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## METHODS OF LINEAR ORDERING IN ESTIMATION OF POTENTIAL OF POLISH MARKET OF AGRICULTURAL PROPERTY

*The article presents the use of the taxonomic classification methods in the estimation of Polish market of agricultural property described by the determinants of its development. The authors used the synthetic measure of development in the classification of provinces according to the changeable model. The results of the analysis enabled distinguishing investment areas and the areas deviating from the model requiring further stimulation by local administration.*

*Keywords:* classification analysis; agricultural lands; real estate market; agricultural land price; Poland.

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## МЕТОДИ ЛІНІЙНОГО УПОРЯДКУВАННЯ ПРИ ОЦІНЮВАННІ ПОТЕНЦІАЛУ ПОЛЬСЬКОГО РИНКУ СІЛЬСЬКОГОСПОДАРСЬКОГО МАЙНА

*У статті показано використання методів таксономічної класифікації при оцінюванні польського ринку сільськогосподарського майна, описаного з використанням факторів розвитку. Використано синтетичний параметр розвитку для класифікації провінцій Польщі за моделлю, що змінюється. Результати аналізу дозволили виділити інвестиційні зони та зони зі значними відхиленнями від моделі, що потребують більше уваги з боку органів місцевого самоврядування.*

*Ключові слова:* класифікаційний аналіз; сільськогосподарські землі; ринок нерухомості; ціна на сільськогосподарські землі; Польща.

*Табл. 4. Рис. 1. Форм. 12. Літ. 17.*

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## МЕТОДЫ ЛИНЕЙНОГО УПОРЯДОЧИВАНИЯ В ОЦЕНКЕ ПОТЕНЦИАЛА ПОЛЬСКОГО РЫНКА СЕЛЬСКОХОЗЯЙСТВЕННОЙ СОБСТВЕННОСТИ

*В статье показано использование методов таксономической классификации при оценке польского рынка сельскохозяйственной собственности, описанного с использованием факторов развития. Использован синтетический параметр развития для классификации провинций Польши согласно изменяющейся модели. Результаты анализа позволили выделить инвестиционные зоны и зоны со значительными отклонениями от модели, что требует более пристального внимания со стороны органов местного самоуправления.*

*Ключевые слова:* классификационный анализ; сельскохозяйственные земли; рынок недвижимости; цена на сельскохозяйственные земли; Польша.

### Introduction

Polish real estate market under the market economy can be assessed from the perspective of over 20 years. Market mechanisms in this sector appeared with the systemic changes in the economy in the 1990s. Agricultural lands trade has been an area of particular interest of the state. The fact that before 1990 a large part of agricultur-

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al lands in Poland was the property of the state is still of great importance for the development of the agricultural property market. The privatization of this resource continues through the institution established specifically for this purpose – the Agricultural Property Agency (ANR). Moreover, Polish accession to the European Union was preceded by the legislative actions to protect agricultural land against sales to foreign entities and the exclusion from agricultural production. ANR sells state property through limited and non-limited tenders and through no-bid sale, mainly to leaseholders. The preferential form of sales (including payment by instalments, preferential loans, limiting offers to a selected group of customers) will expire on 31 December 2013. At the same the trade of agricultural properties takes place at the secondary market, which is highly correlated with the trade of real estate belonging to the state.

The research presented in the article shows that for the assessment of Polish market of agricultural property one can use the changes occurring in the scale of provinces by analyzing the determinants of the growth of this market, grouping together those related on the basis of their level of development during economic downturn (Forys, 2007; Forys, 2008). The tools used can support investors in selecting the areas for further investment analyses and decision-makers in looking for the causes of a weaker position of a given province in the national ranking.

#### **The methods of linear ordering in the evaluation of developing real estate market**

The literature on the methods of linear ordering is extensive, however, they are rarely used in the analysis of real estate market. The multidimensional methods of the analysis of the structure of a set of observations may include the discrimination and classification methods (Jajuga, 1993; Milligan, 1989). Among the methods of classification one can distinguish the methods of linear ordering that enable ordering the objects from the best one to the worst one, according to a considered criterion, by creating classes corresponding to the levels of the ordering criterion to which individual objects will be assigned. They encompass the sets of multiple variables and enable the comparison of spatial structures (regional). It requires the selection of an appropriate method of division of the set into disjoint and non-empty groups of similar units according to the subject of consideration. The process of the isolation of homogeneous groups is based on the methods using the features of the synthetic measure of development – SMR (Hellwig, 1968; Borys, 1978; Strahl, 1978; Gordon, 1999; Kendall, 1955; Stevens, 1959; Kruskal, 1964). They are very useful when the subject of a study is the development level of a phenomenon described by a set of many features that can be replaced with one synthetic variable. SMR is determined for each test object, provided that the assumptions are met (Walesiak, 1996):

