Daniel Armeanu[1], Georgeta Vintila[2]

APPLICATION OF MARKOWITZ MODEL FOR OPTIMIZING INVESTMENT PORTFOLIO IN A CREDIT INSTITUTION

The authors use one of the most relevant models in modern financial theory, the Markowitz model, for effective management of loan portfolio in a credit institution in Romania. Starting from the existing structure of the loan portfolio marked by different levels of average yield and associated risk, the minimum risk portfolio and the maximum obtainable yield portfolio were created, considering that at the market the short-selling operations are not authorized. The results show the way in which the Markowitz model of diversifying investments can be applied for optimizing portfolios of financial assets owned by credit institutions.

Keywords: *financial assets portfolio, portfolio yield and risk, minimum risk portfolio structure, maximum yield portfolio structure, Markowitz effective frontier.*

JEL classification: *G10, G30.*

Даніель Армяну, Джорджета Вінтіла

ЗАСТОСУВАННЯ МОДЕЛІ МАРКОВІЦА ДЛЯ ОПТИМІЗАЦІЇ ІНВЕСТИЦІЙНОГО ПОРТФЕЛЯ КРЕДИТНОЇ ОРГАНІЗАЦІЇ

У статті автори використовують одну з найбільш актуальних моделей у сучасній фінансовій теорії — Марковіца — для ефективного управління кредитним портфелем у кредитній установі в Румунії. Виходячи з існуючої структури кредитного портфеля, що відрізняється різними рівнями середньої прибутковості і пов'язаним з цим ризиком, було створено портфель з мінімальним ризиком і портфель з максимальною прибутковістю, з урахуванням того, що в Румунії короткострокові операції продажу заборонені. Результати дослідження показують, яким чином модель Марковіца для диверсифікації інвестицій може бути застосована для оптимізації портфелів фінансових активів, що належать кредитним організаціям.

Ключові слова: *портфель фінансових активів, прибутковість і ризики портфеля, портфель з мінімальною структурою ризиків, портфель із структурою максимальної прибутковості, межа ефективності за Марковіцем.*

Таб. 5. Фор. 4. Рис. 4. Літ. 18.

Даниэль Армяну, Джорджета Винтила

ПРИМЕНЕНИЕ МОДЕЛИ МАРКОВИЦА ДЛЯ ОПТИМИЗАЦИИ ИНВЕСТИЦИОННОГО ПОРТФЕЛЯ КРЕДИТНОЙ ОРГАНИЗАЦИИ

В статье авторы используют одну из наиболее актуальных моделей в современной финансовой теории — модель Марковица — для эффективного управления кредитным портфелем в кредитном учреждении в Румынии. Исходя из существующей структуры кредитного портфеля, отличающегося различными уровнями средней доходности и связанным с этим риском, были созданы портфель с минимальным риском и портфель с максимальной доходностью, с учетом того, что в Румынии краткосрочные операции продажи запрещены. Результаты исследования показывают, каким образом модель

[1] PhD, Professor, Bucharest Academy of Economic Studies, Romania.
[2] PhD, Professor, Bucharest Academy of Economic Studies, Romania.

Марковица для диверсификации инвестиций может быть применена для оптимизации портфелей финансовых активов, принадлежащих кредитным организациям.

Ключевые слова: *портфель финансовых активов, доходность и риски портфеля, портфель с минимальной структурой рисков, портфель со структурой максимальной доходности, граница эффективности по Марковицу.*

1. Introduction. Managing portfolio, as part of the financial and experimental finances theories, has received special attention lately, especially due to the notoriety of H.M. Markowitz who was awarded the Nobel Prize for economics (together with Merton Miller and William Sharpe) and of Vernon Smith after being awarded the Nobel Prize for economics in 2002. Before making a decision in an uncertain situation it is necessary to build a functional model which evaluates the level of satisfaction of the decision maker who assumes risks. If such a functional model is obtained, decisional issues can be solved through searching for the decision that maximizes the level of satisfaction for a decider. The increasing volatility manifested at financial markets as well as the multitude of capital investment possibilities recommends the use of scientific models for grounding investment decisions. The analysis undergone within the expected return — risk taken duality — aims at finding effective portfolios, respectively the portfolios that at a certain level of risk taken by the investor ensures the maximum return obtainable.

