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SPATIAL COMPETITION FOR ROUTING CHOICE AND SUBSIDY DIFFERENTIATION IN A PARATRANSIT DUOPOLY MARKET

The purpose of this paper was to investigate the effects of Fu-Kang bus routing choice strategies on the subsidy differentiation in Taiwan. Building a spatial competition model between the public and the private bus firm and resolving the sub-game perfect Nash equilibrium, it was found that equilibrium of routing choice and price exits. Furthermore, both firms' the profits and social welfare were compared and discussed through numerical analysis. It was concluded that the firms should obtain precise information, so that subsidized transport fare policy and bus routing choices could be possible and easier developed.

Keywords: *demand responsive transport service; subsidy differentiation; duopoly market; Fu-Kang Bus Service; routing choice strategy.*

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КОНКУРЕНЦИЯ ЗА ПРОСТІР ПРИ ВИБОРІ МАРШРУТІВ І ДИФЕРЕНЦІАЦІЯ СУБСИДІЙ НА КОНКУРЕНТНОМУ РИНКУ СОЦІАЛЬНИХ ПЕРЕВЕЗЕНЬ

У статті досліджено вплив діяльності автобусного перевізника "Фу-Кан" на диференціацію субсидій на Тайвані. Побудовано модель конкуренції за простір між державною і приватною автобусними фірмами і вирішено досконалу рівновагу Неша, при цьому було встановлено, що вибір між перевізниками залежить в рівній мірі від маршруту і від вартості. Крім того, обидві фірми були зіставлені за прибутком і рівнем соціального захисту і перевірені методом чисельного аналізу. Фірми отримують точну інформацію, тому політика субсидування транспорту і вибір маршрутів повинні спроститися.

Ключові слова: *транспортні послуги з відгуком на попит, диференціація субсидій, дуопольний ринок, перевізник Фу-Кан, вибір маршрутів.*

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КОНКУРЕНЦИЯ ЗА ПРОСТРАНСТВО ПРИ ВЫБОРЕ МАРШРУТОВ И ДИФФЕРЕНЦИАЦИЯ СУБСИДИЙ НА КОНКУРЕНТНОМ РЫНКЕ СОЦИАЛЬНЫХ ПЕРЕВОЗОК

В статье исследовано влияние деятельности автобусного перевозчика "Фу-Кан" на дифференциацию субсидий на Тайване. Построена модель конкуренции за пространство между государственной и частной автобусными фирмами и решено совершенное равновесие Нэша, при этом было установлено, что выбор между перевозчиками зависит в равной степени от маршрута и от стоимости. Кроме того, обе фирмы были сопоставлены по прибыли и уровню социальной защиты и проверены методом численного анализа. Фирмы получают точную информацию, так что политика субсидирования транспорта и выбор маршрутов должны упроститься.

Ключевые слова: *транспортные услуги с откликом на спрос, дифференциация субсидий, дуопольный рынок, перевозчик Фу-Кан, выбор маршрутов.*

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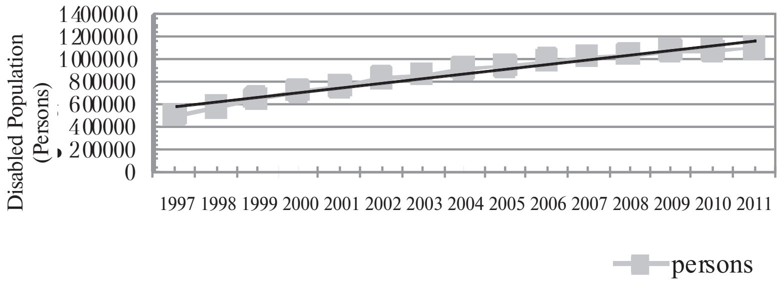
Introduction. Since enactments of the People with Disabilities Rights Protection Act (PDRPA) were announced in 2009, public and private firms launched into DRTS services in Taiwan, in particular, the barrier-free transportation service. Moreover, there are impaired people whose accessibility the government needs to consider. Thus, the Ministry of the Interior (MOI) makes policies with respect to impaired people. The most important policy is "White Paper of People with Disabilities Right Protection Act (WPPDRPA)". According to this policy, the Ministry of the Interior award municipal authorities social welfare funds. Afterward, municipal authorizes entrust the social welfare organization to operate the bus transport services which the impaired people desire to carry them to hospitals or clinic centers, i.e. door-to-door service. In addition, the Ministry of Transportation and Communications (MOTC) gives renewed impetus to the barrier-free bus transportation service. Therefore, bus firms obtain proportional subsidies from municipal authorities.

According to Statistical Yearbook of Interior (MOI, 2011), the black bold trend line of the disabled population is illustrated as Fig. 1. It is implied that the market shares of transport service with respect to impaired people are gradually increasing. However, the government, public and private bus firms are in a predicament. Although the public bus firm had a superiority of vehicle facilities and human resource over the private one, the public transport department had a rough transport planning for Demand Responsive Transport Service (DRTS). The result of performance was that many impaired people cannot obtain an DRTS service (MOI, 2011). On the other hand, the private bus firm considers demographic and geographic factors to invest into facilities and operate their transport business in central business districts (CBD) of the cities. They would rather serve citizens dwelling in the CBD then metropolitan outskirts or outlying communities. Due to development of sustainable transportation in Taiwan (Issac I. C. Chen et al., 2002), the government must tackle the problem with impaired people accessibility. Furthermore, enactments were established, white paper was written and researches were done considerably, the government should actually integrate and wisely use these resources.

Our paper considers the routing choice of origin-destination (OD) points of riders either public buses or private ones and determines the effect of the subsidy differentiation in terms of medical service quality, government's fiscal budget, and social welfare. To do so, in this paper, we establish a spatial competition model with respect to paratransit duopoly market for the vanpool taxi service and Fu-Kang bus service. The purpose of this paper, therefore, is to analyze riders' choice of routing and decisions about subsidy differentiation under the asymmetric paratransit duopoly market that comprises both a public firm and a private firm, and we examine changes in terms of quality of service (QOS), profits, and social welfare.