- the subject of the analysis is a non-empty and finite set of objects and a finite set of variables relevant to the study, presented at least on the ordinal scale, comparable as a result of the normalization;
- there is a synthetic criterion for organizing objects which do not undergo direct measurement and the ordering relation is the greater than the relation.

For each test object a set of diagnostic variables is constructed (the matrix of observations  $X = [x_{ij}]$ , where  $i = 1, \dots, n$  objects and  $j = 1, \dots, m$  variables). Diagnostic features  $x_{ij}$  should be universal, measurable, of high quality, interpretable, with vari-

ous means of interaction, and the method of obtaining them should guarantee economic efficiency of the study. To assess the variability of measurable features in the ratio scale one can use the coefficient of variation (it is required that the variability of the feature is greater than 10%) and the asymmetry coefficient (a significant feature is characterized by a strong right-sided asymmetry). The final choice of variables is preceded by the procedure of the elimination of the variables which are excessively correlated. For this purpose one can apply the Hellwig's parametrical method of variables selection (Hellwig, 1977).

The next step is to standardize the nature of the variables, that is to convert the destimulants into stimulants<sup>3</sup>. The typical formulas for this conversion are as follows:

$$- \text{quotient } x_{ij} = b \times D_{ij}^{-1}, \text{ where } b > 0 \quad (1)$$

$$- \text{differential } x_{ij} = \alpha - c \times D_{ij}^{-1}, \text{ where } c > 0 \quad (2)$$

and  $D_{ij}$  is the value of the  $j$ -th destimulant observed in the  $i$ -th object,  $\alpha$  – a constant (in special cases  $\alpha = 0$  or  $\alpha = \max_i\{D_{ij}\}$ ),  $b$  – a constant (in special cases  $b = \min_i\{D_{ij}\}$  or  $b = 1$ ),  $c$  – a constant (in special cases  $c = 1$ ). The diagnostic variables have to satisfy the condition of comparability, which can be achieved by the normalization transformation, and the general formula can be expressed as:

$$x'_{ij} = b \times x_{ij} + \alpha, \text{ where } b > 0 \quad (3)$$

and the choice of the formula depends on the scale in which variables are measured and the characteristics of the standardized variables distribution (the arithmetic mean, the standard deviation, the range), which should confirm the unification of the order of magnitude and the variability of these variables. One can achieve the comparability of dissimilar variables also through the normalization according to the general formula:

$$x'_{ij} = \left( \frac{x_{ij} - A}{B} \right)^p, \text{ for } j = 1, 2, \dots, n \quad (4)$$

where:  $[x_{ij}, x'_{ij}]$  – the output and the standardized values of the  $i$ -th realization of the  $j$ -th variable;  $n$  – the number of observations;  $A, B, p$  – the parameters which assume different values depending on the method of normalization.

While analyzing stimulants, one can use the formula below:

$$x'_{ij} = \frac{x_{ij} - x_j}{S_j} \quad (5)$$

where  $S_j$  – the standard deviation of the  $j$ -th variable.

These methods require the determination of the weight system ( $w_j$ , where  $j = 1, 2, \dots, m$ ), unless the variables are considered as equally important for the purpose of the study. Regardless the adopted procedure, the determination of weights must meet two basic requirements: they have to be non-negative and their sum has to be equal 1 (or the value of  $m$ ).

The aggregation of the variable values is based on the model or non-model formulas, and their use is reduced by the scale of variables measurement. The model methods assume the existence of a model object, in relation to which taxonomic dis-

<sup>3</sup> The variables proposed in the study are stimulants or destimulants measured in the ratio scale, thus, there is no need for conducting the normalization transformation.

tances of other test objects are calculated  $q_i = d(x_i, x_0)$ . Distinguished distances enable ordering the considered objects from the one which is the most developed (closest to the model) to the object which is the least developed (furthest from the model). In most cases, to measure the distance of the object from the development model the Minkowski's metrics is applied, a special case of which is the Euclidean distance:

$$q_i = \left[ \sum_{j=1}^m \frac{1}{m} (x'_{ij} - x_{0j})^2 \right]^{1/2}, \text{ for } i = 1, 2, \dots, n \quad (6)$$

where  $x'_{ij}$  – the normalized values of the  $j$ -th diagnostic variable for the  $i$ -th object.

