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BENEFITS OF ASSETS DIVERSIFICATION: APPLICATION OF MEAN VARIANCE EFFICIENT FRONTIER TO ROMANIAN STOCK MARKET

Our paper investigates the efficiency of portfolio diversification at Romanian stock market. In order to do this, we use data mining techniques to study the effect that the number of assets has on the total portfolio variance. We use the liquidity of transactions during Jan. 1st 2011 and May 11th 2012 to rank 42 of the most active stocks traded at Bucharest Stock Exchange, and after that we simulate portfolios and compute the Markowitz efficient frontier for combinations of assets that start with only the most liquid company and continue until all the 42 stocks are included. Our results show that portfolio risk decreases as the number of assets increases, both for the minimum variance portfolio (MVP) and for other discretionary types of portfolios. We also find that the marginal benefits of diversification are high at the earlier stages and decreases as the number of assets increase. We continue by simulating different combinations between the 42 stocks universe of risky portfolios and the risk free asset, and we confirm that best choice of capital market line (CML) portfolios, for all levels of risk aversion, are found when the number of assets in the portfolio is higher.

Keywords: *stock returns; Markowitz efficient frontier; portfolio diversification; emerging markets; data mining.*

JEL Classification: *G01, G11, G12, G14, G15, G17, G32.*

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ПЕРЕВАГИ ДИВЕРСИФІКАЦІЇ АКТИВІВ: ЗАСТОСУВАННЯ СЕРЕДНЬОГО ВІДХИЛЕННЯ ЕФЕКТИВНОЇ МЕЖІ ДО РУМУНСЬКОГО ФОНДОВОГО РИНКУ

У статті досліджено ефективність портфельної диверсифікації на румунському фондовому ринку. Для вивчення впливу кількості активів на загальні портфельні коливання використано технології аналізу даних. Ліквідність трансакцій 1 січня 2011 р. і 11 травня 2012 р. використано для ранжування 42 найбільш активних пакетів акцій на фондовій біржі Бухаресту, після цього зроблено моделювання портфелів і обчислено ефективна межа Марковіца для поєднань активів, що починаються з найбільш ліквідної компанії і продовжуються доти, поки не будуть включені всі 42 пакети акцій. Результати показали, що портфельний ризик зменшується зі збільшенням кількості активів, як для портфеля з мінімальними коливаннями, так і для інших дискретних типів портфелів. Також виявлено, що граничні переваги диверсифікації високі на початкових стадіях і знижуються зі зменшенням кількості активів. Моделювання різних комбінацій 42 пакетів акцій ризикових портфелів і безризикових активів підтвердило, що найкращий вибір портфелів лінії ринку капіталу, для всіх рівнів ризику, відбувається у тому випадку, якщо кількість активів у потфелі вища.

Ключові слова: *прибуток з акцій; ефективний кордон Марковіца; портфельна диверсифікація; ринки, що розвиваються; аналіз даних.*

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ПРЕИМУЩЕСТВА ДИВЕРСИФИКАЦИИ АКТИВОВ: ПРИМЕНЕНИЕ СРЕДНЕГО ОТКЛОНЕНИЯ ЭФФЕКТИВНОЙ ГРАНИЦЫ К РУМЫНСКОМУ ФОНДОВОМУ РЫНКУ

В статье исследована эффективность портфельной диверсификации на румынском фондовом рынке. Для изучения влияния количества активов на общие портфельные колебания использованы технологии анализа данных. Ликвидность трансакций 1 января 2011 г. и 11 мая 2012 г. использована для ранжирования 42 наиболее активных пакетов акций на фондовой бирже Бухареста, после этого произведено моделирование портфелей и вычислена эффективная граница Марковица для сочетаний активов, начинающихся с наиболее ликвидной компании и продолжающихся до тех пор, пока не будут включены все 42 пакета акций. Результаты показали, что портфельный риск уменьшается с увеличением количества активов, как для портфеля с минимальными колебаниями, так и для других дискретных типов портфелей. Также обнаружено, что предельные преимущества диверсификации высоки на начальных стадиях и снижаются по мере того, как уменьшается количество активов. Моделирование различных комбинаций 42 пакетов акций рисковых портфелей и безрисковых активов подтвердило, что наилучший выбор портфелей линии рынка капитала, для всех уровней риска, получается в том случае, если количество активов в портфеле выше.