Remaining sections of the paper are: the "Related literature" section introduces the definition and applications of DRTS and gives an overview of the related literature. In "Model" section, we present the model of a paratransit duopoly market in which each bus firm have one type of service configurations. In "Competition between both firms" section, we solve the public and private firms with respect to riders' expenses, subsidies and quality of service and equilibrium findings. In "Social optimum" we investigate firm's profits and social welfare when the public firm receives subsidies from the government and provides one type of service configurations. "Concluding remarks" section presents conclusion and discussion.



Source: MOI, 2011.

Figure 1. **The disabled population from 1997 to 2011 in Taiwan**

Related Literature.

1. Concepts: the original transport concept of DRTS service is similarly to jitneys and paratransit service. The jitneys services were private vehicle on the main transport routes for hire. The paratransit services were widespread inclusions that are dial-a-ride, shared-ride taxis, commuter vans, shuttles, vanpools and buspools. Certainly, jitneys and DRTS were also included. DRTS services consist of varying vehicles, communication technologies and service configuration. The varying vehicles that include passenger car, vans and small buses are operating in response to phone calls or personal computers through Internet from passengers. The service configuration is composed of OD points that includes many-to-many, many-to-one, few-to-one and one-to-one (Cervero, 1997).

2. Application: The nature of DRTS service was gradually connected with low productivity and higher per trip costs, an previous report of DRTS and other paratransit services interpreted the researching findings that reducing costs and increasing productivity was significant as transportation operations entry the new industry (TRB,1976).

3. Characteristic: Before executing a DRTS service, demand, environment and bureaucracy need to be considered (Cho et al., 2009).

(a) Demand: The riders studied by Popper et al. (1976) the transportation demand for the rural elderly and presented the methods base on attitudinal surveys, comparative trip rates, and participation frequencies. Because of economies of agglomerations, most transport facilities are located on CBD. However, the riders who live in poor neighborhoods need more transportable method for work or medical needs, in particular, older Americans are neglected due to their loneliness and inadequate attention to medical needs.

(b) Enviroment: Some national cases described should be. Jones (2002) observed and demonstrated that the practical experience of DRT in the UK was widely I spread with rapid introduction of applications in diverse rural areas. Similarly, Japan administrative organization, Ministry of Land, Infrastructure,

Transport and Tourism (MLIT) promotes the DRTS services in Japan basing on "Activation and Rehabilitation of Regional Public Transportation Act (2009)". Hundreds of case studies relative to DRTS services were successfully executed in Japan, in particular, in outlying communities. DRTS services can not only complement the low density areas which have rare transport vehicles but also bring out new paratransit industry to improve their economic life.

(c)Bureaucracy: Polities plays a significant role in transit and spending, and transit officials may accordingly make ineffective and even irresponsible decisions on transit investment (Hess and Lombardi, 2005). The authority maintains and improves transit service through transit subsidy policy; especially, in the part concerning the outlying residential suburbs and the central city employment cores (Wachs, 1989).

4. Research.

(a) Fu-Kang bus in Taiwan: DRTS also named Fu-Kang bus service system in Taiwan. The public transportation includes the Fu-Kang bus from the survey of utilization rates of public transportation on 2010 (MOTC, 2011). The MOTC announced a Law of People with Disabilities Having a Discount Rate to Ride the Public Transport Vehicle to support the people with disabilities with a favor of 50% discount rate to ride a domestic transport vehicle and a discount rate of one-way trip with the regulated taxi rate between 50% and 66.67% to ride a Fu-Kang bus (MOTC, 1998).

(b) Previous and Present Studies: the previous transportation studies included two fields. One field is optimization. The transport routing planning that included taxi transport services and Fu-Kang were designed and the solutions were optimized so that the improvement of operational efficiency of vehicle fleets for operators and promotion of service quality for users were analyzed (Wei et al., 2007). In addition, Mass Rapid Transit (MRT) service is indispensable when it concerns accessibility. Chang (2004) studied integration of possibilities between MRT and Fu-Kang bus services. Furthermore, Huang and Hsu (2009), Shyr, Tao and Hsu (2009) found the optimization of service quality, operational cost, efficiency and demand under the carpool condition. The other field is the users' behavior at the transport market. Recent studies on the vehicle choices of elderly people include Gi (2010) and Lin (2005). The literature on taxi services using the DEMATEL method for analyzing taxi market policies includes Chang, Wu and Chen (2009). Recent study on the relationship between the choice behavior of riders who are willing to pay fee and the operational cost of taxi was discussed through Chang and Chu (2008). The approach of these papers is not only case studies but also empirical evidences. On the other hand, Chen, Lee and Chang (2012) studied the possibility of executing DRTS in Taiwan though routing bus transport of partial compatibility, supporting services.

Our present study concentrates on the theoretical analysis with respect to paratrasit economy, in particular, Fu-Kang bus service and vanpool taxi service. Furthermore, our model is analogous to the concept of fiscal competition because we can interpret Fu-Kang bus service or vanpool taxi service as a certain type of public good. Hence, our research is the first step of such studies at the paratransit market, and it also contributes to the analysis of the role of public and private firms.

The model: *1. Service Configuration:* Service configuration means here the routing type. Generally, it can be classified into 5 routing types, i.e. few-to-one, few-to-few, many-to-one, many-to-few, many-to-many (Cervero, 1997). "Few-to-one" routing type denotes having few origins and only one destination like a local community hospital. "Few-to-few" denotes the demand having few origins and few destinations like a local community hospital and a metropolitan hospital. "Many-to-one" has many random origins and only one destination like an academic medical hospital. "Many-to-many" means many random origins and many destinations like a clinic center, a local community hospital, a metropolitan hospitals and an academic medical center.

