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USING DYNAMIC PROGRAMMING FOR OPTIMIZING THE INVESTMENT STRATEGY FOR AN AGRICULTURAL SHEEP HOLDING: AN INVESTMENT CASE SIMULATION

Nowadays, for achieving the best solution in valuing the economic potential, many investors appeal to econometrical models in optimizing investment decisions. Allocating financial resources in production, in agricultural capacities is for many entrepreneurs the best way to achieve economic performance by using in a superior manner the available resources applying the efficiency criteria. This paper presents a model of optimizing the strategy to develop an agricultural holding, through investments, using dynamic programming for a sheep farm, in the context of best decision-making.

Keywords: *investments, dynamic programming, efficiency, decision-making.*

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ВИКОРИСТАННЯ ДИНАМІЧНОГО ПРОГРАМУВАННЯ ДЛЯ ОПТИМІЗАЦІЇ ІНВЕСТИЦІЙНОЇ СТРАТЕГІЇ ВІВЦЕФЕРМИ: МОДЕЛЮВАННЯ ІНВЕСТИЦІЙНОЇ ПРИВАБЛИВОСТІ

У статті показано, що на даний час для ухвалення найкращих рішень при оцінюванні економічного потенціалу багато інвесторів використовують економетричні моделі для оптимізації інвестиційних рішень. Розподіл фінансових ресурсів у сільськогосподарському виробництві для багатьох підприємців є оптимальним способом покращення економічних показників із використанням наявних ресурсів за вживання критеріїв ефективності. Представлено модель оптимізації стратегії розвитку сільськогосподарського холдингу за рахунок інвестицій за допомогою динамічного програмування в контексті ухвалення найкращого рішення про інвестування.

Ключові слова: *інвестиції, динамічне програмування, ефективність, прийняття рішень.*

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ИСПОЛЬЗОВАНИЕ ДИНАМИЧЕСКОГО ПРОГРАММИРОВАНИЯ ДЛЯ ОПТИМИЗАЦИИ ИНВЕСТИЦИОННОЙ СТРАТЕГИИ ОВЦЕФЕРМЫ: МОДЕЛИРОВАНИЕ ИНВЕСТИЦИОННОЙ ПРИВЛЕКАТЕЛЬНОСТИ

В статье показано, что в настоящее время для принятия наилучших решений при оценке экономического потенциала многие инвесторы используют эконометрические модели для оптимизации инвестиционных решений. Распределение финансовых ресурсов в сельскохозяйственном производстве для многих предпринимателей становится оптимальным способом улучшения экономических показателей с использованием имеющихся ресурсов с применением критериев эффективности. Представлена модель оптимизации стратегии развития сельскохозяйственного холдинга за счет инвестиций с помощью динамического программирования в контексте принятия лучшего решения об инвестировании.

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Ключевые слова: *инвестиции, динамическое программирование, эффективность, принятие решений.*

Introduction. Investments in development and modernization of production capacities are, in the context of market economy, an important lever in creating benefits and new opportunities for achieving prosperity. Recovery of economic potential requires not only a proper identification of allocated resources but also implies a good definition of the investment destination and adopting the best decision. Planning an investment program involves the use of appropriate methods of mathematical programming in obtaining the best investment options with adequate financial efforts in the context of limited resources. Using dynamic programming in this sense can be an important tool in attainment of all objectives in a proposed investment.

Dynamic programming offers a whole range of decisions with major impact, especially in terms of optimizing the allocated resources in the context of a coherent decision, which aims to maximize revenues achieving the ultimate efficiency criteria. A coherent mode of dynamic programming is to make decisions related to successive stages, depending on the decisions taken in respective sequences and whose results reflect these decisions.

As Romanu and Vasilescu (1997), Vasilescu et al. (2000), Ionita and Blidaru (1999) or Cicea et al (2008), Claudiu et al. (2008), Bagher (2011) and Andrei (2010) suggest, developing a dynamic programming model of investment is based on the total fund of investment knowledge, the years of performance, the production capacity for each department and economic efficiency indicators characterizing the composition of each section of a farm. Also in literature, Gittinger (1972), Vasilescu et al. (2003), Stoica et al. (2007), Subic et al. (2008) and Vasiljevic (2006), Iarca et al. (2010), argue that this activity involves defining the production capacity which has to be built for every year, the quantities of raw materials necessary for functioning, domestic consumption or sales outside an agricultural unit, so that the profit obtained during the realization of the economic objective of production volume reaches the to maximum.