One of the major events that have marked the history of financial theory was publishing in 1952 of the essay "Portfolio Selection" written by American professor Harry M. Markowitz. He states that the investment decision is based on the analysis of the return — risk couple and that the diminishing of risk entails diversifying investments made at financial markets. In all fairness theoreticians and practitioners from the finance domain were aware for quite some time that basing the investment decision has to take into consideration the efficiency and risk of assets but this realization was done at an instinctual, latent level. The novelty brought by H.M. Markowitz in 1952 was proposing a mathematical selection model, in uncertainty conditions, of the assets that make up the optimum portfolios — meaning those portfolios that at a certain level of profitability bare minimum risks.

2. Experimental Section. Using as starting point the theory developed by H.M. Markowitz (1950), J. Treynor (1962), W. Sharpe (1964), J. Lintner (1965) and J. Mossin (1966) came up with the famous CAPM evaluation model of financial assets (capital asset pricing model). The literature has shown several shortcomings of the Markowitz and CAPM models. First of all, the hypothesis that the profitability of financial assets is normally distributed is invalidated by financial reality (numerous studies have shown that in many cases the profitability of financial assets have probable distributions characterized by negative asymmetry). Thus, the probability that the profitability of financial assets would register high deviations from the average is higher in practice than the theoretical normal probability distribution model would predict. Another issue is the use of the variance as the measure of a financial asset's risk. Critics of 2 models claim that volatility is a historic parameter, while the expected efficiency is anticipation for future. On another hand, both models assume that investors have access to free and indiscriminate financial infor-

mation and that they have the same anticipations in regards to profitability and asset risk, which, in most cases does not happen. Also, it is assumed that there are no taxes and transaction costs, which again is false. In addition, market portfolio is generally made of financial assets, since these are transacted on regulated markets and there is sufficient information regarding prices history. At least at the theoretical level, market portfolio should comprise all the types of assets that the individuals or organizations own as investment (including real estate, art objects and so on). Finally, another important problem of the Markowitz and CAPM models derives from the supposition that all investors have a price taker type of behavior, being unable to influence the rates at a market. If in case of small investors, this assumption may be considered reasonable; in case of institutional investors the hypothesis is surely wrong.

3. Results and Discussion. Within this article we use one of the already classical models in modern financial theory, more specific, the Markowitz model for effectively managing the credit portfolio in a Romanian commercial bank. Thus, the credit portfolio of a commercial bank presents the structure in Table 1 and it is characterized by certain average profitability and risk rates (measured using the average square deviation):

Table 1. **The structure of the credit portfolio of a commercial bank**

Asset	Total balance (mln EUR)	Weight in portfolio (x_P), %	Average yield (R), %	Volatility
Loans to banks	254.21	7.27	3.92	1.65
Mortgage loans	1,225.86	35.07	5.02	1.24
Consumer loans	979.23	28.02	12.06	1.84
Credit cards	11.65	0.33	25.14	2.92
Loans to SMEs	425.23	12.17	7.89	1.56
Loans to large companies	598.92	17.14	10.17	1.40
TOTAL	**3,495.10**	**100.00%**	**8.21%**	-

Source: Authors' calculations.

The yield and risk of current portfolio of the bank is computed using the following calculus:

$$R_P = x_P^T R,$$
$$\sigma_P = \sqrt{x_P^T \Omega x_P},$$

where: x_P = structure of credit portofolio, $R=(R_1\ R_2\ ...\ R_N)^T$ is the vector of yields for considered assets, σ_P = risk of portofolio and Ω = corelation matrix.

After computing, the following results are obtained: R_P = 8.21% and σ_P = 0.75%.

We now set to determine the yield, volatility and structure of R minimum risk portfolio that the bank is able to constitute. We will therefore use the following formula:

$$R_R = \frac{B}{A},\ \sigma_R = \sqrt{\frac{1}{A}},\ x_R = \frac{1}{A}\Omega^{-1}U,$$

where:

$A = U^T \Omega^{-1} U$, $B = R^T \Omega^{-1} U = U^T \Omega^{-1} R$ (because $R^T \Omega^{-1} U$ is scalary), $U = (1 \quad 1...1)^T$ is the unit vector. After computing, the following results are obtained: $R_R = 8.36\%$ and $\sigma_R = 0.65\%$.

