Serhiy V. Melnikov[1]

DYNAMIC MONOPOLY PRICING UNDER THE PURCHASE PRICE EFFECT

This paper investigates the dynamic monopoly pricing under the purchase price effect. For this purpose, a model of monopoly with a difference demand function is constructed. Three criteria of efficient, discrete and equilibrium solutions are considered.

Keywords: *monopoly; dynamic pricing; purchase price; difference demand function.*

Сергій В. Мельников

ДИНАМІЧНЕ ЦІНОУТВОРЕННЯ МОНОПОЛІЇ З УРАХУВАННЯМ ЕФЕКТУ ЦІНИ ПОКУПКИ

У статті досліджено проблеми динамічного ціноутворення монополії в умовах дії ефекту ціни покупки. З цією метою побудовано модель монополії з різницевою функцією попиту. Розглянуто три критерії ефективності, знайдено дискретні та рівноважні рішення.

Ключові слова: *монополія; динамічне ціноутворення; ціна покупки; різницева функція попиту.*

Форм. 15. Рис. 3. Табл. 1. Літ. 10.

Сергей В. Мельников

ДИНАМИЧЕСКОЕ ЦЕНООБРАЗОВАНИЕ МОНОПОЛИИ С УЧЕТОМ ЭФФЕКТА ЦЕНЫ ПОКУПКИ

В статье исследованы проблемы динамического ценообразования монополии в условиях действия эффекта цены покупки. С этой целью построена модель монополии с разностной функцией спроса. Рассмотрены три критерия эффективности, найдены дискретные и равновесные решения.

Ключевые слова: *монополия; динамическое ценообразование; цена покупки; разностная функция спроса.*

Problem statement. One of the important aspects of the theory and practice of monopolies is to develop an optimal dynamic pricing strategy. Urgency of the problem stems from the fact that in practice the costs of exploring all of the demand curve can be quite substantial. Therefore, modelling of dynamic pricing strategies is certainly of a great theoretical and practical interest.

Recent research and publications analysis. A number of works have been dedicated to monopoly pricing strategies. P. Aghion, P. Bolton and B. Jullien (1988), W.J. Baumol and R.E. Quandt (1964), E.P. Lazear (1986) investigated deterministic demand curves. J.M. Leonard, L. Samuelson and A. Urbano (1993), Y. Levin, J. McGill and M. Nediak (2010), A. McLennan (1984), M. Rothschild (1974) investigated the models with stochastic demand. P. McAfee and V. Velde (2008) investigated the dynamic monopoly pricing under constant elasticity of demand.

Research objectives. When modelling the demand curve linear functions have often been used. An important characteristic of the demand curve is its elasticity, defined by the modelling function. It is based on the value of price elasticity of pric-

[1] Odesa National Maritime University, Ukraine.

ing decisions taken. Linear function is good for modelling of the demand curve, but has certain disadvantages.

Marketing literature describes the dynamic effect of purchase price (Simon, 1997). According to this, consumer responds not only to the new value price, but also to the ratio of the new and the previous price. Typically, consumers use the previous price as the "purchase price". The effect is that the reaction of consumers to price increases and decreases is asymmetric. For example, if to raise price for some initial demand volume, and then to reduce it by the same amount, the new demand volume will be less than the initial demand volume. Price increase will have a strong influence on the volume of demand than the decline.

When using the linear demand function, this effect is absent, the reaction of consumers to price increases and decreases is symmetric. Under the symmetric influence of prices the monopolist can achieve the global optimum regardless the previous price because consumers do not care what was the "purchase price".

In modelling the effect of the purchase price represent the demand curve in the form of difference equation.

Thus, the **goal of this article** is to explore the dynamic monopoly pricing taking into account the marketing effect of purchase price.

Key research findings. We investigate the model in the short-run time period, during which the price elasticity of demand is constant. Demand function with constant price elasticity given in the form of the difference equation is

$$Q_t = Q_{t-1} \times \left(1 - E \times \left(\frac{P_t}{P_{t-1}} - 1\right)\right), \tag{1}$$

where P_t and Q_t – the price and the volume of demand in discrete time t ($t \in N$, $N = \{1, 2, ..., T\}$ – set of moments); $E > 0$ – the absolute value of price elasticity of demand (we do not consider the zero elasticity case). Non-negativity condition demand $P_t \le P_{t-1} \times \frac{E+1}{E}$.

We consider 3 criteria: maximum profit at each time point (I), the maximum profit at the last time point (II), the maximum profit for all time (III).