Numerous studies, such as Andric (1991), Vasilescu et al. (200), Subic (2003), Botezatu and Andrei (2009), Done et al. (2009) state that calculations on the allocation of investment funds is made only for the duration of execution (of) the economic purpose. It aims to divide them by departments. This model is used with priority for integrated production units and the finished product of a polling station is further down the blank for it. Production capacities of the final sections are designed.

As a result of the project on modernization-development of a farm sheep, the "PKB" — OPOVO agricultural holding (South Banat, Republic of Serbia) questions were prepared for an optimal plan of sheep-breeding. The technical-economic documentation to achieve this investment indicates that initially the agricultural holding has a number of sheep, which is divided into 3 sections, namely:

- sheep up to 3 months (lambs);
- sheep aged between 3 months and 12 months (sheep);
- sheep aged over 12 months (ewes and rams).

We note that, both sheep from the second section as well as sheep in the third section, after a year has an average number of offsprings in age from 0 to 3 months, which are part of the first section. Of these sheep, some are retained for breeding and breeding material, others are sold. It is required that the problem is solved by setting up a plan for breeding and selling sheep, such as in the execution period (*d*) the gain a maximum profit from selling sheep, and the number of sheep after this time period to be at least equal to the final number, designed, respectively, for 3 sections.

Materials and methods. The data available to us are: the total allocated funds of fixed capital investment is 110,000 EUR which is to be achieved in 3 years, in the first year the investments amounting to 34,000 EUR, in the second year — 36,000 EUR and in the third year — 40,000 EUR. The investment funds necessary for sheep-breeding in section I are of 11,000 EUR, for sheep breeding in section II of 38.500 EUR and sheep breeding in section III of 60.500 EUR.

The investment fund is composed of its own resources (meaning 20% from the sale of animals) and attracted sources of non-refundable character (respectively 80%, of which 25% of IBRD sources and 55% received capital from the SAPARD program. Under the terms imposed by the IBRD and SAPARD (the static programming approach) it is required that the investment fund spent in the first year to be at least 24,000 EUR. Sheep farm is characterized by technical-economic indicators, which are presented in Table 1.

Table 1. **Technical-economic indicators characterizing the 3 sections of sheep**

Section	Symbol	Number of animals	Medium quantity (kg/animal)	Unit price (Ђ/kg)	Production capacity		Production cost		Specific investment
					physical (kg)	value (Ђ)	unitary (Ђ/kg)	total (Ђ)	S=I/q
lambs	X	175	12	2,8	2,100	5,880	2,3	4,830	62,86
sheep	Y	130	40	4,2	5,200	21,840	3,6	18,720	296,15
ewes and rams	Z	560	68	2,5	38,080	95,200	2,2	83,776	108,04

Source: investments' indicators.

Marking with the symbols X, Y and Z the 3 production sections, in which sheep are divided, the links between them can be presented schematically as shown in Fig. 1.

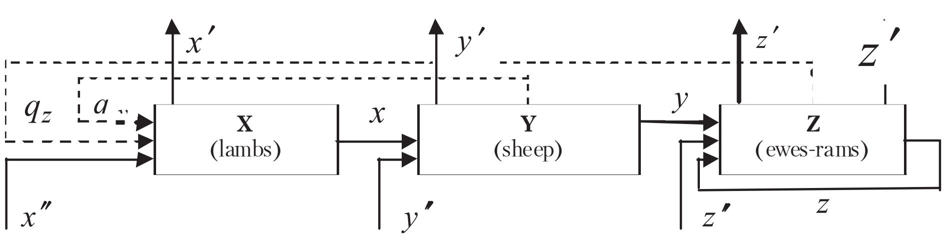

Figure 1. **The scheme of the links between the agricultural holding's sections**

The symbols used in Figure 1 have the following meanings:

X — production capacity achieved in the first division;

Y — production capacity achieved in the second division;

Z — production capacity achieved in the third division;

x", y", z" — animals purchased from outside the first division, second and third, respectively;

x, y, z — production flows between departments, respectively the animals delivered by the first section to the second section (x), this one to the third one (y) and self-delivery in the third section (z);

x', y', z' — the sales of animals for each department;

q_y, q_z — breeding flows, respectively animals in the form of offsprings of the second and third sections delivered to the first section.

