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A SINGLE-PERIOD INVENTORY MANAGEMENT MODEL WITH A CONTINUOUS FUZZY RANDOM DEMAND

The paper describes an algorithm for searching the optimal enterprise inventory capacity, where the demand for this resource is a fuzzy random variable. In particular, the case of continuously distributed demand with the expected value which is a triangular fuzzy number has been discussed. A numerical example is given to illustrate the model.
Keywords: *newsvendor problem; fuzzy random variable (FRV); graded mean integration representation method; fuzzy stochastic optimization.*

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ОДНОПЕРІОДНА МОДЕЛЬ УПРАВЛІННЯ ЗАПАСАМИ З НЕПЕРЕРВНИМ НЕЧІТКИМ ВИПАДКОВИМ ПОПИТОМ*

У статті описано алгоритм пошуку оптимального обсягу запасу ресурсу підприємства, в якому попит на цей ресурс є нечіткою випадковою величиною. Зокрема, розглянуто випадок, коли закон розподілу випадкового попиту відомий або може бути оцінений на основі статистичних даних і математичне сподівання якого — нечітке трикутне число. Теоретичний матеріал проілюстровано числовим прикладом.
Ключові слова: *задача «газетяра», нечітка випадкова величина, метод інтегрування градуйованого середнього, нечітка стохастична оптимізація.*
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ОДНОПЕРИОДНАЯ МОДЕЛЬ УПРАВЛЕНИЯ ЗАПАСАМИ С НЕПРЕРЫВНЫМ НЕЧЕТКИМ СЛУЧАЙНЫМ СПРОСОМ

В статье описан алгоритм поиска оптимального размера запаса ресурса предприятия, спрос на который является нечеткой случайной величиной. В частности, рассмотрен случай, когда закон распределения спроса известен или может быть оценен на основании статистических данных и математическое ожидание которого — нечеткое треугольное число. Теоретический материал проиллюстрирован числовым примером.
Ключевые слова: *задача «газетчика», нечеткая случайная величина, метод интегрирования градуированного среднего, нечеткая стохастическая оптимизация.*

1. Introduction. A single-period inventory model, also well known as the newsvendor model, attempts to solve the problem of finding the optimal order quantity that maximizes the expected profit or minimizes the expected inventory costs. This model has been used for perishable products, such as flowers, food and for inventory of items with a short or restricted term of selling. For example, spare parts for some car models which are not produced any more, fashion or seasonal goods, newspapers etc. are associated with the problem. Extensions of the newsvendor model are used in multiperiod models and inventory control models. Therefore, the single-period inventory model is very important both for practical use and theoretical research.

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Main assumptions of the classical model are the following: relatively short selling season, commit to purchase before a season starts, demand distribution known or estimated, significant lost sales costs, significant excess inventory cost. Most of the single-period models have been made in probabilistic framework. It means that uncertainty of future demand can be described by a probability distribution. But in real situations such approach can be inadequate due the lack of statistical data or the need to consider the environmental influence.

After creating the fuzzy logic by L.A. Zadeh (1965) the attempts to introduce the fuzzy set theory for solving different inventory problems were made. In these models a demand was described linguistically such as "demand will be about 400 items". So it represented the judgments of experts for forecasting the future demand, in particular for solving the newsvendor problem. However, most appropriate approach for seeking optimal solution of inventory models is considering the two sources of uncertainty: randomness and fuzziness. In this paper the model in which a demand is a fuzzy random variable is constructed and an optimal solution is found.

2. Latest research and publications analysis. First note about the single-period problem was made by Edgeworth (1888) in the context of a bank setting the level of its cash reserves to cover the demands of its customers.

The term "newsboy" to name this problem was firstly proposed by Morse and Kimball (1951). They found implicitly the optimal fractile solution. Arrow et al. (1951) formulated a more general problem and also implicitly provided the critical fractile solution within a special case of their solution. Whitin (1953) explicitly provided the critical fractile solution.

New approach for solving the problem under assumption that there were not enough statistical data for estimating the demand distribution was proposed by Scarf (1958). His min-max order formula was obtained by maximizing the minimum expected profit among all demand distributions with the given mean and variance.

