applying this type of scale one of the most frequently used measure is the arithmetic mean (Graham, Harvey 2001). However, due to the fact that this ratio is not a very reliable in case of asymmetric and multimodal distributions (Walesiak 1996) it was abandoned. As an alternative it was proposed an assessment of the probability of the events. Using the classical definition of probability (Papoulis 1972), the probability assessment can be defined as:

$$\overline{P(A_i)} = w_i, \tag{12}$$

where: w_i – relative frequency defined by the formula:

$$w_i = \frac{n_i}{n}, \tag{13}$$

n_i – number of the events A_i $i=1,2,3,4$,

n – number of the surveyed units.

For this purpose there were calculated the relative frequencies of the events A_1, A_2, A_3, A_4, A_5, where:

- event A1 – indication on the scale "4",
- event A2 – indication on the scale "3",
- event A3 – indication on the scale "2",
- event A4 – indication on the scale "1",
- event A5 – indication on the scale "0".

and the sum of events:

- $A_1 \cup A_2$,
- $A_1 \cup A_2 \cup A_3$,

for all the enterprises and subgroups discrete in the cluster analysis. The sum of the assessment of events probability in general can be written as (Хальд, 1956):

$$\begin{aligned} \overline{P(A_1 \cup A_2 \cup \ldots \cup A_m)} &= \overline{P(A_1)} + \overline{P(A_2)} + \ldots \overline{P(A_m)} - \overline{P(A_1 \cap A_2)} - \overline{P(A_1 \cap A_3)} - \ldots \\ &\ldots - \overline{P(A_{m-1} \cap A_m)} + \overline{P(A_1 \cap A_2 \cap A_3)} + \overline{P(A_1 \cap A_2 \cap A_4)} + \ldots \\ &\ldots + \overline{P(A_{m-2} \cap A_{m-1} \cap A_m} + \ldots + (-1)^{m-1} \overline{P(A_1 \cap A_2 \cap \ldots \cap A_m)} \end{aligned} \tag{14}$$

Thus, respectively for the sum of events A1 A2 and A1 A2 A3, the formula 14 will be the following:

$$\overline{P(A_1 \cup A_2)} = \overline{P(A_1)} + \overline{P(A_2)} - \overline{P(A_1 \cap A_2)} \tag{15}$$

$$\begin{aligned} \overline{P(A_1 \cup A_2 \cup A_3)} &= \overline{P(A_1)} + \overline{P(A_2)} + \overline{P(A_3)} - \overline{P(A_1 \cap A_2)} - \overline{P(A_1 \cap A_3)} - \\ &- \overline{P(A_2 \cap A_3)} + \overline{P(A_1 \cap A_2 \cap A_3)} \end{aligned} \tag{16}$$

As in the survey form there are independent events, the formulas will be (McClave, Benson 1988):

$$\overline{P(A_1 \cup A_2)} = \overline{P(A_1)} + \overline{P(A_2)} \tag{17}$$

$$\overline{P(A_1 \cup A_2 \cup A_3)} = \overline{P(A_1)} + \overline{P(A_2)} + \overline{P(A_3)} \tag{18}$$

III. Research results

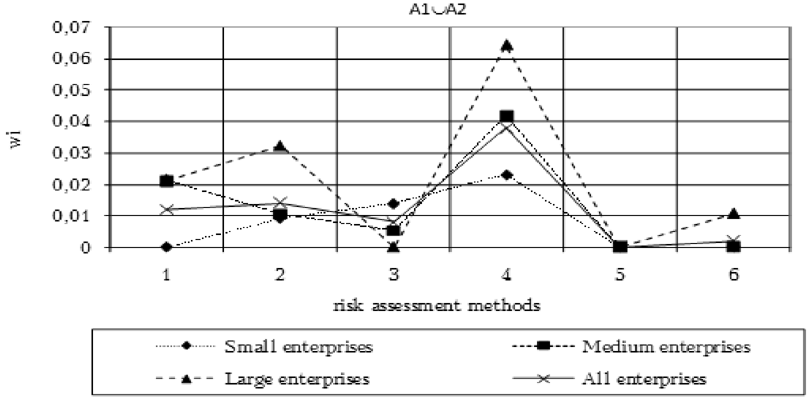
The surveyed entities had five risk assessment methods and "others" to choose from. These methods were numbered as follows:

1) decision trees;

2) variance, standard deviation;
3) coefficient of variation;
4) simulations;
5) game theory;
6) others.

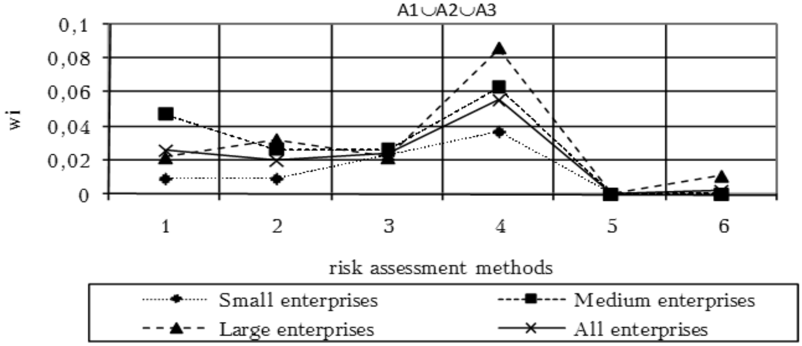
Unfortunately, the results of research conducted by the authors are not optimistic. The distributions of all responses were characterized by a very strong right-sided asymmetry indicating a very poor use of such methods by the surveyed companies.