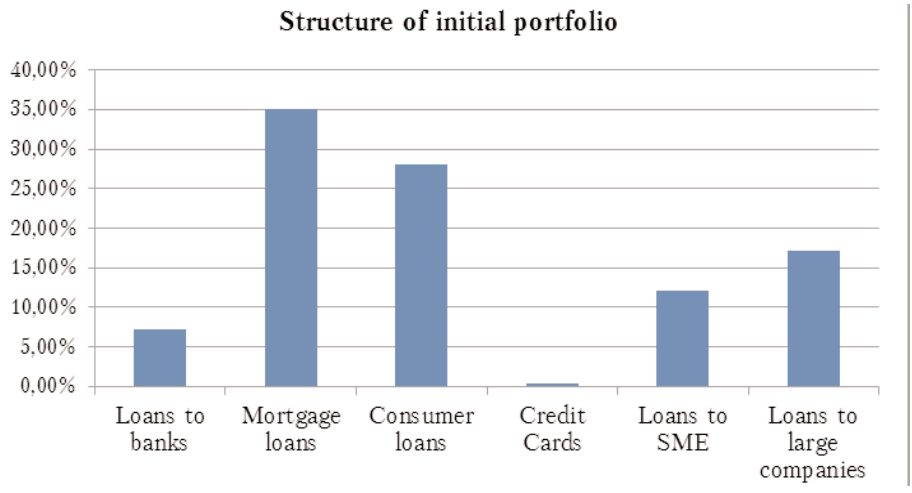
The structure of the xR minimum risk portfolio will be given by the elements shown in Table 2.

Table 2. **The structure of the minimum risk portfolio**

Asset	Total balance (mln EUR)	Weight in portfolio (x_P), %	Variation (mln EUR)
Loans to banks	545.956	15.62%	291.7455
Mortgage loans	966.678	27.66%	-259.182
Consumer loans	439.025	12.56%	-540.205
Credit Cards	174.325	4.99%	162.6749
Loans to SMEs	610.768	17.47%	185.5375
Loans to large companies	758.349	21.70%	159.4289
TOTAL	**3,495.10**	**100.00%**	0

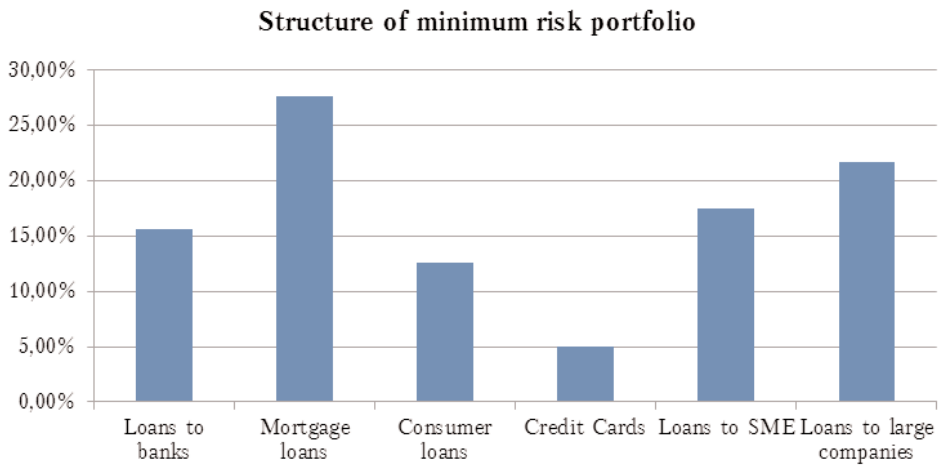
Source: Authors' calculations.

It is observed that the bank is not situated on the ineffective frontier, as the yield of the current portfolio is below the minimum risk portfolio, considering that the risk taken is higher than the risk of U portfolio. In Table 1 we have shown, besides the weight of each type of loan in the minimum variance portfolio, the differences in absolute sums of the current structure of the commercial bank portfolio. Thus, it is to be noticed that the bank has to reduce the investments in mortgage loans by almost 259 mln EUR and in consumer loans by almost 540 mln EUR. Also, the bank has to increase the exposure to loans granted to banking institutions by approximately 291 mln EUR, credit cards by 163 mln EUR, loans to SME by almost 185 mln EUR and the loans to large companies by 159 mln EUR. The comparative structures of the 2 portfolios, respectively the initial one of the commercial bank from Chart 1 and the one minimum risk one in Chart 2 are presented comparatively in below charts:



Source: Authors' calculations.

Chart 1. **Structure of initial portfolio**



Source: Authors' calculations

Chart 2. **Structure of minimum risk portfolio**

We now set out to compute the yield, risk and structure of U portfolio, meaning of the portfolio characterized by maximum yield considering that on the market the short selling operations are not authorized, as it is in Romania. In order to do this, we will use the following calculus relations:

$$R_U = \frac{C}{B},$$

$$\sigma_U = \frac{\sqrt{C}}{B},$$

$$x_U = \frac{1}{B}\Omega^{-1}R,$$

where: $C = R^T\Omega^{-1}R$.