Ключевые слова: *прибыль по акциям; эффективная граница Марковица; портфельная диверсификация; развивающиеся рынки; анализ данных.*

1. Introduction. The minimization of return variance is one of the classical topics of portfolio theory. According to the traditional approach in portfolio theory, one of the main difficulties in variance minimization is that the necessary input factors, the individual values for variance and covariance of the assets from the investment universe are unknown. Most of the time, researchers try to estimate the values for variance and covariance using samples of time series of assets returns. Subsequently, they incorporate these estimates into optimization as if they were the true parameters. Disappointed with the performance of market weighted benchmark portfolios, yet skeptical about the merits of active portfolio management, in recent years investors turned to alternative index definitions. Minimum variance investing is one of these popular rule driven, new passive concepts.

Taking into account traditional and modern portfolio theories, our study focuses on expanding the portfolio by gradually including stocks with a lower level of liquidity, stocks that normally should have a smaller capitalization. With time, many researchers were interested in the behavior of portfolio variance related to the number of assets included in a portfolio. A great number of such studies, both very recent and very old, show that the variance of portfolio is decreasing as number of assets is increasing. Still, we didn't have until now a very relevant and extensive study to confirm that for Romanian stock market. This is why we consider our study to be an original and useful contribution in this field of research, its conclusions being of interest both for the research community and for financial practitioners.

Also, our study uses the most actual data available and offers a different approach in comparison with all the previous related studies, as we construct our universe of risky assets (from which the portfolios are build) taking into consideration liquidity of the assets. We favored this approach because we wanted to offer portfolio managers and finance practitioners a useful tool for their day-to-day investment decisions, knowing from practice that investors consider liquidity as one of the most important and valued characteristic of a financial asset or investment opportunity.

To sustain our conclusions, we simulate different approaches to the process of portfolio management: first we look at the evolution of the variance of the minimum variance portfolios (MVP), than we investigate the behavior of the Markowitz efficient frontier as the number of risky assets in the universe is increasing, after that we study how the expected return of an efficient fixed variance portfolio is influenced by the number of assets included, and in the end we look at the risk of the optimal portfolio from the capital market line (CML) for a specified level of risk aversion.

Our results generally confirm, for Romanian stock market, the conclusions of the modern portfolio theory and the findings of other related studies conducted on other developed or emerging stock exchanges. At the same time we emphasize some particularities of the diversification efficiency at Romanian stock market and we recommend an optimum level of diversification in direct relation with the liquidity of the assets included in the portfolio.

2. Literature review. In the past decade wide range of literature appeared regarding the interdependence between a certain portfolio's associated risk and its stocks' diversity. The main empirically proven finding behind this relation is that as diversification of portfolio increases (and most of the studies are considering international diversification), we witness gradual reduction of the total portfolio risk and better ratio between the expected return and this estimated risk.

Solnik (1974) found that international equity diversification reduces risk, but other authors have competing views regarding this theory: one view states that international diversification reduces risk, while the second view agrees that diversification is beneficial, but the additional qualitative risks of investing in foreign securities outweigh potential returns (Rockefeller, 2001).

The first references to this subject found their sources of inspiration in the theory of portfolio diversification in finance, starting with Markowitz's famous study (1952) showing that the main strategy to reduce the risk of a portfolio of securities is to invest in assets whose returns were uncorrelated (or not highly positively correlated). Markowitz has demonstrated his findings using the statistical and mathematical models (before that his mains ideas were accepted intuitively by the finance community) and has introduced the concept of efficient frontier of portfolios. According to this theory, a rational investor should select a portfolio that lies on the efficient frontier. The main conclusions of the Markowitz's study were that diversification does not rely on individual risks being uncorrelated, but for diversification to show benefits it is only necessary that assets are imperfectly correlated. This means that the risk reduction from diversification is limited by the extent to which the correlations of the assets included in the portfolio are lower or even negative.

A few years later, Tobin (1958) expanded on the Markowitz's study including a risk-free asset to the analysis. This made it possible to leverage or deleverage portfolios on the efficient frontier, generating the notions of super-efficient portfolio and the capital market line (CML).