We note that production flows between departments (x, y, z), as well as the sales of animals made by each section (*x', y', z'*) must be established as reduced by animal mortality. Also, breeding streams (q_y, q_z) should be reduced with offspring stillbirths. A final statement which refers to the period of execution is putting the condition through which: the agricultural holding is not allowed to sell and buy at the same time, the same animal and the agricultural holding is not allowed to sell animals from the initial endowment.

Results and discussion. Based on these factors, we now can proceed to developing the mathematical model of dynamic programming. In our case, the model of dynamic programming of investment is composed of 3 groups of recurrence relations (constraints) and one objective function (the economic efficiency criterion):

$$\begin{cases} x_2'' = X_2 \\ y_2'' = Y_2 \\ z_2'' = Z_2 \\ k_y \cdot \left(Y_{n-1} + y_n'' - y_n'\right) + k_z \cdot \left(Z_{n-1} + z_n'' - z_n'\right) + x_{n+1}'' - x_{n+1}' = X_{n+1} \\ X_{n-1} + x_n'' - x_n' + y_{n+1}'' - y_{n+1}' = Y_{n+1} \\ Y_{n-1} + y_n'' - y_n' + Z_{n-1} + z_n'' - z_n' + z_{n+1}'' - z_{n+1}' = Z_{n+1} \\ k_y \cdot \left(Y_{d-1} + y_d'' - y_d'\right) + k_z \cdot \left(Z_{d-1} + z_d'' - z_d''\right) + x_{d+1}'' - x_{d+1}' \geq X_f \\ X_{d-1} + x_d'' - x_d' + y_{d+1}'' - y_{d+1}' \geq Y_f \\ Z_{d-1} + z_d'' - z_d' + Y_{d-1} + y_d'' - y_d' + z_{d+1}'' - z_{d+1}' \geq Z_f \\ s_X \cdot X_{n+1} + s_Y \cdot Y_{n+1} + s_Z \cdot Z_{n+1} \leq \sum_{n=1}^{d} I_n \\ x \geq 0,\ y \geq 0,\ z \geq 0 \end{cases} \quad , n = 1, 2, 3, \dots, d$$

$$\max F(x, y, z) = \sum_{n=1}^{d} \left[A_{n+1} \cdot p_A - B_{n+1} \cdot p_B - \left(C_{n+1} \cdot c_C + B_{n+1} \cdot c_B - A_{n+1} \cdot c_A\right)\right],$$ that means:

$$\max F(x, y, z) = \sum_{n=1}^{d} \left\{\left(x_{n+1}' \cdot p_x + y_{n+1}' \cdot p_y + z_{n+1}' \cdot p_z\right) - \left(x_{n+1}'' \cdot p_x + y_{n+1}'' \cdot p_y + z_{n+1}'' \cdot p_z\right) - \right.$$
$$\left. - \left[\left(X_n + x_{n+1}'' - x_{n+1}'\right) c_x + \left(Y_n + y_{n+1}'' - y_{n+1}'\right) c_y + \left(Z_n + z_{n+1}'' - z_{n+1}'\right) c_z\right]\right\}$$

where:

x_n, y_n, z_n — the existing animal effectives (which come from the previous section) at the beginning of the year *n*;

x'_n, y'_n, z'_n — the animals sold at the beginning of the year n;

x''_n, y''_n, z''_n — the animals bought at the beginning of the year $n+2$;

$x_{n+1}, y_{n+1}, z_{n+1}$ — the existing animals (which are delivered to the next section) at the beginning of the year $n+1$;

k_y, z_y — the average number of animal offspring from the second and third sections;

X_f, Y_f, Z_f — final capacity (projected) for the 3 sections of animals;

$\sigma_X, \sigma_Y, \sigma_Z$ — the production from a monetary unit of fixed capital achieved in the X, Y and Z department;

I_n — the investment in the year n;

A — the sales from 01.01., from 8^{00}-24^{00};

B — the purchases from 01.01., from 8^{00}-24^{00};

C — the existent from 01.01., from 8^{00} (the animal's flow from the previous section, or the existing animal capacity);

p — the sale-purchase unit price;

c — the annual unit cost for the growth (breeding) of animals.