By applying these relations we will obtain: $R_U = 11.07\%$ and $\sigma_U = 0.75\%$.

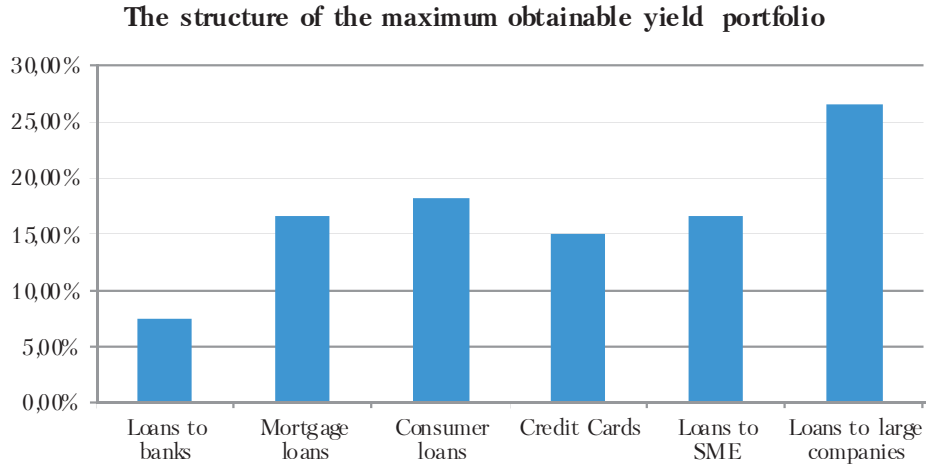
The structure of the U portfolio, as well as the adjustments that are necessary on the current bank portfolio in order to get to this structure are presented in Table 3.

Table 3. **The structure of the maximum obtainable yield portfolio**

Asset	Total balance (mln EUR)	Weight in portfolio (x_P), %	Variation (mln EUR)
Loans to banks	256.1528	7.33	1.9428386
Mortgage loans	580.8197	16.62	-645.0403
Consumer loans	633.7131	18.13	-345.5169
Credit cards	524.5424	15.01	512.89236
Loans to SME	576.7782	16.50	151.54815
Loans to large companies	923.0939	26.41	324.17389
TOTAL	**3,495.10**	**100.00**	0

In order to reach maximum obtainable yield (11.07%) the bank has to reduce the exposure on some products with low yield (mortgage loans by 645 mln EUR and consumer loans by almost 345 mln EUR) and in the same time to increase the investments in credit cards (by approximately 512 mln EUR), in loans to large companies

(by approximately 324 mln EUR) and in loans to SMEs (by 151 mln EUR). Obviously, this modification of the portfolio structure also assumes an increase in the risk taken by the bank, since an increase of the expected yield is accompanied by an increase in risk (this is one of the fundamental hypotheses on which the Markowitz model of diversifying financial assets portfolio is based). Chart 3 shows the structure of the U maximum obtainable yield portfolio:



Source: Authors' calculations.

Chart 3. **The structure of the maximum obtainable yield portfolio**

Next, let's assume that for next year the commercial bank aims an increase of loan portfolio up to the sum of 4,000 mln EUR. Let's consider now that the bank aims for a return of 10%. The following formula is applied in order to determine the risk of the portfolio with the given yield R* = 10%.

$$\sigma_P = \sqrt{\sigma_P^2} = \sqrt{\frac{1}{D}\left(AR^{*2} - 2BR^* + C\right)}, \quad (3)$$

where:

$D = AC - B^2$,

$R^* = E(R_p)$ — expected yield of portfolio which is given.

An expected volatility of 0,689% is obtained. In order to determine the structure of the new portfolio we will use the following formula:

$$x_P = \frac{1}{D}\left[\left(AR^* - B\right)\Omega^{-1}R + \left(C - BR^*\right)\Omega^{-1}U\right] \quad (4)$$

We obtain the structure of the portfolio that ensures the yield of 10% at the risk of 0.689% as shown in Table 4.

So, in order to obtain the yield of 10% in the context of the loan portfolio increasing to the sum of 4,000 mln EUR, the bank will have to make additional investments in loans to banks (approximately 169 mln EUR), in loans to SMEs (250 mln EUR) and in credit cards (430 mln EUR) and in corporate loans (383 mln EUR) and reduce the exposure in mortgage loans (387 mln EUR) and consumer loans (341 mln EUR). Considering this, the estimated risk of the portfolio will be at 0.689%.