We presume two firms, a public and a private, are located at the two ends of a city; both own the varying routing type. The location of firm $i \in (1,2)$. Each of firm can choose one routing type which is composed of OD points. The firm can decide the routing type of its supplies and its quality. Routing type is a simultaneous choice, between 0 and 1, denoted by $x_i \in (1,2)$. For instance, a new routing type is that a firm could provide a rider a one-way trip, e.g. few-to-few type. Another new routing type is that the firm could provide *N* riders one way trip, e.g. many-to-few type. The routing type of public firm, Fu-Kang bus, is denoted by *x* and the routing type of private firm, vanpool taxi, is denoted by *1 - x*. Both firms whose routing type differentiations their own are located on $0 \le x \le 1$.

2. Quality of Service: The characteristics of service quality in bus transportation include frequency of service, rider's comfort, travel time, and reliability of service (Baltes, 2003). Overall, the riders' satisfaction is interpreted. In particular, riders attach to their travel time from their service configuration of routing choices among transport service that differ by time and money spent. Furthermore, passengers consider 5 conditions with respect to transit availability, i.e. near one's OD point, at or near the required time, available information with respect to transit service, sufficient capacity. After a general analysis we assume that operation cost is $\omega\sqrt{Q}$ to provide one unit of the quality of service *Q*, where $\omega > 0$. The service of choice $\omega\sqrt{Q}_i$ is denoted and its market share is m_i=[1,2].

3. Firm's Revenues: The public firm's revenue derived from subsidies is fixed in that Fu-Kang bus fare is regulated. However, the private firm's revenue is derived from bus subscriptions or operations. If service configuration is subsidizer-supported, i.e. by the government, subsidizing revenue per rider is v^*_{sub} fixed.

4. Riders preferences: The *N* potential riders' preferences are assumed to be evenly distributed along a string of unit interval $x \in (0,1)$, so at each point lay a single rider. Hence, total riders at the transport market stand for 1. Actually, all riders who use the Fu-Kang buses are impaired people, we assume only two transport firms, the public and private firm, serve the para transit duopoly market, and they are located at the extremes of the unit length string, as shown in Fig. 2.

If a rider chooses one type of service configuration which is not preferred the service configurations, then a rider pays the transportation cost *tx* and equal to $t > 0$

per unit of length. Furthermore, riders presuppose subsidies from the government when they ride the Fu-Kang, so they choose the public firm; however, the vehicles of Fu-Kang bus are not sufficient to satisfy demands. In reality, some riders make appointments more than in a month. Hereinto, we assume another transportation cost and attach a parameter α. The overall transportation cost is αtx. Each rider chooses from public and private firms one unit of transport service and gains benefits equal to $v+\sqrt{Q}_i$ receiving the transport service of firm i. v denotes the reservation benefits when receiving transport service and we assume that v is sufficiently large. When riders choose to use the routing type of service i, a rider x obtains benefits:

$$U_x = \begin{cases} v+\sqrt{Q_1}-\alpha tx-\beta P_1. \\ v+\sqrt{Q_2}-t(1-x)-P_2. \end{cases}, \tag{1}$$

where P_i represents the regulated fare with respect to P_1, P_2. In case of subscriptions, riders do not pay for riding Fu-Kang bus or vanpool taxi, hence $P_i=0$. If the fare of Fu-Kang bus is subsided from the government, the riders should not pay the total expense, therefore, we attach a subsidy rate β.

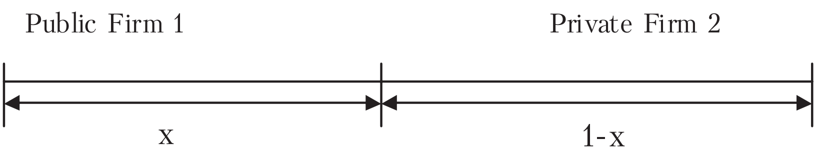


Figure 2. **Riders' preference with two firms**

5. Competition between Both firms: The Competition Game. We establish the competition game as follow:

(1) Stage 1: The riders choose the quality of service simultaneously.

(2) Stage 2: There are two possible cases.

(a) Public firm: riders obtain the government-subsidized that service is equal to v^*_{sub}.

(b) Private firm: riders choose to pay the expenses.

Finally, we search for the sub-game perfect equilibrium in both firms' games under aforementioned consideration and solve the firms' games through backward induction, and then consider the competition game from the third stage to the first stage. The public and private firm: Stage 2: price competition, βP_1 denotes the subsided fare when the rider use the Fu-Kang bus; P_2 denotes that the rider is willing to pay the expense.

The marginal rider is defined by:

$$v+\sqrt{Q_1}-\alpha tx-\beta P_1 = v+\sqrt{Q_2}-t(1-x)-P_2, \tag{2}$$

then

$$x^* = \frac{1}{1+\alpha}+\frac{\sqrt{Q_1}-\sqrt{Q_2}}{(1+\alpha)t}+\frac{P_2-\beta P_1}{(1+\alpha)t}. \tag{3}$$

In case of $x^* \in (0,1)$, the riders shares of service configuration are $m_1=x^*$ and $m_2=1-x^*$ respectively. Each firm i maximizes:

$$\Pi_i^* = P_i N m_i - \omega Q_i \ . \tag{4}$$

For simplicity calculation, N is neglected, and then with respect to P_2, we solve the reaction function:

$$P_1 = R_1^P(P_2) = \frac{t}{2\beta} + \frac{\sqrt{Q_1}-\sqrt{Q_2}}{2\beta} + \frac{P_2}{2\beta}. \tag{5}$$

$$P_2 = R_2^P(P_1) = \frac{\alpha t}{2} + \frac{\sqrt{Q_2}-\sqrt{Q_1}}{2} + \frac{\beta P_1}{2}. \tag{6}$$