We will first set the values to the symbols above, as follows:

I_t=110.000 EUR; I_1=34.000 EUR, I_2=36.000 EUR, I_3=40.000 EUR

X_4=175 heads, Y_4=130 heads, Z_4=560 heads;

s_X=62,86 EUR/animal, s_Y=296,15 EUR/animal, s_Z=108,04 EUR/animal;

k_y=1,3 offspring; k_z=1,5 offspring;

p_x=2,8 EUR/kg, p_y=4,2 EUR/kg, p_z=2,5 EUR/kg;

c_x=2,3 EUR/kg, c_y=3,6 EUR/kg, c_z=2,2 EUR/kg.

Now we can write the recurrence relations in 3 stages because the implementation period is 3 years (d = 3), and in the first year the production capacity operates. Therefore, the recurrence relations are:

- n = 1

$$\begin{cases} x''_2 = X_2 \\ y''_2 = Y_2 \\ z''_2 = Z_2 \end{cases}$$

- n = 2

$$\begin{cases} k_y \cdot y''_2 + k_z \cdot z''_2 + x''_3 - x'_3 = X_3 \\ x''_2 + y''_3 - y'_3 = Y_3 \\ z''_2 + y''_2 + z''_3 - z'_3 = Z_3 \end{cases}$$

- n = 3

$$\begin{cases} k_y \cdot (Y_2 + y''_3 - y'_3) + k_z \cdot (Z_2 + z''_3 - z'_3) + x''_4 - x'_4 \geq X_4 \\ X_2 + x''_3 - x'_3 + y''_4 - y'_4 \geq Y_4 \\ Y_2 + y''_3 - y'_3 + Z_2 + z''_3 - z'_3 + z''_4 - z'_4 \geq Z_4 \end{cases}$$

We substitute with the known values and we get:

$$\begin{cases} x_2'' - X_2 = 0 \\ y_2'' - Y_2 = 0 \\ z_2'' - Z_2 = 0 \end{cases}$$

$$\begin{cases} 1{,}3 \cdot y_2'' + 1{,}5 \cdot z_2'' + x_3'' - x_3' - X_3 = 0 \\ x_2'' + y_3'' - y_3' - Y_3 = 0 \\ z_2'' + y_2'' + z_3'' - z_3' - Z_3 = 0 \end{cases}$$

$$\begin{cases} 1{,}3 \cdot (Y_2 + y_3'' - y_3') + 1{,}5 \cdot (Z_2 + z_3'' - z_3') + x_4'' - x_4' \geq 175 \\ X_2 + x_3'' - x_3' + y_4'' - y_4' \geq 130 \\ Y_2 + y_3'' - y_3' + Z_2 + z_3'' - z_3' + z_4'' - z_4' \geq 580 \end{cases}$$

As the investment funds are limited, we still have a group of restrictions, namely:

$$\begin{cases} S_X \cdot X_2 + S_Y \cdot Y_2 + S_Z \cdot Z_2 \leq I_1 \\ S_X \cdot X_3 + S_Y \cdot Y_3 + S_Z \cdot Z_3 \leq I_2 \\ S_X \cdot X_4 + S_Y \cdot Y_4 + S_Z \cdot X_4 \leq I_3 \end{cases}$$

Substituting the known values, the above system becomes:

$$\begin{cases} 62{,}86 \cdot X_2 + 338{,}46 \cdot Y_2 + 98{,}21 \cdot Z_2 \leq 34.000 \\ 62{,}86 \cdot X_3 + 338{,}46 \cdot Y_3 + 98{,}21 \cdot Z_3 \leq 70.000 \\ 62{,}86 \cdot 175 + 338{,}46 \cdot 130 + 98{,}21 \cdot 560 \leq 110.000 \end{cases}$$

It is noted that the last inequality contains only the known data, so giving up on it we note only the first two inequalities, thus:

$$\begin{cases} 62{,}86 \cdot X_2 + 296{,}15 \cdot Y_2 + 108{,}04 \cdot Z_2 \leq 34.000 \\ 62{,}86 \cdot X_3 + 296{,}15 \cdot Y_3 + 108{,}04 \cdot Z_3 \leq 70.000 \end{cases}$$