Dvoretzky et al. (1952) and Karlin (1958) included nonproportional (nonlinear) costs in their analyses. Karlin (1958) also argued for the possibility that the overage cost might be a nonlinear function of the overage.

Some new approach for constructing a single-period model is the so called mean-variance approach. The mean-risk formulation pioneered by Markovitz (1959) for risk management in portfolio investment has been applied for the analysis of inventory problems. With this perspective, a number of papers have been devoted to risk analysis of supply chain models. Recent studies on the newsvendor problem using the mean-variance framework include Lau (1980), Eeckhoudt et al. (1995), Lau and Lau (1999), Chen and Federgruen (2000). For the extensive review of the literature on this matter one can refer to Choi and Chiu (2012).

All of the reviewed papers are based on the purely probabilistic framework. As an alternative to it there is also a research built on the linguistic basis. In the literature, the fuzzy set theory has been applied to inventory problems to handle the uncertainties related to the demand or cost coefficients. Petrovic et al. (1996) developed two fuzzy models to handle uncertainty in the single-period inventory problem under discrete fuzzy demand. Ishii and Konno (1998) introduced a fuzzy newsboy model restricted to shortage cost that is given by an L-shape fuzzy number while the demand is still stochastic. Li et al. (2002) studied the single-period inventory problem in two different cases where in the first one the demand is probabilistic while the cost components are fuzzy and in the other one costs are deterministic but the demand is fuzzy.

The mixed uncertainty has been introduced to the newsvendor problem in recent years by Dutta et al. (2005), Dey and Chakraborty (2008), Nagare and Dutta (2012). These reviewed works are devoted to the problem with a discrete fuzzy random demand. But in most real inventory situations, it can be computationally more advantageous to consider a fuzzy random variable following a continuous distribution rather than a discrete one. Dey and Chakraborty (2012) proposed the model in which the annual customer demand has been assumed to be a fuzzy random variable following normal or truncated normal distributions.

In this article it has been assumed that on the basis of the historical background a probabilistic nature of demand is known (following a continuous distribution). But influence of the environment leads to changing the demand mean as represented in an experts' judgment. Thus, the single-period inventory model has been considered with the fuzzy random customer demand.

3. Preliminary concepts.

3.1. Triangular fuzzy number. A fuzzy number $\tilde{A}=(a,b,c)$ is said to be triangular if its membership function is given by

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & -\infty<x<a \\ L(x)=\dfrac{x-a}{b-a}, & a\le x\le b \\ R(x)=\dfrac{x-c}{b-c}, & b\le x\le c \\ 0, & c<x<+\infty \end{cases}, \quad (1)$$

where $-\infty<a\le b\le c<+\infty$.

Let $\tilde{A}_1=(a_1,b_1,c_1)$ and $\tilde{A}_2=(a_2,b_2,c_2)$ be triangular fuzzy numbers. Then, according to the function principle (Chen, 1985), linear operations on $\tilde{A}_1$ and $\tilde{A}_2$ can be expressed as follows:

1) The addition of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1+\tilde{A}_2=(a_1+a_2,b_1+b_2,c_1+c_2)$;

2) The subtraction of $\tilde{A}_1$ and $\tilde{A}_2$ is $\tilde{A}_1-\tilde{A}_2=(a_1-c_2,b_1-b_2,c_1-a_2)$;

3) The scalar multiplication of $\tilde{A}_1$ by λ is $\lambda\tilde{A}_1=(\lambda a_1,\lambda b_1,\lambda c_1)$ if $\lambda\ge 0$ and $\lambda\tilde{A}_1=(\lambda c_1,\lambda b_1,\lambda a_1)$ if $\lambda\le 0$.

Before proceeding to the definitions which are needed to formulate the model, we will define the fundamental function in fuzzy logic, the α-cut.