The Nash equilibrium of the sub-game satisfies $P^*_i = R^P_i(P^*_j)$, which implies that:

$$P_1^* = \frac{(2+\alpha)t}{3\beta} + \frac{\sqrt{Q_1}-\sqrt{Q_2}}{3\beta}. \tag{7}$$

$$P_2^* = \frac{(1+2\alpha)t}{3} + \frac{\sqrt{Q_2}-\sqrt{Q_1}}{3}. \tag{8}$$

We consider the price competition, whose effects of Π_1, Π_2 on P_1, P_2, and substitute Equation(3), Equation(5) and Equation(6) into Equation(4). The profit function of firm i=1,2 can be written as:

$$\Pi_1 = \left(\frac{t}{2\beta} + \frac{\sqrt{Q_1}-\sqrt{Q_2}}{2\beta} + \frac{P_2}{2\beta}\right) * \left(\frac{1}{1+\alpha} + \frac{\sqrt{Q_1}-\sqrt{Q_2}}{(1+\alpha)t} + \frac{P_2-\beta P_1}{(1+\alpha)t}\right) - \omega Q_1. \tag{9}$$

$$\Pi_2 = \left(\frac{\alpha t}{2} + \frac{\sqrt{Q_2}-\sqrt{Q_1}}{2} + \frac{\beta P_1}{2}\right) * \left(\frac{\alpha}{1+\alpha} + \frac{\sqrt{Q_2}-\sqrt{Q_1}}{(1+\alpha)t} + \frac{\beta P_1 - P_2}{(1+\alpha)t}\right) - \omega Q_2. \tag{10}$$

The firm i=1,2 maximizes themselves profit Π_1, Π_2 with respect to P_1, P_2. Therefore, the first-order condition for firm i=1,2 is:

$$\frac{\partial \Pi_1}{\partial P_1} = \frac{1}{4t(1+\alpha)}(-\beta P_1). \tag{11}$$

$$\frac{\partial \Pi_2}{\partial P_2} = \frac{1}{4t(1+\alpha)}(-P_2). \tag{12}$$

We investigate the effects of Π_1, Π_2 on P_1, P_2 by comparative static analysis. Therefore, the second-order condition for firm i=1,2 is

$$\partial^2 \Pi_1 / \partial P_1^2 = -\beta / 4t(1+\alpha) < 0,$$

$$\partial^2 \Pi_2 / \partial P_2^2 = -1/4t(1+\alpha) < 0$$

$$\partial^2 \Pi_1 / \partial P_1 P_2 = 0$$

$$\partial^2 \Pi_1 / \partial P_2 P_1 = 0$$

Furthermore, this condition $\partial^2 \Pi_1 / \partial P_1^2 * \partial^2 \Pi_2 / \partial P_2^2 - \partial^2 \Pi_1 / \partial P_1 P_2 * \partial^2 \Pi_1 / \partial P_2 P_1 > 0$ is satisfied.

Proposition 1. If this condition is satisfied then second-order conditions for maximization are always convinced.

We also consider the service of quality, whose effects of Π_1, Π_2 on Q_1, Q_2, and substitute Equation(3), Equation(5) and Equation(6) into Equation(4). The profit function of firm i=1,2 can be written as Equation (9) and Equation (10).

The firm $i=1,2$ maximizes the profit Π_1, Π_2 with respect to Q_1, Q_2. Therefore, the first-order condition for firm $i=1,2$ is:

$$\frac{\partial \Pi_1}{\partial Q_1} = \frac{1}{9\beta t(1+\alpha)\sqrt{Q_1}}[2t + \alpha t + \sqrt{Q_1} - \sqrt{Q_2} - 9\omega\beta t(1+\alpha)\sqrt{Q_1}]. \quad (13)$$

$$\frac{\partial \Pi_2}{\partial Q_2} = \frac{1}{9t(1+\alpha)\sqrt{Q_2}}[2\alpha t + t + \sqrt{Q_2} - \sqrt{Q_1} - 9\omega t(1+\alpha)\sqrt{Q_2}]. \quad (14)$$

We investigate the effects of Π_1, Π_2 on Q_1, Q_2 by comparative static analysis. Therefore, the second-order condition for firm $i=1,2$ is:

$$\frac{\partial^2 \Pi_1}{\partial Q_1^2} = \frac{3\omega t\{\beta(2\alpha+1) - t[(\alpha+2)(\alpha+1)]\} - 1 + t + \alpha t}{3\omega t(\alpha+2) - 1}. \quad (15)$$

$$\frac{\partial^2 \Pi_1}{\partial Q_1 Q_2} = \frac{1}{36\beta t(1+\alpha)} \frac{\{3\omega[9\beta\omega t(1+\alpha) - \beta - 1]\}^4}{[3\omega t(\alpha+2) - 1]^3 [3\beta\omega t(2\alpha+1) - 1]}. \quad (16)$$

$$\frac{\partial^2 \Pi_2}{\partial Q_2^2} = \frac{t\{(2\alpha+1)[1 - 3\beta t(2\alpha+1)] + 3\omega(\alpha+2)\} - 1}{3\beta t(\alpha+2) - 1}. \quad (17)$$

$$\frac{\partial^2 \Pi_2}{\partial Q_2 Q_1} = \frac{1}{36t(1+\alpha)} \frac{\{3\omega[9\beta\omega t(1+\alpha) - \beta - 1]\}^4}{[3\beta\omega t(2\alpha+1) - 1]^3 [3\omega t(\alpha+2) - 1]}. \quad (18)$$

We also investigate the effects of Π_1, Π_2 on Q_1, Q_2 by comparative static analysis. Therefore, the second-order condition with respect to firm $i=1,2$ is written as

$$\partial^2 \Pi_1 / \partial Q_1^2 < 0 \text{ and } \partial^2 \Pi_2 / \partial Q_2^2 < 0.$$

Furthermore, this condition $\partial^2 \Pi_1 / \partial Q_1^2 * \partial^2 \Pi_2 / \partial Q_2^2 - \partial^2 \Pi_1 / \partial Q_1 Q_2 * \partial^2 \Pi_1 / \partial Q_2 Q_1 > 0$ is as satisfied.