Due to the fact that, for the sources attracted, the condition of spending at least 24,000 EUR in the first year was imposed, the system above is transcribed as follows:

$$\begin{cases} 62{,}86 \cdot X_2 + 296{,}15 \cdot Y_2 + 108{,}04 \cdot Z_2 \leq 34.000 \\ 62{,}86 \cdot X_2 + 296{,}15 \cdot Y_2 + 108{,}04 \cdot Z_2 \geq 24.000 \\ 62{,}86 \cdot X_3 + 296{,}15 \cdot Y_3 + 108{,}04 \cdot Z_3 \leq 70.000 \end{cases}$$

Under these conditions, the 3 groups of recurrence relations (constraints) form a system of equations and inequality written as follows:

$$\begin{cases} x_2'' - X_2 = 0 \\ y_2'' - Y_2 = 0 \\ z_2'' - Z_2 = 0 \\ 1{,}3 \cdot y_2'' + 1{,}5 \cdot z_2'' + x_3'' - x_3' - X_3 = 0 \\ x_2'' + y_3'' - y_3' - Y_3 = 0 \\ z_2'' + y_2'' + z_3'' - z_3' - Z_3 = 0 \\ 1{,}3 \cdot (Y_2 + y_3'' - y_3') + 1{,}5 \cdot (Z_2 + z_3'' - z_3') + x_4'' - x_4' \ge 175 \\ X_2 + x_3'' - x_3' + y_4'' - y_4' \ge 130 \\ Z_2 + z_3'' - z_3' + Y_2 + y_3'' - y_3' + z_4'' - z_4' \ge 560 \\ 62{,}86 \cdot X_2 + 296{,}15 \cdot Y_2 + 108{,}04 \cdot Z_2 \le 34.000 \\ 62{,}86 \cdot X_2 + 296{,}15 \cdot Y_2 + 108{,}04 \cdot Z_2 \ge 24.000 \\ 62{,}86 \cdot X_3 + 296{,}15 \cdot Y_3 + 108{,}04 \cdot Z_3 \le 70.000 \\ x \ge 0,\ y \ge 0,\ z \ge 0 \end{cases}$$

The recurrence relations of the objective function at the 3 stages are:

$$F_1(x,y,z) = 0$$

$$F_2(x,y,z) = -\left(x_2'' \cdot p_x + y_2'' \cdot p_y + z_2'' \cdot p_z\right) - \left(x_2'' \cdot c_x + y_2'' \cdot c_y + z_2'' \cdot c_z\right)$$

$$F_3(x,y,z) = \left(x_3' \cdot p_x + y_3' \cdot p_y + z_3' \cdot p_z\right) - \left(x_3'' \cdot p_x + y_3'' \cdot p_y + z_3'' \cdot p_z\right) -$$
$$-\left[\left(X_2 + x_3'' - x_3'\right) \cdot c_x + \left(Y_2 + y_3'' - y_3'\right) \cdot c_y + \left(Z_2 + z_3'' - z_3'\right) \cdot c_z\right]$$

$$F_4(x,y,z) = \left(x_4' \cdot p_x + y_4' \cdot p_y + z_4' \cdot p_z\right) - \left(x_4'' \cdot p_x + y_4'' \cdot p_y + z_4'' \cdot p_z\right) -$$
$$-\left[\left(X_3 + x_4'' - x_4'\right) \cdot c_x + \left(Y_3 + y_4'' - y_4'\right) \cdot c_y + \left(Z_3 + z_4'' - z_4'\right) \cdot c_z\right]$$

Thus, for the approached case the objective function is written as:

$$\max F(x,y,z) = F_2(x,y,z) + F_3(x,y,z) + F_4(x,y,z)$$

Replacing with the known values, the objective function becomes:

$$F_2(x,y,z) = -\begin{bmatrix} (2{,}8 + 2{,}3) \cdot x_2'' + \\ (4{,}2 + 3{,}6) \cdot y_2'' + (2{,}5 + 2{,}2) \cdot z_2'' \end{bmatrix} = -5{,}1 \cdot x_2'' - 7{,}8 \cdot y_2'' - 4{,}7 \cdot z_2''$$