Very related to Tobin's work, the capital asset pricing model (CAPM) of Sharpe (1964), Lintner (1965), Mossin (1966) represents the birth of the asset pricing theory, still being widely used nowadays in estimating the cost of capital for firms and evaluating the performance of managed portfolios.

The CAPM model uses various assumptions about markets and investor behavior to give a set of equilibrium conditions that allow us to predict the return of an asset in relation to its level of systematic (or no diversifiable) risk.

Solnik (1974) used stock returns from 8 different countries over 6 years, in order to demonstrate which are the benefits obtained from diversification across different countries. He used weekly data from Belgium (20 stocks), France (65 stocks), Germany (40 stocks), Italy (30 stocks), Netherlands (25 stocks), Switzerland (15 stocks), the United Kingdom (50 stocks) and the United States (65 stocks). One of Solnik's hypothesis assumed that investor has no ability to select profitable investments and he implemented this no-skill assumption by selecting stocks randomly and assigning each stock an equal weight. He then calculated the proportion of variance that could be eliminated from portfolios by increasing the number of randomly selected stocks. The main conclusion of Solnik's investigation with random diversification is that a portfolio does not need more than about three dozen common stocks to achieve substantial benefits from either domestic or international diversification.

According to Solnik (2000), emerging market equities generally have higher average returns, lower correlations with developed markets, greater serial correlation and greater volatility. Slavescu and Panait (2011) studied volatility, correlations and causality among developed and emerging stock markets during 2007-2011 and reached the same conclusions.

Other empirical studies document that equity portfolios constructed to have the lowest possible risk have surprisingly high average returns. R. Clarke, H. de Silva and S. Thorley (2011) derive an analytic solution for the long-only minimum variance portfolio under the assumption of a single-factor covariance matrix. The analytic and empirical results of their study suggest that minimum variance portfolio performance is largely a function of the long-standing empirical critique of the traditional CAPM that low beta stocks have relatively high average returns.

In 1967 Haugen discovered a market abnormality according to which the low-risk portfolios focused on the left side of Markowitz's efficient frontier would offer investors the best returns and the lowest variance in the long run. This led him to come up with the minimum variance investment theory, which was since then taken into consideration by many investment funds[1]. This discovery stood against the doctrine of that time because the accepted paradigm was Eugene Fama's efficient market hypothesis. His opinion is that due to the effect of mean reversion, those companies that are more profitable during a given period will lose their relative profitability in future and that in all lines of business, success tends to be temporary. The fact that low risk stocks have higher expected returns is an anomaly in the field of finance. Baker and Haugen (2012) argued that this anomaly extends to all the equity markets in the world.

The common investment perception is that small capitalized stocks are usually avoided when constructing a portfolio based on mean variance. An investor may dis-

[1] http://citywire.co.uk/global/minimum-variance-inventor-explains-why-it-can-continue-to-outperform/a382783

like small caps — whose illiquidity/volatility increase when the market is bear - because they fail to provide liquidity when investors may want to exit positions. Similarly, an investor may prefer large capitalized stocks to small capitalized stocks because the returns on the latter fall when market volatility/illiquidity is high and therefore fail to provide insurance against market volatility. Hence small caps command higher expected returns than large caps, which happen to have low variance risk, i.e. positive co-skewness. Additionally, also co-kurtosis could be priced, if an investor dislikes assets whose risk increases at volatile markets. We will test and observe if the same conclusions can be made to our study based on Romanian frontier stock market.

The study conducted by Gabriel Frahm and Christoph Memmel (2010) developed estimators for the minimum variance portfolio that dominate the traditional estimators that are often applied. The new estimators lead to a smaller out-of-sample return variance than the traditional estimators and represent a weighted average of the traditional estimator and of a reference portfolio, for instance of a portfolio with equal shares for all assets (naïvely diversified portfolio). They derive two shrinkage estimators for a global minimum variance portfolio that dominates the traditional estimator with respect to the out of sample variance of the portfolio return.

Fletcher (2009) examined the performance of global minimum variance (GMV) and minimum tracking error variance (TEV) portfolios in the UK stock returns using different models of the covariance matrix and finds that both GMV and TEV portfolios deliver portfolio risk reduction benefits in terms of significantly lower volatility and tracking error volatility relative to passive benchmarks for every model of the covariance matrix used. However, the GMV (TEV) portfolios do not provide significantly superior Sharpe (1966) performance relative to passive benchmarks except for the restricted GMV portfolios.