Proposition 2. If the conditions of

$$\omega t^* \gtrless 1/3(\alpha+2) \text{ and } \beta^* \gtrless (\alpha+2)/(2\alpha+1)$$

are satisfied, then the second-order conditions $\partial^2 \Pi_2 / \partial Q_2^2$ with respect to firm 1 for maximization always convinced. Here, we define the cost synergy.

Proposition 2-1. If the conditions of

$$\beta t^* \gtrless 1/3(2\alpha+1) \text{ and } \omega t^* \gtrless 1/3(\alpha+2)$$

are satisfied, then the second-order conditions $\partial^2 \Pi_2 / \partial Q_2^2$ with respect to firm 2 for maximization always convinced. Here, we also define the ωt^* named cost synergy.

Stage 1: Quality of service, we substitute equilibrium prices given Equation (7) and Equation (8) in the profit functions, in addition, we derive the profit functions of the quality of the service stage. The reaction functions at this stage are defined by the first-order conditions and solved by the Nash equilibrium of the sub-game, we obtain:

$$Q_1^* = \left(\frac{3\omega t(\alpha+2) - 1}{3\omega[9\beta\omega t(1+\alpha) - \beta - 1]} \right)^2. \quad (19)$$

$$Q_2^* = \left(\frac{3\beta\omega t(2\alpha+1) - 1}{3\omega[9\beta\omega t(1+\alpha) - \beta - 1]} \right)^2. \quad (20)$$

The second-order conditions for profit maximization are convinced if as regards Q^*_1,

$$\omega t \gtrless 1/3(\alpha+2) \text{ and } \beta \gtrless (\alpha+2)/(2\alpha+1)$$

are satisfied, as regards Q^*_2,

$$\omega t \gtrless 1/3(\alpha+2) \text{ and } \beta \gtrless 1/3t(2\alpha+1)$$

are satisfied.

Proposition 2-2. If ω is constant, the scale of t is $0<t<1$ and

$$\beta = 1/(1-t)((\alpha+2)/(2\alpha+1)) - (1/3(2\alpha+1))$$

under the conditions that include

$$0.667 \le \alpha \le 0.999, 0.889 \le \beta \le 0.999$$

When α increases, then β decreases.

We stipulate the scale of

$$0.667 \le \alpha \le 0.999, 0.889 \le \beta \le 0.999$$

in order to ensure $\sqrt{Q_1^*}, \sqrt{Q_2^*} > 0$. As reveal α and β in Figure 3, α increases with a unit of transportation cost and β decreases a unit of subsidy rate. Since

$$\beta \gtrless 1/(1-t)((\alpha+2)/(2\alpha+1)) - (1/3(2\alpha+1))$$

is tenable, so is $Q_1^* \gtrless Q_2^*$. As it can be seen in Figure 3, the curve is depicted as $Q^*_1 = Q^*_2$ under the regulated scales α and β. As curve is divided into two parts, the upper right is $Q^*_1 > Q^*_2$ and the lower left is $Q^*_1 < Q^*_2$.

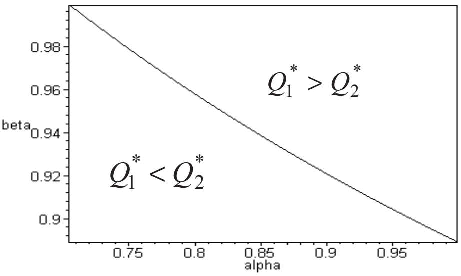


Figure 3.

$0.667 \le \alpha \le 0.999$; $0.889 \le \beta \le 0.999$

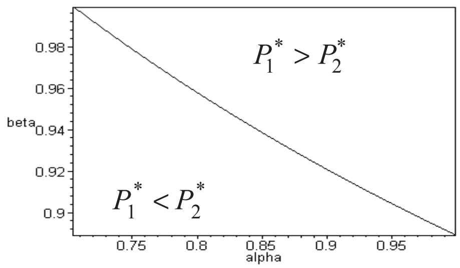


Figure 4.

$0.704 \le \alpha \le 0.999$; $0.889 \le \beta \le 0.999$

According to Equations (19) and (20) and proposition 2-2, we obtain the following effects of Q_1, Q_2 on α, β:

$$\frac{\partial Q_1}{\partial \alpha} = \frac{2t[\beta(2-9t\omega)-1][3t\omega(\alpha+2)-1]}{3\omega[9\beta t\omega(1+\alpha)-\beta-1]^3}. \tag{21}$$

$$\frac{\partial Q_1}{\partial \beta} = \frac{2[3\omega t(\alpha+2)-1]^2[1-27\omega^2 t(1+\alpha)]}{27\omega^3[9\beta\omega t(1+\alpha)-\beta-1]^3}. \tag{22}$$

$$\frac{\partial Q_2}{\partial \alpha} = \frac{2\beta t\{3\beta\omega t[9\beta\omega t(2\alpha+1)+2(\beta+1)]-(1-2\beta)(1-6\alpha\beta\omega t)\}}{3\omega[9\beta\omega t(1+\alpha)-\beta-1]^3}. \tag{23}$$

$$\frac{\partial Q_2}{\partial \beta}=\frac{2[3\omega t(\alpha+2)-1]}{9\omega^2[9\beta\omega t(1+\alpha)-\beta-1]^3}. \quad (24)$$

For simplification of Equations (21) to (24), we establish the following proposition

Proposition 3.