Criterion I – the maximum profit at each time point. We assume that the monopolist is in the moment $t - 1$ and wants to maximize profit in the next moment t.

The monopoly's profit is

$$F_t = (P_t - z) \times Q_t = (P_t - z) \times Q_{t-1} \times \left(1 - E \times \left(\frac{P_t}{P_{t-1}} - 1\right)\right) \to \max_{P_t}. \tag{2}$$

Equating the first derivative to zero $\frac{dF_t}{dP_t} = 0$, we find the optimal price

$$P_t^* = z + \frac{P_{t-1} \times (E+1) - z \times E}{2 \times E}, \quad t \in N. \tag{3}$$

Second derivative $\frac{d^2 F_t}{dP_t^2} = -\frac{2 \times E \times Q_{t-1}}{P_{t-1}} < 0$, i.e. at the price (3) we get the maximum profit. Here are the optimums of the monopoly with the initial conditions: $P_0 = 10$, $Q_0 = 22$, $E = 0.83$, $z = 6$, $t = 1, 2, 3$ (Figure 1).

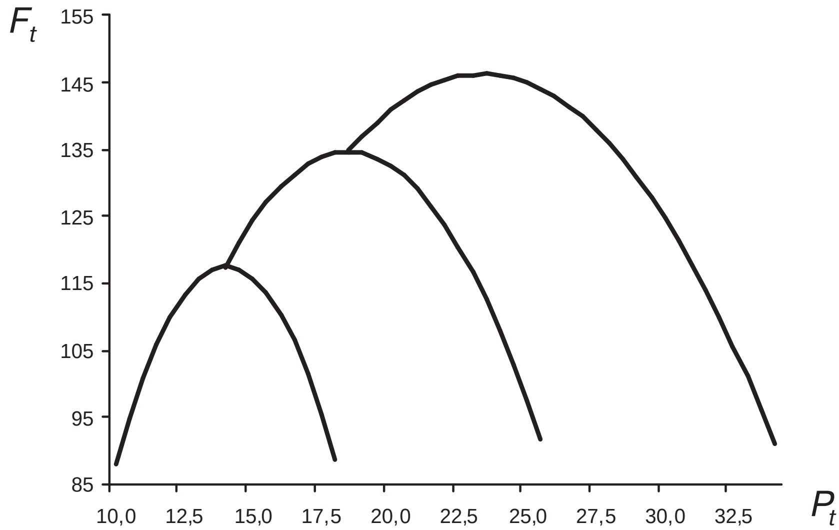

Figure 1. **Monopoly's optimums,** *author's*

As we know, when demand is inelastic ($E<1$), the monopolist is advised to raise the price. At constant elasticity, which is less than unity, in theory, the price in the next moment of time (t) can reach infinity, because consumers do not change their behavior. But with the difference demand function, this situation can not arise outside the increase or decrease in price. Price increase above the optimum with inelastic demand may lead to lower profits. This is due to the effect of "purchase price" which is an important the ratio of new and the previous price.

Note that (1) is a linear nonhomogeneous first-order difference equation. The general solution for this discrete difference equation with the initial condition $P_t = P_0$ for $t = 0$.

For $$E=1,\ P_t^* = P_0 + \frac{z\times t}{2},\ t\in N. \tag{4}$$

For $$E\neq 1,\ P_t^* = \left(\frac{1+E}{2\times E}\right)^t \times \left(P_0 + z\times\frac{E}{1-E}\right) - z\times\frac{E}{1-E},\ \ t\in N. \tag{5}$$

Using the equilibrium condition $P_t = P_{t-1}$, we find the equilibrium solution of equation (3):

$$P^e = z\times\left(1+\frac{E}{E-1}\right),\ E>1. \tag{6}$$

Thus, the equilibrium price (6) is invariant with respect to the initial price, and coincides with the optimum price monopolist in statics (McAfee and Johnson, 2005).

Criterion II – the maximum profit at the last time point.

The monopoly's profit is

$$F_T = (P_T - z)\times Q_T = (P_T - z)\times Q_0 \times \prod_{t\in N}\left(1-E\times\left(\frac{P_t}{P_{t-1}}-1\right)\right) \to \max_{\{P_t\},t\in N}. \tag{7}$$

Then we find partial derivatives and equate them to zero.