Contributing to the existing literature, Bodnar and Schmid (2009) constructed the exact tests and confidence intervals for the 3 parameters of the efficient frontier. Their result consisted in the derivation of a confidence region of the whole efficient frontier for a finite sample. It turns out that this region is bordered by 5 parabolas.

3. Data and methodology. In our study we used the most liquid 41 companies traded at Bucharest Stock Exchange. In order to establish the liquidity hierarchy, we have compared the total value of the transactions for each of the 78 companies traded at Bucharest Stock Exchange during Jan. 1st 2011 and May 11th 2012. The list of the most liquid 41 companies, in the ascending order of the total value of the transactions for this period is presented in Table 1 at the end of this article. For all those 41 companies we have obtained official daily stock prices during the period Jan 1st 2011 — May 11th 2012. All the data was provided by the Bucharest Stock Exchange, via Bloomberg Professional service platform.

We were very careful to adjust all the prices with the corporate events that took place during the investigated period for some of the companies included in our study (mainly dividends and share capital increases).

Also we were very careful to align the date in perfect chronologic order. For all the situations when an individual stock was not traded at all during any particular day but the stock exchange was open (so we had a price for the index at the end of that day), we filled the "blank" with the last available price from previous trading sessions.

After all this preparations were accomplished, in order to eliminate the obvious non-stationarity from our data we have transformed the price time series into return time series for all the 41 individual stocks.

Regarding the returns estimation, as Strong (1992, p.353) pointed out "there are both theoretical and empirical reasons for preferring logarithmic returns. Theoretically, logarithmic returns are analytically more tractable when linking together sub-period returns to form returns over long intervals. Empirically, logarithmic returns are more likely to be normally distributed and so conform to the assumptions of the standard statistical techniques." This is why we decided to use logarithmic returns in our study since one of our objectives was to test whether the daily returns were normally distributed or, instead, showed signs of asymmetry (skewness). The computation formula of the daily returns is as follows:
where $R_{i,t}$ is the return of asset *i* in period *t*; $P_{i,t}$ is the price of asset *i* in period *t* and

$$R_{i,t} = Ln(\frac{P_{i,t}}{P_{i,t-1}}),$$

$P_{i,t-1}$ is the price of asset *i* in period *t-1*. As already mentioned above, according to this methodology of computing the returns, the prices of the assets must be adjusted for corporate events such as dividends, splits, consolidations and share capital increases (mainly in case of individual stocks because indices are already adjusted).

As a result of this initial data gathering we obtained 41 time series of log-returns, each with 344 daily observations.

For those 41 time series we have computed the mean daily log-returns, the daily sample variances and standard deviations. Also we have computed 41 rows by 41 columns variance-covariance matrix.

For a financial time series the mean represents the simple mathematical average of all the observations within the sample. It is obtained by adding up the series and dividing the result by the number of observations.

The standard deviation of a financial time series is a measure of dispersion or spread in the series. The standard deviation is computed by:
where N is the sample size, R_i represents the individual observations of daily returns,

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}\left(R_i - \overline{R}\right)^2}{N-1}},$$

and *R* represents the sample mean computed as above. The standard deviation is actually the square root of the variance.

Using Matlab we have simulated 41 possible universes of risky assets, the first being composed only by the most liquid assets (FP in our case) and then going forward by including one more active at a time, in the priority of their position in the liquidity hierarchy. As a result, the second risky asset universe was composed by the first two stocks in terms of liquidity, the third universe was composed by the first three stocks in terms of liquidity, and so on until the 41th universe of risky assets that included all the 41 stocks used in our study.