(1) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $\beta \gtrless \frac{\alpha+2}{2\alpha+1}$ then $\frac{\partial Q_1}{\partial \alpha} \gtrless 0$.

(2) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $\beta \gtrless \frac{\alpha+2}{2\alpha+1}$ then $\frac{\partial Q_1}{\partial \beta} \gtrless 0$.

(3) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $\beta t \gtrless \frac{\alpha+1}{3(2\alpha+1)}$ then $\frac{\partial Q_2}{\partial \alpha} \gtrless 0$.

(4) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $\beta t \gtrless \frac{\alpha+1}{3(2\alpha+1)}$ then $\frac{\partial Q_2}{\partial \beta} \gtrless 0$.

In addition, we consider the condition α is sufficiently large, β is within the regulated range $0.889 \le \beta \le 0.999$. The plots indicate that even though α is sufficiently large, the value of β is approximately 0.889.

Finally, the objective functions of the stage are the equilibrium profits of Stage 1. Our model includes both firms which are located at the extremes of the city in our model and both firms are located at the extreme profiles interval; furthermore, we derive the equilibrium prices and quality of service. Afterward, we substitute Equation (19) and Equation (20) into Equation (7) and Equation (8) then we derive Equations (25) and (26):

$$P_1^*=\frac{t(1+\alpha)[3\omega t(2+\alpha)-1]}{9\beta\omega t(1+\alpha)-\beta-1}, \quad (25)$$

$$P_2^*=\frac{t(1+\alpha)[3\beta\omega t(2\alpha+1)-1]}{9\beta\omega t(1+\alpha)-\beta-1}. \quad (26)$$

Proposition 3. If $t\omega=1/3(\alpha+2)$, $0<t\omega<1$ and $\beta=(\alpha+2)/(2\alpha+1)$ under 2 conditions that include $0.704 \le \alpha \le 0.999, 0.889 \le \beta \le 0.999$.

When α increases, then β decreases.

We regulate the scale of

$$0.704 \le \alpha \le 0.999, 0.889 \le \beta \le 0.999$$

in order to convince P^*_1, $P^*_2>0$. As shown in Figure 4, α increases with a unit of transportation cost and β decreases a unit of subsidy rate. Since

$$\beta \gtrless ((\alpha+2)/(2\alpha+1))-(1/3(\alpha+2))-\omega t$$

holds, so does $P_1^* \gtrless P_2^*$ As it can be seen in Figure 6, the curve is depicted as $P^*_1=P^*_2$ under the regulated scales α and β. As curve is split into two areas, the upper right is $P^*_1>P^*_2$ and the lower left is $P^*_1<P^*_2$.

According to Equations (25) and (26), we obtain the following effects of P_1, P_2 on α, β:

$$\frac{\partial P_1}{\partial \alpha}=\frac{t\{27\beta\omega^2 t^2(\alpha+1)^2+(1+2\beta)[1-3\omega t(2\alpha+3)]\}}{[9\beta\omega t(1+\alpha)-2\beta-1]^2}, \quad (27)$$

$$\frac{\partial P_1}{\partial \beta}=\frac{t[3\omega t(2+\alpha)-1][2-9\omega t(1+\alpha)]}{[9\beta\omega t(1+\alpha)-2\beta-1]^2}, \tag{28}$$

$$\frac{\partial P_2}{\partial \alpha}=\frac{t\{27\beta^2\omega^2 t[t(1+\alpha)+1]-(2\beta+1)[\beta\omega t(1+2\alpha)+(1-3\beta\omega)]\}}{[9\beta\omega t(1+\alpha)-2\beta-1]^2}, \tag{29}$$

$$\frac{\partial P_2}{\partial \beta}=\frac{3\omega t(1+\alpha)[3-2(1+2\alpha)\text{-}2}{[9\beta\omega t(1+\alpha)-2\beta-1]^2}. \tag{30}$$

Proposition 3.1.

(1) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $\beta \gtrless \frac{\alpha+2}{2\alpha+1}$ t then $\frac{\partial P_1}{\partial \alpha} \gtrless 0$.

(2) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $\beta \gtrless \frac{\alpha+2}{2\alpha+1}$ then $\frac{\partial P_1}{\partial \beta} \gtrless 0$.

(3) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $\beta t \gtrless \frac{\alpha+1}{3(2\alpha+1)}$ then $\frac{\partial P_2}{\partial \alpha} \gtrless 0$.

(4) when $\omega t \gtrless \frac{1}{3(\alpha+2)}$ and $t \gtrless \frac{2(2+\alpha)}{3(1+\alpha)(1+2\alpha)}$ then $\frac{\partial P_2}{\partial \beta} \gtrless 0$.

We substitute Equation (19), Equation (20), Equation (25) and Equation (26) into Equation (3) to obtain Equation (31):

$$x^*=\frac{\beta(9\omega t-1)}{9\beta t\omega(1+\alpha)-\beta-1}. \tag{31}$$

x^* is the equilibrium choice of the riders different between the two firms.