Table 4. **The structure of a portfolio that ensures the yield of 10% at the risk of 0.689%**

Asset	Total balance (mln EUR)	Weight in portfolio (x_P), %	Variation (mln EUR)
Loans to banks	423.8094844	10.60	169.5994844
Mortgage loans	838.6826266	20.97	-387.1773734
Consumer loans	637.4869307	15.94	-341.7430693
Credit cards	442.427413	11.06	430.777413
Loans to SMEs	675.422749	16.89	250.192749
Loans to large companies	982.1707962	24.55	383.2507962
TOTAL	**4,000.00**	**100.00**	504.9

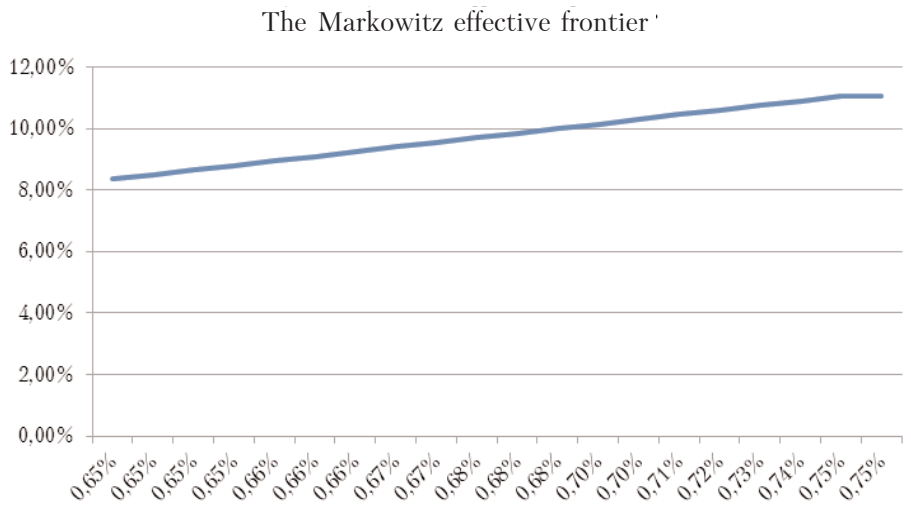
In order to determine the effective Markowitz frontier of double portfolios (considering that at the market there aren't allowed any short-selling operations) we will simulate the yield-risk keeping in mind that the desired yield of the portfolio may take values from the [RR,RU] interval, meaning it will be between the minimum risk portfolio yield and the maximum yield portfolio as long as it doesn't have short sell. In Table 5, we show the values for desired yield and volatility of portfolio, considering the increments of 0.15% for yield and computing the associated risk with the aid of the formula used for determining the risk of the portfolio with the given yield of R*.

Table 5. **The Markowitz effective frontier**

Risk, %	Yield – efficient portfolios, %
0.65213	8.35
0.65243	8.50
0.6534	8.65
0.65497	8.80
0.65720	8.95
0.66006	9.10
0.66355	9.25
0.66765	9.40
0.67235	9.55
0.67764	9.70
0.68351	9.85
0.68000	10.00
0.69692	10.15
0.70443	10.30
0.71245	10.45
0.72096	10.60
0.72996	10.75
0.73942	10.90
0.74932	11.05
0.75062	11.07

Source: Authors' calculations.

The graphical representation of the Markowitz effective frontier is shown in Chart 4.



Source: Authors' calculations

Chart 4. **Graphical representation of the Markowitz frontier**

4. Conclusions. In this article we have presented the way in which the Markowitz model of diversifying investments can be applied to optimize the financial assets portfolios owned by credit institutions. This is why we have considered the loan portfolio of a commercial bank (containing 6 categories of loans) and yields, variances and their historic covariances. Based on these inputs we have applied the Markowitz model for determining the minimum risk portfolios, the maximum obtainable yield considering that at the market there are no open sales allowed as well as the portfolios which ensure a desired level of yield or risk. From the analysis of the initial portfolio it is shown that if the bank would apply the Markowitz model it have obtain the portfolio yield of 8.21% at the risk of 0.75%. In case the bank would apply the model it has the opportunity of obtaining to the same exposure to risk of a much higher level (almost 11%). So this model can be used by commercial banks for loan portfolio optimization.

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