The obtained values of the synthetic variable  $q_i$  are converted to achieve the synthetic measure of development  $q'_i$  for the  $i$ -th object:

$$q'_i = q_i / \|Q\|, \text{ for } i = 1, 2, \dots, n \quad (7)$$

where  $\|Q\|$  is the norm of the synthetic variable.

In practice, the formula (7) is converted to determine  $1 - q'_i$ , which leads to the change in variable preferences (for stimulants greater values mean higher level of the examined phenomenon). And as the  $\|Q\|$  norm one most often adopt (Zelias, 1991):

- the maximum  $\|Q\| = \max_i \{q_i\}$  (8)

- the statistical maximum  $\|Q\| = q + \alpha \times S_q$  (9)

- the sum of the values of the synthetic variable  $\|Q\| = \sum_{j=1}^m w_j \times x'_{ij}$  (10)

- the range  $\|Q\| = \max_i \{q_i\} - \min_i \{q_i\}$  (11)

where  $q$  is the arithmetic mean,  $S_q$  – the standard deviation of the synthetic variable, whereas,  $\alpha$  – any value fixed at the level complying with the requirements  $q'_i \in (0; 1)$  and  $q_i > 0$ .

Using the equation (9) one can determine such a value of  $\alpha$  that satisfies the inequality:

$$\alpha \geq \frac{q_{i \max} - \bar{q}}{S_q} \quad (12)$$

where  $q_{i \max}$  is the maximum of the  $q_i$  indicator (eg. =2 due to the rule of 2 or 3 standard deviations). Synthetic measures differ not only in the presented aggregate function but also in the selection of the model procedure (determination of coordinates) and the method of features normalization. A shortcoming of the model formulas is their limited interpretation, meaning merely the interpretations relevant to interval scales, which, however, does not detract from their usefulness to assess the results of the study.

The last step can be grouping the objects in  $k$ -classes characterized by the similarity of the structure, where as a classification criterion one recognizes the determined values of the synthetic variable ( $z_i = x'_{ij}$ ). The number of classes is fixed arbitrarily or using other available methods of grouping. For a changeable model one can use the division into 4 groups according to the formula:

$$\begin{aligned}
 G1: z_i \in \left( \bar{z} + S_z; \max\{z_i\} \right), G2: z_i \in \left( \bar{z}; \bar{z} + S_z \right) \\
 G3: z_i \in \left( \bar{z} - S_z; \bar{z} \right) G4: z_i \in \left( \min\{z_i\}; \bar{z} - S_z \right)
 \end{aligned}
 \tag{13}$$

where  $z_i$  – the synthetic variable;  $\bar{z}$  – the arithmetic mean of the synthetic variable;  $S_z$  – the standard deviation of the synthetic variable.

An advantage of the proposed methods is the possibility to determine the position of the given market compared to other provinces. It enables distinguishing the areas that differ from the development model, require support and acceleration of the development processes. As a result, it is possible to determine the path of development of a given phenomenon, to assess the delays in the development of objects and, above all, to choose the optimal development strategy.

### The classification of regional markets of agricultural properties through the synthetic measure of development

The subjects of grouping are provinces, described by a set of characteristics which are important for the development of the agricultural property market, including social, economic and resource factors. The assumed period of analysis resulted from the established purpose of the study, that is the assessment of the level of development of Polish market during the economic crisis. The study included 16 objects  $O_i$  (provinces)  $i = 1, 2, \dots, 16$  in 2008 and 2010. The adopted period for analysis is explained by the completeness of the statistical data and by the stage of the development of Polish agricultural property market. The included variables are the effects of the access to information and the completeness of the data. The variables, which were constant at the assumed level of accuracy in the analyzed period were eliminated on the assumption that the next steps of analysis will eliminate the variables with low variation. These assumptions lead to the formation of 4 groups of variables (Table 1).