The α-cut of a fuzzy number $\tilde{U}$, with $0<\alpha\le 1$ is the crisp set

$$[\tilde{U}]_\alpha=\{x\in R\,|\,\mu_{\tilde{U}}(x)\ge\alpha\}. \quad (2)$$

In particular, explicit expression for α-cut of a triangular fuzzy number $\tilde{A}=(a,b,c)$ is

$$[\tilde{A}]_\alpha=[(b-a)\alpha+a,-(c-b)\alpha+c]. \quad (3)$$

3.2. Defuzzification: Graded Mean Integration Representation method (GMIR). The fuzzy numbers with given membership function are not easy to compare based on the fuzzy form. Hence, it's necessary to defuzzify the fuzzy numbers under comparison. There are many defuzzification methods such as weighted fuzzy mean, mean of maximum etc. (Leekwijck and Kerre, 1999). In this research, the graded mean integration representation method (Chen and Hsieh, 1998) is selected to represent fuzzy numbers because of its simplicity and accuracy.

The GMIR method can be described as follows. Let L^{-1} and R^{-1} be the inverse functions of the functions L and R, respectively. Then the graded mean h-level value of the triangular fuzzy number $\tilde{A}=(a,b,c)$ is given by $h[L^{-1}(h)+R^{-1}(h)]/2$. Therefore, the GMIR of $\tilde{A}$ is

$$G(\tilde{A})=\frac{\int_0^1 (h[L^{-1}(h)+R^{-1}(h)]/2)dh}{\int_0^1 hdh}=\frac{a+4b+c}{6}. \qquad (4)$$

3.3. Ranking of fuzzy numbers. There are different ways to rank fuzzy numbers. Some of these methods were reviewed by Chen and Hwang (1972). However, some of them are computationally complex and difficult to implement. Thus, in this article we use the method based on the GMIR (Ding, 2011) as a less complicated and more powerful one.

Let $\tilde{A}_i=(a_i,b_i,c_i)$, $i=1,2,\dots,n$, be n triangular fuzzy numbers. Suppose $G(\tilde{A}_i)$ and $G(\tilde{A}_j)$ are the GMIR values of the triangular fuzzy numbers $\tilde{A}_i$ and $\tilde{A}_j$, respectively. We define:

1) $\tilde{A}_i \prec \tilde{A}_j \Leftrightarrow G(\tilde{A}_i)<G(\tilde{A}_j)$;
2) $\tilde{A}_i \succ \tilde{A}_j \Leftrightarrow G(\tilde{A}_i)>G(\tilde{A}_j)$;
3) $\tilde{A}_i = \tilde{A}_j \Leftrightarrow G(\tilde{A}_i)=G(\tilde{A}_j)$.

3.4. Fuzzy random variable (FRV). Kwakernaak (1978) conceptualized a FRV as a vague perception of a crisp but unobservable RV. His definition of it is given as follows.

Definition (Kwakernaak, 1978). Let $(\Omega,\Im,P)$ be the probability space and $F(R)$ is the set of all fuzzy numbers on R. The fuzzy random variable is mapping $\chi:\Omega\to F(R)$ such that for any $\alpha\in[0,1]$ and all $w\in\Omega$, the real valued mappings $\chi_\alpha^-:\Omega\to R$ and $\chi_\alpha^+:\Omega\to R$, where for $\alpha\neq 0$ $[\chi(w)_\alpha^-;\chi(w)_\alpha^+]$ is α-cut of $\chi(w)$ and for α = 0 it is closure of the support of $\chi(w)$, are real-valued random variables.

The expectation of the fuzzy random variable $\tilde{X}$ is a unique fuzzy number whose α-cuts are given by

$$[E(\tilde{X})]_\alpha=E[\tilde{X}]_\alpha=[E(\tilde{X}_\alpha^-),E(\tilde{X}_\alpha^+)],\ \alpha\in(0,1]. \qquad (5)$$

4. Research methodology. The mathematical model presented in this study has the following assumptions:

1. A single-period inventory problem is considered.

2. The demand in previous periods is continuously distributed with cumulative distribution function $F_0(x)$, it is estimated on the basis of statistical data.