$$F_3(x,y,z) = (2{,}8 + 2{,}3) \cdot x_3' + (4{,}2 + 3{,}6) \cdot y_3' + (2{,}5 + 2{,}2) \cdot z_3' -$$
$$-(2{,}8 + 2{,}3) \cdot x_3'' + (4{,}2 + 3{,}6) \cdot y_3'' + (2{,}5 + 2{,}2) \cdot z_3'' - 2{,}3 \cdot X_2 - 3{,}6 \cdot Y_2 - 2{,}2 \cdot Z_2 =$$
$$= 5{,}1 \cdot x_3' + 7{,}8 \cdot y_3' + 4{,}7 \cdot z_3' - 5{,}1 \cdot x_3'' - 7{,}8 \cdot y_3'' - 4{,}7 \cdot z_3'' - 2{,}3 \cdot X_2 - 3{,}6 \cdot Y_2 - 2{,}2 \cdot Z_2$$

$$F_4(x,y,z) = (2{,}8 + 2{,}3) \cdot x_4' + (4{,}2 + 3{,}6) \cdot y_4' + (2{,}5 + 2{,}2) \cdot z_4' -$$
$$-(2{,}8 + 2{,}3) \cdot x_4'' + (4{,}2 + 3{,}6) \cdot y_4'' + (2{,}5 + 2{,}2) \cdot z_4'' - 2{,}3 \cdot X_3 - 3{,}6 \cdot Y_3 - 2{,}2 \cdot Z_3 =$$
$$= 5{,}1 \cdot x_4' + 7{,}8 \cdot y_4' + 4{,}7 \cdot z_4' - 5{,}1 \cdot x_4'' - 7{,}8 \cdot y_4'' - 4{,}7 \cdot z_4'' - 2{,}3 \cdot X_3 - 3{,}6 \cdot Y_3 - 2{,}2 \cdot Z_3$$

After the calculations it results:

$$\max F(x,y,z) = -5{,}1 \cdot x_2'' - 7{,}8 \cdot y_2'' - 4{,}7 \cdot z_2'' +$$
$$5{,}1 \cdot x_3' + 7{,}8 \cdot y_3' + 4{,}7 \cdot z_3' - 5{,}1 \cdot x_3'' - 7{,}8 \cdot y_3'' - 4{,}7 \cdot z_3'' +$$
$$+5{,}1 \cdot x_4' + 7{,}8 \cdot y_4' + 4{,}7 \cdot z_4' - 5{,}1 \cdot x_4'' - 7{,}8 \cdot y_4'' - 4{,}7 \cdot z_4'' -$$
$$-2{,}3 \cdot X_2 - 3{,}6 \cdot Y_2 - 2{,}2 \cdot Z_2 - 2{,}3 \cdot X_3 - 3{,}6 \cdot Y_3 - 2{,}2 \cdot Z_3 =$$
$$= \left(-x_2'' + x_3' - x_3'' + x_4' - x_4''\right) \cdot 5{,}1 + \left(-y_2'' + y_3' - y_3'' + y_4' - y_4''\right) \cdot 7{,}8 +$$
$$+\left(-z_2'' + z_3' - z_3'' + z_4' - z_4''\right) \cdot 4{,}7 - \left(X_2 + X_3\right) \cdot 2{,}3 - \left(Y_2 + Y_3\right) \cdot 3{,}6 - \left(Z_2 + Z_3\right) \cdot 2{,}2$$

We state that it was the condition that the agricultural holding cannot sell and cannot buy at the same time the same sheep. In these circumstances, we can write that there is x' and there is not x'', or vice versa (there is no x' and there is x''). As the x, x' єі x'' variables, and the other variables cannot be negative, the above condition is written as follows: when $x'=0$, then $x''\neq0$ and vice versa, when $x'\neq0$, then $x''=0$.