**Table 1. The variables which characterize the determinants of the development of the agricultural property market in Poland**

1. REAL ESTATE RESOURCES	
X11	Agricultural lands (% of the total area)
X12	The average farm area above 1 ha of agricultural land (ha)
X13	Fallow arable lands (% of the total arable lands)
2. SOCIAL POTENTIAL	
X21	Rural population (% of the total population)
X22	Employment in agriculture, hunting, forestry and fishing (% of the total employment)
X23	Number of students per school in rural areas (person)
3. CONDITIONS AND RESULTS OF PRODUCTION	
X31	The use of mineral and chemical fertilizers per 1 ha of agricultural lands (kg/ ha)
X32	The agricultural area per one tractor (ha)
X33	Agricultural lands in the private sector (% of the total agricultural lands)
X34	Agricultural production (in constant prices from the previous year)
X35	Cattle per 100 ha of agricultural lands (unit)
X36	Procurement of agricultural products per 1 ha of agricultural lands (kg)
4. REAL ESTATE TRADING	
X41	The average price of agricultural lands (zł/ ha) ANR
X42	The average price for arable lands in general in the fourth quarter (zł/ ha) GUS
X43	The average price of good lands in the fourth quarter (zł/ ha) GUS
X44	The average price of average lands in the fourth quarter (zł/ ha) GUS
X45	The average price of poor lands in the fourth quarter (zł/ ha) GUS

Source: Based on the GUS and ANR data.

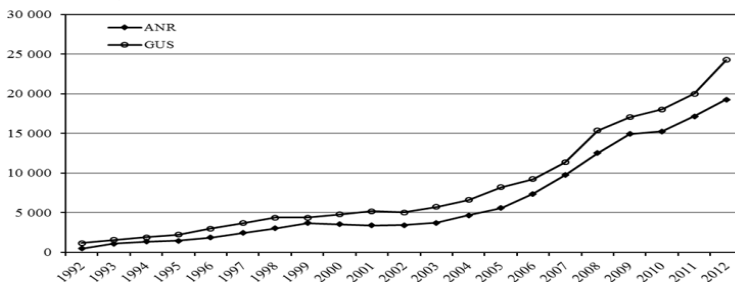
To each variable the symbol  $X_{ij}$  is assigned, where  $i$  stands for the number of the group ( $i = 1, 2, \dots, 4$ ), whereas  $j$  denotes the number of the variable in the group ( $j = 1, 2, \dots, 6$ ).

The first group consists of the variables characterizing agricultural resources, of which the variables  $X_{11}$  and  $X_{12}$  stimulate the market development, while the  $X_{13}$  is a destimulant. Among the researched provinces both in 2008 and 2010, the highest percentage of the agricultural land was in Lodz Province – 72.1% and 71.3% respectively, and the lowest was in Lubuskie Province (41.1% and 40.8%). Farms with the highest average area were located in Warmia-Masuria Province (19.8 ha), whereas the smallest ones in Podkarpacie Province (4.1 ha), which is the result of the centuries-old agrarian relations in the region. In 2010 in West Pomerania Province there was the biggest average farm (33.73 ha) and the smallest one was located in Malopolska Province (4.76 ha).

The second group includes the variables describing the social potential of the regions. The largest percentage of people living in rural areas in 2008 and in 2010 was in Podkarpacie Province (59.30% and 58.54%, respectively). The largest percentage of the population working in agriculture, hunting, forestry and fishing was in Lublin Province (36.2% in 2008 and 28.60% in 2010). In 2008 the largest number of pupils per one school were in Pomerania Province (130 children, and then 129 children in 2010).

In the third group, there are the variables concerning conditions and effects, particularly of crop production. In the two periods under study, farmers in Opole Province used the biggest amount of fertilizers (196 kg/ha and 184 kg/ha, respectively) and the smallest amount was noted in 2008 in Podkarpacie Province (66 kg/ha), and in 2010 in Malopolska Province (63 kg/ha). The largest area to plow with one tractor had the farmers living in West Pomerania (25.7 ha in 2008, 33.13 ha in 2010), whereas the least area for both years was in Malopolska Province (less than 6 ha).

The fourth group consists of the nominal transaction prices, divided into agricultural land sold by the ANR and arable lands sold at the secondary market. The latter information is gathered by GUS and presented with the division into 3 groups of arable lands: good (classes I, II, IIIa), average (classes IIIb and IV) and poor (classes V and VI). Since the information by provinces is published quarterly, the data used for the analysis is the data of the fourth quarter of each year. The figure below shows the change in transaction prices of the ANR resources and at the secondary market (according to GUS) in 1992–2012.