3. Expected demand in the next period is forecasted by experts.

4. The following notations are used: c – purchase or production cost per unit; p – selling price per unit; h – holding cost per unit; s – shortage cost per unit; Q – order quantity at the start of the period. Costs of the inventory system are independent on time.

This situation can be modelled as follows. Let the expectation value of demand in the next period be a triangular fuzzy number. Then we can define the future demand as FRV $\tilde{D}=(D-\Delta_1, D, D+\Delta_2)$, where Δ_1, Δ_2 are the set by decision maker and D is the random variable with cumulative distribution function $F(x)$, which has the same type as F_0, but the mean of D can be distinct from the mean of the demand in previous periods.

Then the expected fuzzy value of profit in the next period is

$$\tilde{P}(\tilde{D},Q)=\begin{cases} p\tilde{D}-cQ-h(Q-\tilde{D}), & \tilde{D}\preceq Q \\ (p-c)Q-s(\tilde{D}-Q), & \tilde{D}\succeq Q \end{cases}. \quad (6)$$

For solving the problem of mean profit maximization we defuzzify $\tilde{P}(\tilde{D},Q)$ by GMIR method. Let us consider two cases.

1. When $\tilde{D}\preceq Q$ or, as it noted in 3.3:

$$G(\tilde{D})=\frac{D-\Delta_1+4D+D+\Delta_2}{6}=D+\frac{\Delta_2-\Delta_1}{6}\le Q. \quad (7)$$

In this case $\tilde{P}(\tilde{D},Q)$ is the triangular fuzzy number $(\underline{P},P,\overline{P})$ with

$$\begin{aligned} P&=pD-cQ-h(Q-D); \\ \underline{P}&=p(D-\Delta_1)-cQ-h(Q-D+\Delta_1); \\ \overline{P}&=p(D+\Delta_2)-cQ-h(Q-D-\Delta_2). \end{aligned} \quad (8)$$

Therefore, the GMIR value of fuzzy profit in this case is

$$\begin{aligned} G[\tilde{P}(D,Q)]&=\frac{1}{6}\left[(p+h)(4D+D-\Delta_1+D+\Delta_2)-6cQ-6hQ\right]= \\ &=(p+h)\left(D+\frac{\Delta_2-\Delta_1}{6}\right)-(c+h)Q. \end{aligned} \quad (9)$$

2. When $\tilde{D}\succeq Q$ or $D+\frac{\Delta_2-\Delta_1}{6}\ge Q$.

Then the profit is the triangular fuzzy number $(\underline{P},P,\overline{P})$ with

$$\begin{aligned} P&=(p-c)Q-s(D-Q); \\ \underline{P}&=(p-c)Q-s(D+\Delta_2-Q); \\ \overline{P}&=(p-c)Q-s(D-\Delta_1-Q). \end{aligned} \quad (10)$$

So, using the GMIR method the defuzzified value of the profit is obtained:

$$G[\tilde{P}(D,Q)]=(p-c)Q+s\left(Q-D-\frac{\Delta_2-\Delta_1}{6}\right). \quad (11)$$

Considering the two cases the defuzzified value of the profit takes the following form:

$$G[\tilde{P}(D,Q)]=\begin{cases} (p+h)\left(D+\frac{\Delta_2-\Delta_1}{6}\right)-(c+h)Q, & D\le Q-\frac{\Delta_2-\Delta_1}{6} \\ (p-c)Q+s\left(Q-D-\frac{\Delta_2-\Delta_1}{6}\right), & D\ge Q-\frac{\Delta_2-\Delta_1}{6} \end{cases}. \quad (12)$$

Then the expectation value of a random variable function $G[\tilde{P}(D,Q)]$ is

$$\begin{aligned} E_Q[G(\tilde{P}(D))]&=(p+h)\int_0^{Q-\frac{\Delta_2-\Delta_1}{6}} Df(D)dD+\left((p+h)\frac{\Delta_2-\Delta_1}{6}-(c+h)Q\right)\int_0^{Q-\frac{\Delta_2-\Delta_1}{6}} f(D)dD- \\ &-s\int_{Q-\frac{\Delta_2-\Delta_1}{6}}^{+\infty} Df(D)dD+\left((p-c+s)Q-s\frac{\Delta_2-\Delta_1}{6}\right)\int_{Q-\frac{\Delta_2-\Delta_1}{6}}^{+\infty} f(D)dD. \end{aligned} \quad (13)$$