For $t = 1, 2, ..., T-1$

$$\frac{\partial F_T}{\partial P_t} = \left(\frac{P_{t+1}}{P_t^2} - \frac{1}{P_{t-1}}\right) \times E \times (E+1) \times (P_T - z) \times Q_0 \times \prod_{\substack{i=1, \\ i \neq t, t+1}}^{T} \left(1 - E \times \left(\frac{P_i}{P_{i-1}} - 1\right)\right) = 0; \quad (8)$$

$$P_t^* = \sqrt{P_{t-1}^* \times P_{t+1}^*}. \quad (9)$$

The second partial derivative is:

$$\frac{\partial^2 F_T}{\partial P_t^2} = -\frac{2 \times P_{t+1}}{P_t^3} \cdot E \times (E+1) \times (P_T - z) \times Q_0 \times \prod_{\substack{i=1, \\ i \neq t, t+1}}^{T} \left(1 - E \times \left(\frac{P_i}{P_{i-1}} - 1\right)\right) < 0, \quad (10)$$

i.e. relation (9) is the condition of maximum profit.

$$\frac{\partial F_T}{\partial P_T} = Q_0 \times \left(1 - E \times \left(\frac{P_T}{P_{T-1}} - 1\right) - (P_T - z) \times \frac{E}{P_{T-1}}\right) = 0; \quad (11)$$

$$P_T^* = z + \frac{P_{T-1} \times (E+1) - z \times E}{2 \times E}. \quad (12)$$

Thus, the optimal prices P_t^*, $t = 1, 2, ..., T-1$ are the members of a geometric progression. The optimal price P_T^* coincides with (3). From (9) we obtain another property of the geometric progression

$$P_1 = \sqrt{P_0 \times P_2}; \;\; P_2 = \sqrt[3]{P_0 \times P_3^2}; \;\; P_3 = \sqrt[4]{P_0 \times P_4^3}; \; ...; \; P_t = \sqrt[t+1]{P_0 \times P_{t+1}^t}. \quad (13)$$

Substituting (12) in (13), we obtain the optimality condition for price P_{T-1}

$$P_{T-1}^* = \sqrt[T]{P_0 \times \left(\frac{P_{T-1}^* \times (E+1) + z \times E}{2 \times E}\right)^{T-1}}. \quad (14)$$

P_T^* and P_t^*, $t = 1, 2, ..., T-2$ find from (12) and (13). From the equilibrium condition we obtain that the equilibrium solution coincides with (6).

Criterion III – the maximum profit for all time.

The monopoly's profit is

$$F = \sum_{t \in N} (P_t - z) \times Q_t \to \max_{\{P_t\}, t \in N}. \quad (15)$$

Since qualitative analysis of the first partial derivatives of (15) is problematic, then we will search for the optimal pricing strategy using the tool "Solver" in Microsoft Excel.

Under the conditions $P_0 = 7.5$, $Q_0 = 25$, $E = 1.52$, $z = 6$, $t = \overline{0,100}$ the optimal pricing strategies and trajectories corresponding to profits are shown in Figures 2 and 3. The equilibrium price is equal $P^e = 17.5$. Table 1 shows the values of criteria indicators.

Table 1. **Numerical example**

criterion / profit	I	II	III
F_T	63.5	**78.3**	76.3
$\sum_{t \in N} F_t$	6321.1	6650.9	**7363.5**