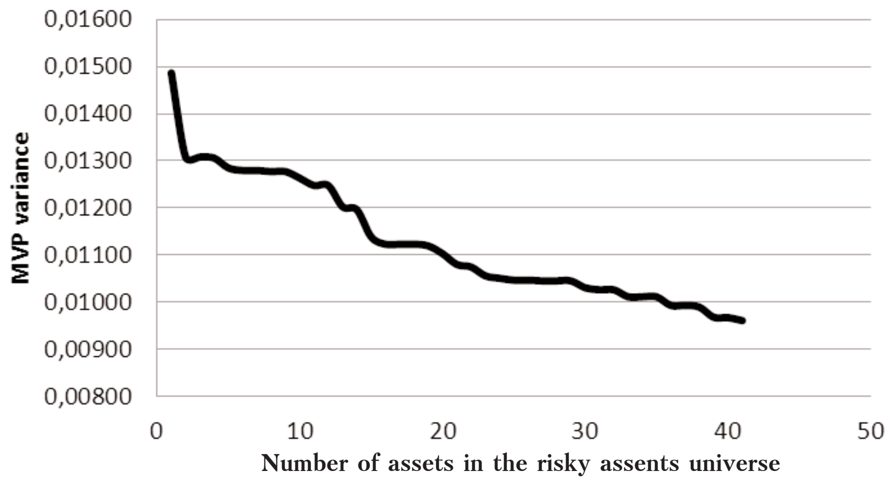
For each of the 41 simulated universes of risky assets we have identified the minimum variance portfolio and measured its variance and expected return. Also, we have plotted each of 41 efficient frontiers in order to investigate their transition induced by the size of the universe (the number of assets included).

The next step in our research was to identify the efficient portfolios, from each of 41 efficient frontiers, which had specific value of variance. We have determined the expected returns for these portfolios and compared them in order to draw conclusions regarding the evolution of the expected return for an efficient portfolio with a predetermined risk, as the number of the assets in the risky universe is increasing.

We concluded our study of the efficiency of the diversification at Bucharest Stock Exchange by introducing the risk free asset and identifying the unique Capital Market Line related to each of the 41 efficient frontiers previously generated. The risk free rate was considered to be the current (May 11th 2012) daily continuously compounded interest rate paid by the medium term Romanian sovereign bonds issued in local currency. Considering different levels of investor risk aversion, and assuming that the utility function of the investment is the one introduced by Arrow (1965) and Pratt (1964) we have determined the optimal portfolios for those predetermined levels of risk aversion on each of 41 capital market lines. Subsequently, we studied the transition of the expected return and risk for those optimal portfolios as the number of assets in the risky asset universe increased.

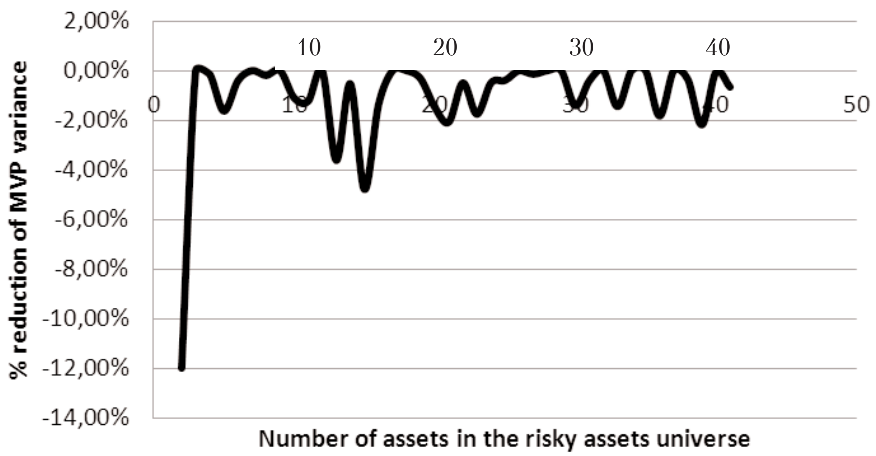
4. Results and interpretations. As mentioned above, we used Matlab's Financial Toolbox to simulate 41 possible universes of risky assets from 41 companies selected for our study. The first universe was composed only by the most liquid asset, in our case FP (see Table 1 at the end of the article). The second universe was constructed by adding to the first universe the next company in terms of liquidity, in our case BRD. As a conclusion the second universe of risky assets is made from FP and BRD. Going forward, the third universe of risky assets is made from FP, BRD and SIF5. The fourth universe of risky assets is made from FP, BRD, SIF5 and SIF2, and so on, until the 41th universe of risky assets which is made from all the 41 companies included in our study. Using Matlab functions we have identified the minimum variance portfolio (MVP) for each of the 41 universes of risky assets and we computed the variance for each of those 41 minimum variance portfolios. The result is presented below in Figure 1.