The studies showed that companies mostly apply simulation methods, are implemented mainly by the biggest companies (Fig. 1, 2).



Sourse: Developed by the author's.

Figure1. **Relative frequency of the use of risk assessment methods – the sum of events A1, A2**



Sourse: Developed by the author's.

Figure2. **Relative frequency of the use of risk assessment methods – the sum of events A1, A2, A3**

It is understandable as the complexity of the simulation methods is considerable, which requires either to have the right department in the company which deals with this type of analysis, which only the big companies can afford, or outsource the analyses to the third parties, which is associated with significant costs. Probably smaller companies that use these methods rarely decide to look the analyses in case of large and significant investments.

Due to the little use of risk assessment methods, the surveyed companies were asked about the ways to reduce risk. Respondents on the 5-point scale from 0 to 4 (where 0 – no, four – always) indicated the degree of limitation among:

- risk avoidance and reduction of its sources and effects,
- risk transfer (insurance, quarantees etc.),
- reserves use,
- others.

The research indicated very great care of all enterprises expressed in risk avoidance, which the image is left-side asymmetry of the distribution of this variable (Table 1).

Table 1. **Asymmetric distribution coefficients of measures to reduce the negative effects of risk in the surveyed enterprises**

Specification	All enterprises	Large enterprises	Medium enterprises	Small enterprises
Avoiding risk	-0,569257	-0,514526	-0,656320	-0,520286
Risk transfer	0,199356	-0,054519	0,144590	0,357318
Reserves use	0,651151	0,415676	0,563400	0,852445

Sourse: Developed by the author's.

For large companies, it is important to use the opportunities of risk transfer as it is indicated by the left-side asymmetry (AS = -0.05), and the analysis of probability assessment of sum of events A1 A2 and A1 A2 A3 – w_i – equal respectively to 0.46 and 0.57 (Table 2).

Table 2. **The relative frequencies of negative risk effects reduction**

	Reduction of negative risk effects	relative frequency						
		$w_{i(A1)}$	$w_{i(A2)}$	$w_{i(A3)}$	$w_{i(A4)}$	$w_{i(A5)}$	$w_{i(A1vA2)}$	$w_{i(A1vA2vA3)}$
Large enterprises	1. avoiding risk	0,290	0,269	0,204	0,129	0,108	0,559	0,763
	2. risk transfer	0,258	0,204	0,108	0,215	0,215	0,462	0,570
	3.reserves	0,054	0,204	0,194	0,118	0,430	0,258	0,452
	4.others	0,000	0,000	0,011	0,000	0,989	0,000	0,011
Medium enterprises	1. avoiding risk	0,279	0,300	0,216	0,053	0,153	0,579	0,795
	2. risk transfer	0,174	0,189	0,184	0,137	0,316	0,363	0,547
	3.reserves	0,105	0,111	0,221	0,200	0,363	0,216	0,437
	4.others	0,000	0,005	0,005	0,011	0,979	0,005	0,011
Small enterprises	1. avoiding risk	0,286	0,253	0,226	0,074	0,161	0,539	0,765
	2. risk transfer	0,161	0,134	0,198	0,166	0,341	0,295	0,493
	3.reserves	0,083	0,101	0,157	0,207	0,452	0,184	0,341
	4.others	0,005	0,000	0,009	0,009	0,977	0,005	0,014
All enterprises	1. avoiding risk	0,284	0,274	0,218	0,076	0,148	0,558	0,776
	2. risk transfer	0,184	0,168	0,176	0,164	0,308	0,352	0,528
	3.reserves	0,086	0,124	0,188	0,188	0,414	0,210	0,398
	4.others	0,002	0,002	0,008	0,008	0,980	0,004	0,012

Sourse: Developed by the author's.

Also, medium and small companies often use this tool, which can be some kind of a surprise – w_i equal respectively 0,36 and 0,29 for the sum of events A1 A2 and 0.55 and 0.49 for the sum of events A1 A2 A3. It can be concluded that the majority of businesses, both large and small ones prefer to "be better safe than sorry" with the reserves than to take a risk and try to limit the negative effects of other methods.

IV. Conclusions

The success of the implemented investment project is essential both for the company implementing the project, entities involved in its financing, as well as for the entire economy. Uncertainty and the related risk foster the companies to conduct proper economic calculations in the planning of investment, which is obviously a factor that increases the chances of the investment success. The degree of the use of the methods of identification and the investment risk assessment among the surveyed companies is very small. It was discovered that among 500 surveyed, 358 (72%) had never applied any assessment methods. The use of probabilistic and statistical methods, which as the only methods allow to assess the degree of risk borne by the investor directly is even smaller. They are mostly used by the largest with the biggest money resources but also those, which undertake the investment projects with high money commitment. The most common statistical and probabilistic methods are simulation methods which, although difficult and costly to develop, are characterized by high reliability.

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