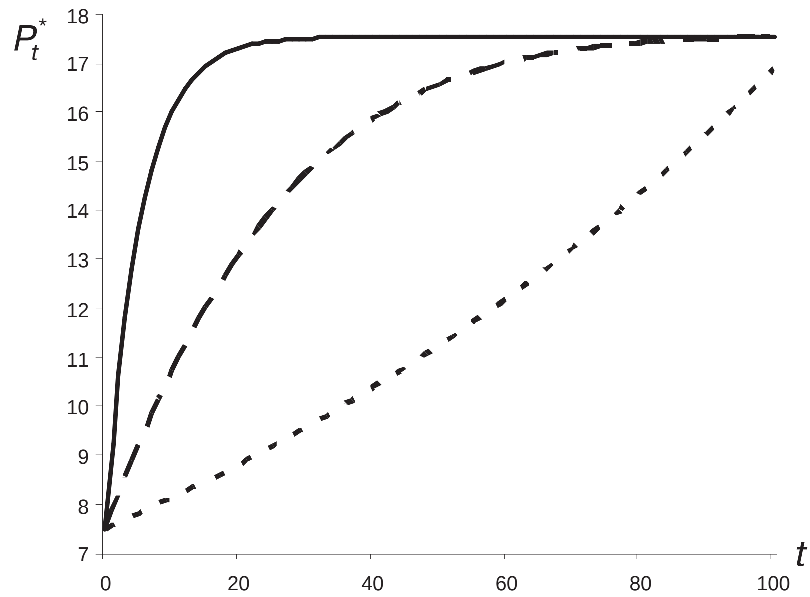

Figure 2. **Monopoly's optimal price strategies,** *author's*

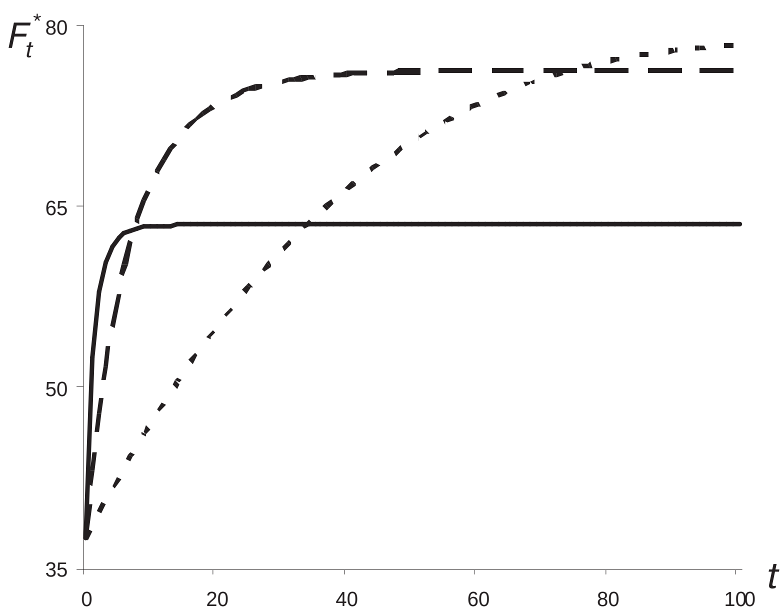

Figure 3. **Profit dynamic by criteria,** *author's*

In Figures 2 and 3 we see that the strategy by criterion III asymptotically approaches the equilibrium state by other criteria. For local profit maximization (criterion I) becomes an unreachable global maximum (criterion III). From a certain point (in our case at $t = 8$) local optimization strategy exhausts itself. If the monopolist will try to change at $t = 8$ pricing strategy I to the strategy II or III, then it will not be possible to increase profits significantly. Pricing strategy by criterion II is characterized by low, compared with other strategies, growth. This is consistent with the recommendations of marketers on the gradual increase in prices rather than a single one.

Conclusions and prospects for further research. We have considered the effect of the purchase limits price increase in inelastic demand and price reduction in elastic demand. Global and local criteria correspond to different optimal pricing strategies. For the same price companies may receive different profits – "price" history generates the corresponding customer loyalty. Not only the absolute value becomes important, but also the dynamics of prices. Among the strategies considered, there can't be the best one. The strategy choice will be determined by the objectives of marketing and planning period.

In the future we plan to model the monopoly pricing strategies taking into account other marketing effects.

References:

Aghion, P., Bolton, P., Jullien, B. (1988). Learning through Price Experimentation by a Monopolist Facing Unknown Demand. Cambridge, Mass.: Massachusetts Institute of Technology Harvard Univ. 44 p.

Baumol, W.J., Quandt, R.E. (1964). Rules of Thumb and Optimally Imperfect Decisions. American Economic Review, 54: 23–46.

Lazear, E.P. (1986). Retail Pricing and Clearance Sales. American Economic Review, 76: 14–32.

Leonard, J.M., Samuelson, L., Urbano, A. (1993). Monopoly Experimentation. International Economic Review, 34: 549–563.

Levin, Y., McGill, J., Nediak, M. (2010). Optimal Dynamic Pricing of Perishable Items by a Monopolist Facing Strategic Consumers. Production and Operations Management, 19: 40–60.

McAfee, P., Velde, V. (2008). Dynamic Pricing with Constant Demand Elasticity. Production and Operations Management, 17: 432–438.

McAfee, P.R., Johnson, S.J. (2005). Introduction to Economic Analysis // www.mcafee.cc.

McLennan, A. (1984). Price Dispersion and Incomplete Learning in the Long Run. Journ. Econ. Dynamics a. Control, 7: 331–347.

Rothschild, M. (1974). A Two-Armed Bandit Theory of Market Pricing. Journ. Econ. Theory, 9: 185–202.

Simon, H. (1997). Profit durch Power Pricing: Strategien aktiver Preispolitik. Frankfurt am Main; New York: Campus Verlag.

Стаття надійшла до редакції 28.08.2014.