Our data and calculations presented in Figure 1 confirm that the variance (risk) of the MVP is decreasing as the number of stocks from which it was generated is increasing. This proves the benefits of portfolio diversification even from a perspective where each asset added to the portfolio is chosen for its level of liquidity of transactions. We can observe from Figure 1 that the efficiency of diversification is higher when we diversify portfolios with a small number of assets. As we go forward and have portfolios that are already composed of a large number of assets, if we further diversify them, the efficiency of such action is decreasing.



Source of data: Bucharest Stock Exchange; calculations by the authors.

Figure 1. **Behavior of the variance for MVP as the number of assets in the risky universe is increasing**

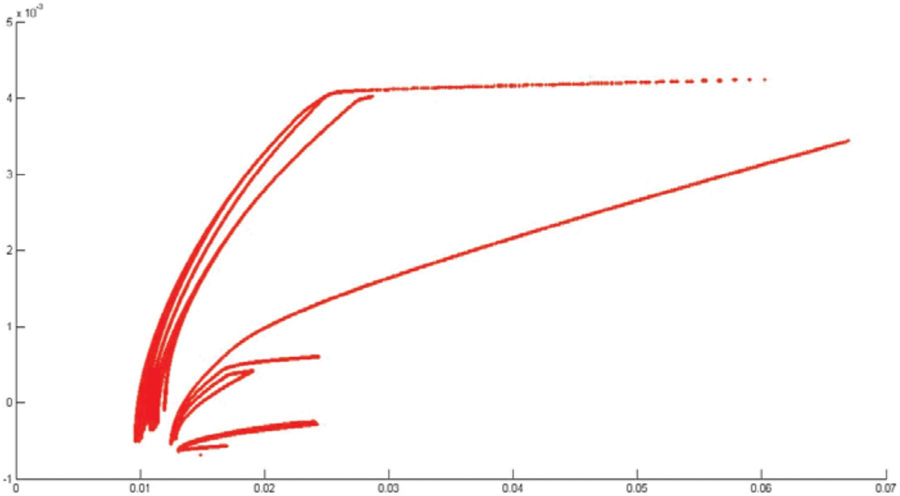


Source of data: Bucharest Stock Exchange; calculations by the authors

Figure 2. **Behavior of the variance for MVP as the number of assets in the risky universe is increasing**

In Figure 2 we show the companies that contribute the most to the reduction of MVP variance when introduced in the pool of risky assets for the first time. As we can observe the most effective are the first 25 actives introduced in the risky assets universe. As the universe of risky assets grow further, from 25 to 41 assets, each of the assets that are introduced contribute to the reduction of the variance of MVP but at much lower rates of impact.

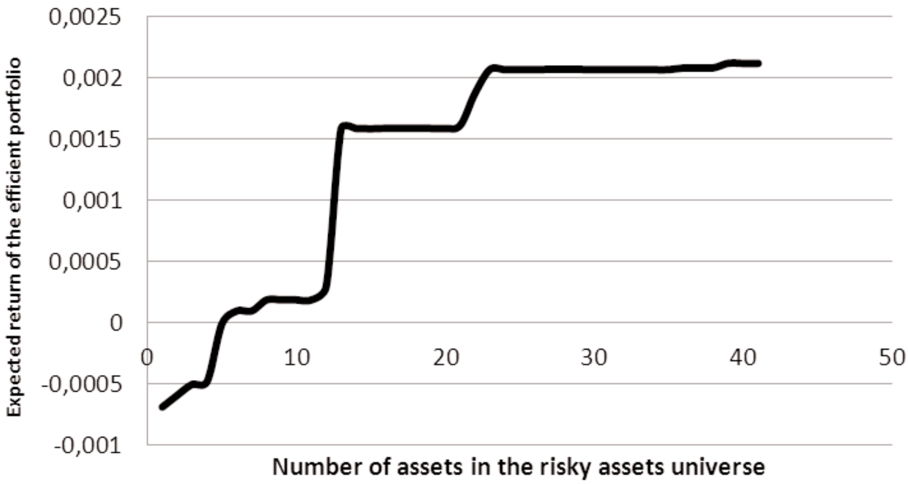
Seeing that the variance of MVP is indeed reduced as the number of the assets in the risky assets universe is increasing, we were interested to see the whole picture and to identify how exactly the entire Markowitz efficient frontier is behaving as we increase the number of stocks in the risky assets universe. The results are presented in Figure 3 (below) which plots all the 41 efficient frontiers related to each of 41 simulated universes of risky assets. As we can observe the Markowitz efficient frontier has a progressive translation to the left and upwards as the number of assets in the risky assets universe is increasing. This proves that an individual investor finds better choices of efficient portfolios (with lower risk and higher expected rates of return) from larger universes of assets than from smaller universes.