Hence, the optimization problem can be formulated as

$$\max_{Q} E_Q[G(\tilde{P}(D))]. \tag{14}$$

The first and the second derivatives of $E_Q[G(\tilde{P}(D))]$ are

$$\frac{d}{dQ}E_Q[G(\tilde{P}(D))] = (p-c+s)-(p+s+h)F\left(Q-\frac{\Delta_2-\Delta_1}{6}\right);$$
$$\frac{d^2}{dQ^2}E_Q[G(\tilde{P}(D))] = -(p+s+h)f\left(Q-\frac{\Delta_2-\Delta_1}{6}\right). \tag{15}$$

Therefore, the target function is concave in Q. The optimal order quantity can be obtained from the first-order condition of an optimum.

Now let $F^{-1}(\cdot)$ be the inverse function of a cumulative distribution function F, which is defined on [0,1). Because of $0<\frac{p-c+s}{p+s+h}<1$ it follows that the optimal solution of of the problem (14) is

$$Q^* = F^{-1}\left(\frac{p-c+s}{p+s+h}\right)+\frac{\Delta_2-\Delta_1}{6}. \tag{16}$$

The relationship with the classical fractile solution is the following. Let the demand be a crisp random variable ($\Delta_1 = \Delta_2 = 0$) and $p - c + s = C_U$, $h + c = C_O$ are the under-ordering and over-ordering cost, respectively. Then the formula (16) is transformed into the classical optimal solution of the newsvendor problem under stochastic uncertainty:

$$Q^* = F^{-1}\left(\frac{C_U}{C_O+C_U}\right). \tag{17}$$

5. Numerical example. For numerical analysis of the newsvendor problem under the hybrid uncertainty we use the following parameters: production cost $c = 30$, selling price $p = 65$, holding cost $h = 10$, shortage penalty cost $s = 20$. Let the distribution of demand in previous periods be normal with parameters $\mu = 400$, $\sigma = 80$. Experts suppose the mean demand in the next period is going to be half as much again because of the advertising campaign.

Thus, we have a linguistic value of expected demand which can be transformed into a triangular fuzzy number by decision makers as, for example, (400,600,650). Then the random variable D is normal distributed with $\mu = 600$, $\sigma = 80$ and $\Delta_1 = 200$, $\Delta_2 = 50$.

Using the formula (16) we have an optimal solution:

$$Q^* = F^{-1}(0{,}579) - 25 = \mu + \sigma\Phi^{-1}(0{,}579) - 25 = 591, \tag{18}$$

where $\Phi^{-1}(\cdot)$ is the inverse function of a cumulative distribution function of the standard normal distribution.

Solving the problem for the crisp case, when the demand is normal random variable with $\mu = 400$, $\sigma = 80$ ($\Delta_1 = \Delta_2 = 0$), we obtaine:

$$Q^* = F_0^{-1}(0{,}579) = 416. \tag{19}$$

But the solution under the fuzzy probabilistic framework has more preferences because of taking into consideration not only previous statistical information but environmental impact through the judgment of experts.

6. Conclusion. In this study the newsvendor model under the hybrid uncertainty has been developed. In particular, the formula of the optimal order quantity when the expected value of demand is a fuzzy random variable has been proposed. It can be useful for the cases when probabilistic nature of demand is known (distribution can be estimated) but the mean demand in the future period is supposed to be chosen because of some environmental processes or a firm policy.

It would be interesting to consider the case of not only a fuzzy mean but a fuzzy mean square error of demand. In general, further research in this direction can be devoted to finding the optimal solution of the problem for different demand distributions with fuzzy parameters. Another problem in this regard is solving a multiperiod problem under hybrid uncertainty with different restrictions or extensions. For example, for practical purposes it is important to consider a case when cost parameters are dependent on order quantity.

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