The Social Optimum:

1. Social Welfare: We derive the demand function for firm 1 from the maximization of consumers' utility as follows:

$$D_1=\begin{cases}0 & if\sqrt{Q_1}-\sqrt{Q_2}\le\beta P_1-P_2+[(\alpha-1)x-1]t\\ x^* & if\ \beta P_1-P_2+[(\alpha-1)x-1]t<\sqrt{Q_1}-\sqrt{Q_2}\le\beta P_1-P_2+[(1-\alpha)x+1]t,\\ 1 & if\sqrt{Q_1}-\sqrt{Q_2}>\beta P_1-P_2+[(1-\alpha)x+1]t\end{cases} \tag{32}$$

The demand for firm 2 is

$$D_2=1\text{-}D_1. \tag{33}$$

The two firms' consumer surplus CS_1 and CS_2 are

$$CS_1=\int_0^x(v+\sqrt{Q_1}-\alpha tx-\beta P_1)dx$$
$$=\frac{3\beta\omega(9t\omega-1)\{v[9\beta t\omega(1+\alpha)-\beta-1]-3\alpha\beta t[\omega t(\alpha-3)+1/2(1-3\omega)]+\beta t(1-3\omega t)+t(\alpha+2)-9\beta\omega t+1\}}{3\omega[9\beta t\omega(1+\alpha)-\beta-1]^2}. \tag{34}$$

$$CS_2=\int_x^1(v+\sqrt{Q_2}-t-tx-P_2)dx$$
$$=\frac{[\beta(9t\omega-1)]\{3\omega[9\beta t\omega(1+\alpha)-\beta-1](v-t)+[3\beta t\omega(2\alpha+1)-1][1-t(1+\alpha)]\}}{3\omega[9\beta t\omega(1+\alpha)-\beta-1]^2} \tag{35}$$
$$\frac{-\{1.5t\omega[9\beta t\omega(2+\alpha)-2\beta-1](9\alpha\beta t\omega-1)\}}{3\omega[9\beta t\omega(1+\alpha)-\beta-1]^2}.$$

The two firms' profits are

$$\Pi_1 = \frac{9\beta\omega t(1+\alpha)-2}{18\omega}\frac{[3\alpha t(2+\alpha)-1]^2}{[9\beta t\omega(1+\alpha)-\beta-1]^2}. \quad (36)$$

$$\Pi_2 = \frac{9\omega t(1+\alpha)-1}{9\omega}\frac{[3\beta\alpha t(2\alpha+1)-1]^2}{[9\beta t\omega(1+\alpha)-\beta-1]^2}. \quad (37)$$

The social welfare *SW* is given by

$$SW = CS_1 + CS_2 + \Pi_1 + \Pi_2. \quad (38)$$

The social welfare is decomposed into 5 terms: (i) total benefits of riders in choosing their requisite firm, (ii) transport expenses of Fu-Kang bus are subsidized by the government, (iii) fixed cost of quality of service with respect to routing type, (iv) total transportation cost is of riders who choose their requisite routing type, and (v) welfare gained by accepted routing types of the public firm, or welfare loss by abstained from choosing the public firm. Our paper focuses on the effects of transport firms under the transportation cost α and subsidy rate β conditions. For mathematical manipulation, the social welfare is shown as Equation (38). In next subsection, we substitute the numeral scale with respect to transportation cost and subsidy rate into Equation (38) for precise analysis.

2. Numerical Analysis: In this section, we discuss how the variation of transportation cost α and subsidy rate β affect the total social welfare across two firms. As shown in Table 1, we fix the value scale of subsidy rate $0.889 \leq \beta \leq 0.999$ and calculate the value of transportation cost α under differentiated routing types. Furthermore, we construct 5 charts (Figures 5 through Figure 9). Basing on the results of numerical manipulation, differentiated routing types each has a maximal and minimal value of social welfare (*Min,Max*)=(69.792, 539.118). As shown in Table 2, we fix the transportation cost value $0.704 \leq \alpha \leq 0.999$ and calculate the value of subsidy rate β under differentiated routing types. Furthermore, we construct 5 charts (Figure 10 through 14). Basing on the results of numerical manipulation, differentiate routing types each have a maximal and minimal value of social welfare (*Min,Max*)=(69.972, 539.118).

These findings demonstrate the range of social welfare is the equivalent of 469.326 whatever we fix the subsidy rate or the transportation cost. From a Fu-Kang bus operating perspective, the administrant execute the barrier-free transport with respect to the Fu-Kang bus in Taiwan by the fixed subsidy rate. According to the severity of the physical disability, the impaired people are given a discount between 50 and 66.7% of taxi-based fare for riding a Fu-Kang bus. Most firms actually operate the Fu-Kang bus routing type with "few-to-few", having few origins and few destinations like a few regulated local community hospitals. The firms' bus operation of one routing type "few-to-few" caused few impaired people to benefit the Fu-Kang bus transport service. This perspective is in line with our theoretical results. Nevertheless, we detect some unreasonable results in Table 1: (i) the subsidy rate of routing type "many-to-many" is 0.889 and their social welfare is 194.373; however, maximal social welfare (212.724) of routing type "few-to-few" is larger than minimal social welfare of other routing types, (ii) the Fu-Kang bus operations of "many-to-few" "many-to-many" routing type could not be considered due to flat subsidy rate and dispersion of social welfare. On the other hand, if we fix the transportation cost α, the merits are acquired: (i) the ranges of social welfare with respect to differentiate subsidy rates whatever their transportation cost is flat, (ii) the Fu-Kang bus operations of overall routing types could be executed due to flexibility of subsidy rate and equity of social welfare.

We evidence the spatial competition with respect to routing type and subsidy differentiation between the public and the private firms through variation of transportation cost α and subsidy rate β.

Table 1. **The scale value of α,β and social welfare (|)**

No	Routing Types	α		β (fixed)		SW	
		MIN	MAX	MIN	MAX	MIN	MAX
(1)	few-to-one	0.704	0.739	0.889	0.999	69.792	171.876
(2)	few-to-few	0.740	0.779	0.889	0.999	87.374	212.724
(3)	many-to-one	0.800	0.849	0.889	0.999	121.602	297.142
(4)	many-to-few	0.850	0.899	0.889	0.999	155.425	368.151
(5)	many-to-many	0.900	0.999	0.889	0.999	194.373	539.118

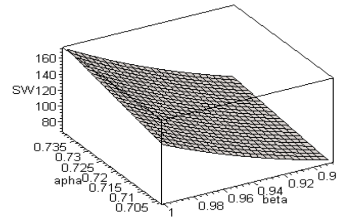


Figure 5.