Source of data: Bucharest Stock Exchange; calculations by the authors.

Figure 3. **The translation of the efficient frontier to left (lower levels of risk) as the number of assets in the risky assets universe is increasing**

In order to consolidate our findings, we tried to approach the benefits of diversification from a different angle. We looked at how the expected rate of return is behaving for efficient portfolios (that is for portfolios that are situated on the efficient frontier) as we increase the number of assets risky from which that particular efficient frontier is determined. We found that at different predefined levels of accepted variance (assumed portfolio risk), the expected rate of return is higher if the portfolio is from an efficient frontier computed from a larger number of assets. For example, one investor who is interested to hold the most liquid asset at the market (in our case FP), will automatically accept the level of risk (variance of that asset). If instead of investing in a single asset he will be interested in a diversified efficient portfolio, than the calculations show that our investors should be entitled to expect a larger average rate of return. The expected rate of return for such an efficient portfolio with a predetermined level variance is higher as he chooses to take into consideration a universe with a larger number of risky assets. The results for this particular simulation are presented in Figure 4.

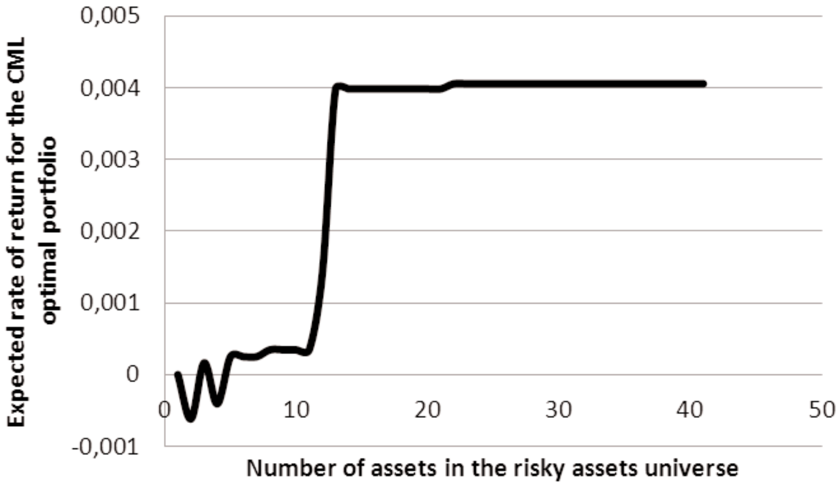


Source of data: Bucharest Stock Exchange; calculations by the authors.

Figure 4. **The expected rate of return for efficient portfolios with a predetermined level of variance equal with the variance of the most liquid asset on the market (FP)**

Going further with our study, we introduce the risk-free rate defined as the daily continuously compounded interest rate paid by Romanian government bonds issued on medium term in local currency. By combining 41 simulated universes of risky assets with the risk-free rate we identify the related 41 capital market lines (CML) that represent possible choices for an investor who is interested in the optimal allocation between risk and risk-free combination of assets. We assumed that the utility function for all the investors at the market is the one proposed by Arrow (1965) and Pratt (1964) and we have computed the optimal portfolio choice for the investors with different levels of risk aversion. We found that most of the time the optimal portfolios formed from larger universes of risky assets are better in terms of expected return in comparison with optimal portfolios formed from smaller a universes of risky assets. In Figure 5 we show the behavior of the expected rate of return for the optimal portfolios that would be chosen by an investor with an Arrow-Pratt coefficient of risk aversion equal 2. As shown, an optimal portfolio CML obtained from a combination of risk free asset and a larger universe of risky assets offers a better expected rate of return. Still, the most important benefits of diversification appears to come from universes made of the first 15-25 most liquid assets.