Similarly, the question is being made for other variables (y', y'', z', z''). As a result, we will have to solve 8 variants of the algorithm (based on $n = 2$ and $n = 3$), each having a system of 6 equations and 6 inequalities, and 21 unknown. Also to each variant are added the no negativity conditions and the objective function. 8 variants of the calculation algorithm are:

- variant I: $x_3''=0$; $y_3''=0$; $z_3''=0$ and respectively $x_4''=0$; $y_4''=0$; $z_4''=0$
- variant II: $x_3''=0$; $y_3'=0$; $z_3''=0$ and respectively $x_4''=0$; $y_4'=0$; $z_4''=0$
- variant III: $x_3''=0$; $y_3''=0$; $z_3'=0$ and respectively $x_4''=0$; $y_4''=0$; $z_4'=0$
- variant IV: $x_3''=0$; $y_3'=0$; $z_3'=0$ and respectively $x_4''=0$; $y_4'=0$; $z_4'=0$
- variant V: $x_3'=0$; $y_3''=0$; $z_3''=0$ and respectively $x_4'=0$; $y_4''=0$; $z_4''=0$
- variant VI: $x_3'=0$; $y_3'=0$; $z_3''=0$ and respectively $x_4'=0$; $y_4'=0$; $z_4''=0$
- variant VII: $x_3'=0$; $y_3''=0$; $z_3'=0$ and respectively $x_4'=0$; $y_4''=0$; $z_4'=0$
- variant VIII: $x_3'=0$; $y_3'=0$; $z_3'=0$ and respectively $x_4'=0$; $y_4'=0$; $z_4'=0$

Solving the algorithm the simplex method, after clearing 8 variants, we will choose the optimal variant, where the agricultural holding's income is maximum.

Conclusions. After the calculations made we conclude that the question submitted to the study has the same solution in 3 variants of calculations, at which is obtained the minimum loss of —1.917,48 EUR. The optimal solution is determined by the variants III and IV (depending on $n = 2$) and respectively the second variant (depending on $n = 3$), Table.2.

According to the optimal variant, in the first year 81.04 sheep will be bought, then in the third year 105.35 lambs will be sold, 566.87 ewes and rams will be bought. Therefore, in the fourth year 780.65 lambs, 87.91 ewes and rams will be sold and 253.35 sheep will be bought. This means that in the second year we will have 81.04 sheep, respectively in the third year 647.91 ewes and rams in addition to the original number. A closer look at the investment funds reveals that the funds needed fit perfectly in the original conditions of the problem, the fact which we enabled us to obtained in the second year a surplus of 10,000 EUR.

Table 2. **The optimal variant of dynamic programming problem**

No.	Decision Variable	Solution Value	Unit Cost or Profit c(j)	Total Contribution	Reduced Cost	Basis Status	Allowable Min. c(j)	Allowable Max. c(j)
1	x2s	0	-5,10	0	-0,75	at bound	-M	-4,35
2	y2s	81,04	-7,80	-632,11	0	basic	-11,32	3,43
3	z2s	0	-4,70	0	0	basic	-M	-2,55
4	x3p	105,35	5,10	537,30	0	basic	2,40	5,10
5	y3p	0	7,80	0	0	basic	-7,21	7,80
6	z3p	0	4,70	0	0	at bound	-M	4,70
7	x3s	0	-5,10	0	0	at bound	-M	-5.10

The End of *Table 2*

8	y3s	0	-7,80	0	0	at bound	-M	-7,80
9	z3s	566,87	-4,70	-2.664,28	0	basic	-8,09	-4,70
10	x4p	780,65	5,10	3.981,34	0	basic	4,51	5,10
11	y4p	0	7,80	0	0	at bound	-M	7,80
12	z4p	87,91	4,70	413,17	0	basic	3,95	4,70
13	x4s	0	-5,10	0	0	at bound	-M	-5,10
14	y4s	235,35	-7,80	-1.835,75	0	basic	-8,38	-7,80
15	z4s	0	-4,70	0	0	at bound	-M	-4,70
16	X2	0	-2,30	0	0	basic	-M	-1,55
17	Y2	81,04	-3,60	-291,74	0	basic	-7,12	7,63
18	Z2	0	-2,20	0	-2,15	at bound	-M	-0,05
19	X3	0	-2,30	0	-2,77	at bound	-M	0,47
20	Y3	0	-3,60	0	-15,01	at bound	-M	11,41
21	Z3	647,91	-2,20	-1.425,40	0	basic	-6,96	M
Objective Function (Max.) = -1.917,48								
Nr.	Constraint	Left Hand Side	Direction	Right Hand Side	Slack or Surplus	Shadow Price	Allowable Min. RHS	Allowable Max. RHS
1	C1	0	=	0	0	-7.88	-184,45	0
2	C2	0	=	0	0	-18,96	-81,04	87,91
3	C3	0	=	0	0	-16,40	0	87,91
4	C4	0	=	0	0	2,70	-M	105,35
5	C5	0	=	0	0	3,53	-87,91	0
6	C6	0	=	0	0	7,65	-87,91	M
7	C7	175,00	>=	175,00	0	-5,10	-M	955,65
8	C8	130,00	>=	130,00	0	-7,80	-105,35	M
9	C9	560,00	>=	560,00	0	-4,70	-M	647,91
10	C10	24.000,00	>=	24.000,00	0	-0,04	0	34.000,00
11	C11	24.000,00	<=	34.000,00	10.000,00	0	24.000,00	M
12	C12	70.000,00	<=	70.000,00	0	0,05	60.502,40	M