$0.704 \le \alpha \le 0.739$; $0.889 \le \beta \le 0.999$

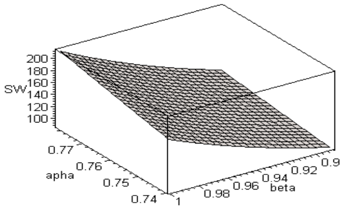


Figure 6.

$0.740 \le \alpha \le 0.779$; $0.889 \le \beta \le 0.999$

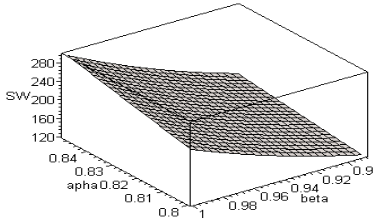


Figure 7.

$0.800 \le \alpha \le 0.849$; $0.889 \le \beta \le 0.999$

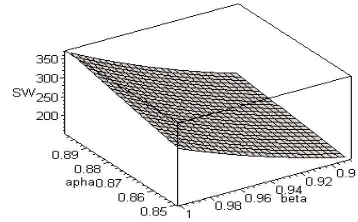


Figure 8.

$0.850 \le \alpha \le 0.899$;$0.889 \le \beta \le 0.999$

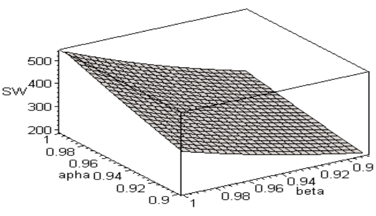


Figure 9.

$0.900 \le \alpha \le 0.999$;$0.889 \le \beta \le 0.999$

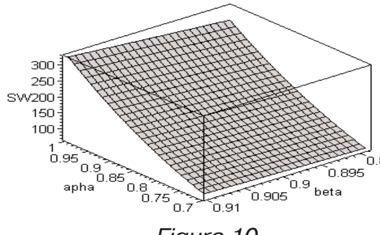


Figure 10.

$0.704 \le \alpha \le 0.999$; $0.889 \le \beta \le 0.910$

Table 2. **The scale value of α, β and social welfare (‖)**

No.	Routing Types	α (fixed)		β		SW	
		MIN	MAX	MIN	MAX	MIN	MAX
(6)	few-to-one	0.704	0.999	0.889	0.910	69.972	326.070
(7)	few-to-few	0.704	0.999	0.911	0.932	80.903	370.703
(8)	many-to-one	0.704	0.999	0.933	0.954	93.255	420.315
(9)	many-to-few	0.704	0.999	0.955	0.976	122.168	475.332
(10)	many-to-many	0.704	0.999	0.977	0.999	122.794	539.118

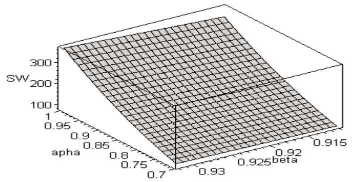


Figure 11.

$0.704 \le \alpha \le 0.999; 0.911 \le \beta \le 0.932$

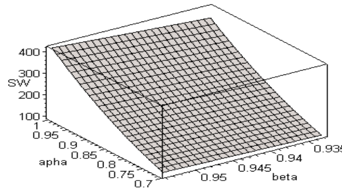


Figure 12.

$0.704 \le \alpha \le 0.999; 0.933 \le \beta \le 0.954$

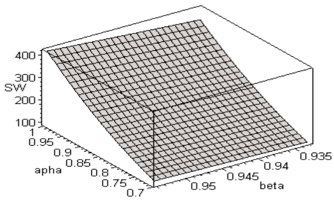


Figure 13.

$0.704 \le \alpha \le 0.999; 0.955 \le \beta \le 0.976$

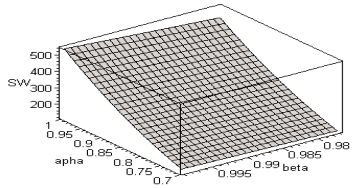


Figure 14.

$0.704 \le \alpha \le 0.999; 0.977 \le \beta \le 0.999$

Concluding remarks: This paper illustrates the effects of Fu-Kang bus routing choice strategies on the subsidy differentiation. We figure out the relation between transportation cost and subsidy rate by Nash equilibrium. Our theoretical spatial competition model with numerical analysis is improved with rough subsidy in Fu-Kang bus policy making in evidence. The results show that the flexible subsidy rate substitutes for the flat subsidy rate, then the bus firm could provide more variable Fu-Kang bus routing type for impaired people. Furthermore, the bus operating directions of the public firm and the private firm are very clear. Due to more subsidies, the private firm could choose bus routing type of the "few-to-one" and "few-to-few" types. Due to more social welfare, the public firm could choose the bus routing type of "many-to-one", "many-to-few" and "many-to-many". Thus, the government could precisely consider how to appropriate funds for the public firm or the private firm. The routing types and subsidy differentiation with respect to demand responsive transport service in this paper is a contribution to barrier-free bus transport policy-making.

Some extensions are as follow. First, in this paper, we established our spatial competition model between the public and the private firm to figure out their relation. Actually, the exact condition is that the municipal authorizes the social welfare organization to operate the Fu-Kang bus transport service in Taiwan. Their executive position is between the public firm and the private firm, and therefore we need to consider the difference among these firms. Second, bus routing could be divided precisely.

For example, the bus routing type "few-to-one" could be divided into "one-to-one", "two-to-one" and "three-to-one " routing types. Because of different distances, we have to consider how to appropriate funds. Lastly, most impaired people were carried to hospitals by Fu-Kang buses. The barrier-free bus transport service should include hospitals. If the destination is only an academic medical center, how many funds they should contribute? These extensions are to be researched in the future.

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Стаття надійшла до редакції 29.08.2012.