5. Conclusions. This study was dedicated to efficiency of portfolio diversification at Bucharest Stock Exchange. We used daily price series during Jan 1st 2011 and May 11th 2012 in order to select the most liquid 41 companies out of the total 78 traded companies. After that, we simulated investment universes by gradually adding companies in the order of their market liquidity (starting with the most liquid ones and continuing with the ones with lower levels of liquidity). We obtained 41 such simulated universes of risky assets and compared between them the values of MVP, the form of the Markowitz efficient frontiers, the expected return for efficient portfolios with different predetermined levels of variance and the expected return for optimal CML portfolios with different predetermined levels of investor risk aversion.



Source of data: Bucharest Stock Exchange; calculations by the authors.

Figure 5. **The expected rate of return for optimal CML portfolios chosen by an investor with an Arrow-Pratt coefficient of risk aversion equal with 2**

Our results show that investors obtain better portfolios (with lower risk, or with higher rate of return), when they made their decision in a universe with a larger number of risky assets. However, we find that, in most cases, marginal benefits of diversification are much lower after the universe of risky assets already includes the most liquid 25 assets. As a result of this finding we consider that a portfolio that includes the most liquid 25 assets at Romanian capital market represents an optimal choice for individual investors who find it difficult to manage portfolios with a large number of assets.

Further studies in this field at Romanian capital market could be realized from different approaches regarding the construction of portfolios. We constructed our universes of assets with a liquidity constrain, but other approaches could be that the universes of assets are randomly constructed, or selected taking into account the size (capitalization of the company), the sector/industry, or the relative market valuation.

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Table 1. **41 companies used in our study to generate 41 universes of risky assets**

Series symbol	Company name	Included in universes
FP	Sc Fondul Proprietatea Sa	1-41
BRD	BRD - GSG	2-41
SIF5	Societatea de Invesititii Financiare 5 Oltenia Sa	3-41
SIF2	Societatea de Invesititii Financiare 2 Moldova Sa	4-41
SNP	OMV Petrom	5-41
TLV	Banca Transilvania Sa	6-41
SIF1	Societatea de Invesititii Financiare 1 Banat Crisana Sa	7-41
SIF4	Societatea de Invesititii Financiare 4 Muntenia Sa	8-41
EBS	Erste Bank	9-41
BVB	Sc Bursa de Valori Bucuresti Sa	10-41
TGN	Sc Transgaz Sa	11-41
OLT	Sc Oltchim Sa Rm. Valcea	12-41
AZO	Sc Azomures Sa Tg. Mures	13-41
TEL	Sc Transelectrica Sa	14-41
RPH	Sc Ropharma Sa	15-41
BIO	Sc Biofarm Sa Bucuresti	16-41
BRK	SIF Broker Sa Cluj	17-41
DAFR	Sc Dafora Sa Medias	18-41
ELGS	Sc Electroarges Sa Curtea de Arges	19-41
ATB	Sc Antibiotice Sa Bucuresti	20-41
ALR	Sc Alro Sa Slatina	21-41
CBC	Sc Carbochim Sa	22-41
STZ	Sc Sinteza Sa Oradea	23-41
BCC	Banca Comerciala Carpatica	24-41

The End of *Table 1*

Series symbol	Company name	Included in universes
IMP	Sc Impact Sa	25-41
COMI	Sc Condmag Sa	26-41
SCD	Sc Sicomed Zentiva Sa	27-41
OIL	Sc Oil Terminal Sa	28-41
CMP	Sc Compa Sa Sibiu	29-41
VESY	Sc Ves Sa Sighisoara	30-41
PPL	Sc Prodplast Sa	31-41
AMO	Sc Amonil Sa	32-41
TRP	Sc Teraplast Sa	33-41
RRC	Sc Rompetrol Rafinare Sa Constanta	34-41
SOCP	Sc Socep Sa Constanta	35-41
VNC	Sc Vrancart Sa	36-41
PTR	Sc Rompetrol Well Services Sa	37-41
TBM	Sc Turbomecanica Sa	38-41
ARS	Sc Aerostar Sa Bacau	39-41
ALT	Sc Altur Sa Slatina	40-41
ALU	Sc Alumil Rom-Industry Sa	41

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