Source: Authors' own computation

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References:

Andrei, J., Botezatu, M. (2009). The Role of Alternative Funding Sources in the Sustainable Development of Companies and the Capitalization of the Economic Potential: A Romanian Perspective in the Context of the Sustainable Development, The 14th IBIMA Conference on Global Business Transformation through Innovation and Knowledge Management, Istanbul, Turkey 23-24 June 2010, pp.227-236.

Andric, J. (1991). The costs and calculations in agricultural production. Faculty of Agriculture Belgrade, Belgrade - Zemun.

Bagher, A. (2011). Regression analysis of effective factor on people participation in protecting and revitalizing of pastures and forests in Ilam province from the view of users, African Journal of Agricultural Research, Vol. 6(2), pp. 416-422

Claudiu, C, Jonel, S, Drago, C. (2008). Beyond Agriculture and Rural Development: Investments, Efficiency, Econometrics. Monograph. 147 p. Institute of Agricultural Economics, Belgrade, Republic of Serbia

Done, I, Subic, J, Vasiljevic, Z, Ionita, I, Jean, A, Mirela, M. (2009). Investments and agriculture development in the city of Belgrade area. Proceedings of The 13th International Business Information Management Association Conference "Knowledge Management and Innovation in Advancing Economies: Analyses & Solutions", November 9-10, 2009 Marrakech - Morocco, International Business Information Management Association (IBIMA), pp. 338-353

Gittinger, P., J. (1972). Economic Analysis of Agricultural Project. The Johns Hopkins University Press, Baltimore and London.

Iarca, I., Zoran, P., Dorel, D, Jonel, S., Jean A, Ioan D. (2010). The Evaluation of Economic Investments' Efficiency in Intensive Breed of Game: A Study Case for European Deer (Cervus Elaphus L.) and Wild Boar (Sus Scrofa L.) in the context of the best investment decision, African Journal Of Business Management, Vol. 5(3), pp. 934-943

Ionita, I., Blidaru, Gh. (1999). Eficienta investitiilor in agricultura, Editura CERES, Bucuresti, Romania

Romanu, I., Vasilescu, I. (1996). Investments' and fix capital efficiency, Didactic and Pedagogic Publishing House, Bucharest, Romania.

Romanu, I., Vasilescu, I. (1997). Investment Management, Margaritar Publishing, Bucharest.

Stoica, M., Ionita, I., Botezatu, M. (1997). Modelarea si Simularea proceselor economice, Editura Economica, Bucuresti, Romania.

Subic, J. (2003). Founding investment decision in agriculture. Proceedings of International Symposium on "Investments and Economic Performance", Academy of Economic Studies Bucharest, Management Faculty, Department of Management, Section V/13 (pp. 1-5).

Subic, J., Umihanic, B., Hamovic, V. (2008). Constitution of investment calculation and its significance for making business plan on agricultural husbandries. The symposium of agro-economists with international participation apropos 45. Anniversary of the Department of Agroeconomics. Agroeconomic science and profession in transition of education and agro-economy. Thematic proceedings. University in Belgrade, Faculty of Agriculture, Institute of Agroeconomics, Belgrade.

Vasiljevic, Z. (2006). Managing the investments. Scripts. University Braca Karic, Faculty of Commerce and Banking - Janicije and Danica Karic, Belgrade, Republic of Serbia.

Vasilescu, I., Romanu, I., Cicea, C. (2000). Investments, Economic Publishing, Bucharest, Romania.

Vasilescu, I., Cicea, C., Dobrea, R.C. (2003). Applied Efficiency of Investment, Lumina Lex Publishing House, Bucharest, Romania.

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