Source: Constructed by the authors basing on the GUS and ANR data.

Figure 1. The dynamics of the nominal price of agricultural lands in Poland according to the ANR and GUS, 1992–2012 (zl/ha)

Taking into consideration the dynamics of prices for agricultural lands in Poland one can notice the 3 phases of growth: 1992–1998, 1989–2008 and 2008–2012 (Figure 1). The first one covers the period of ownership transformations and political changes in Poland. The second, inter alia, includes the accession of Poland to the European Union, restrictions on the acquisition of agricultural lands by foreigners and the boom at the real estate market. The last phase of growth covers the years of economic downturn and the slump in the real estate market. In the 2008–2012 period there are two distinctive inflection points on the curve of transaction prices for agricultural lands: 2008 and 2010, which are the subject of this study.

In the analyzed periods, for all the chosen variables, some basic descriptive characteristics were established –  $\bar{x}, S(x), V_S$ , that were used for the determination of the final diagnostics variables. The variables underwent selection because of the lack of correlation and the value of the variation coefficient, for which  $V_j < 10\%$  were removed from the set. They were considered quasi-permanent and not having discriminating properties. An additional discriminatory criterion was the asymmetry coefficient, which pointed to the asymmetry of the distribution of the variable and the same direction in the analyzed years. Finally, as the final set of diagnostic variables, two disjoint sets of the variable stimulants and destimulants were distinguished:

$$X_S = \{X_{11}, X_{12}, X_{22}, X_{34}, X_{35}, X_{36}, X_{42}, X_{43}\};$$

$$X_D = \{X_{23}\}.$$

Considering the criteria of the selection of the set of variables described above, for further analysis we have chosen the variables, for which the basic statistics in the analyzed period are presented in Table 2.

Table 2. Selected descriptive statistics of the diagnostic variables characterizing the test objects

Variables	X <sub>11</sub>	X <sub>12</sub>	X <sub>22</sub>	X <sub>23</sub>	X <sub>34</sub>	X <sub>35</sub>	X <sub>36</sub>	X <sub>42</sub>	X <sub>43</sub>
2008									
$\bar{x}$	50,54	10,72	17,18	100,88	3930,75	32,46	447,19	14702,25	17861,75
S(x)	9,20	4,74	9,15	13,20	914,78	15,69	329,32	5001,45	6664,40
V <sub>S</sub>	18,20	44,26	53,23	13,09	23,27	48,34	73,64	34,02	37,31
A <sub>S</sub>	-0,13	0,33	0,78	0,56	1,22	0,96	0,52	0,67	0,90
2010									
$\bar{x}$	59,67	14,87	13,79	98,56	4604,75	31,99	541,69	17505,43	21521,80
S(x)	8,34	8,24	7,08	13,17	1084,10	19,01	406,70	5338,85	6853,96
V <sub>S</sub>	13,97	55,41	51,30	13,37	23,54	59,41	75,08	30,50	31,85
A <sub>S</sub>	-0,59	0,68	0,74	0,65	0,88	1,34	0,63	0,44	0,37

$\bar{x}$  – the arithmetic mean, S(x) – the standard deviation, A<sub>S</sub> – the asymmetry coefficient, V<sub>S</sub> – the classical coefficient of variation (%).

Source: Authors' calculations.

The next step to determine the synthetic measure of development SMR, which uses the formula (5) of the standardization of variables and (12) for determining the parameter  $\alpha$  in the formula (9), for  $x_{0j}$  – the maximum value of the diagnostic variable  $X_j$ , determined in each given year (changeable model). The values of the obtained synthetic measures were converted according to the formula (7) to the form  $z_j = 1 - (d_j/d_0)$ , which means that the closer to 1 the value of  $z_j$  is, the higher is the development level of the analyzed market.

**Table 3. The values of the synthetic measure of the development of the real estate market by provinces, 2008 and 2010**

Province	Object	2008	2010
Lower Silesia Province	O <sub>1</sub>	0,1571	0,1524
Kujawy-Pomerania Province	O <sub>2</sub>	0,4700	0,4180
Lublin Province	O <sub>3</sub>	0,2246	0,2160
Lubuskie Province	O <sub>4</sub>	0,0000	0,0000
Lodz Province	O <sub>5</sub>	0,3152	0,3147
Malopolska Province	O <sub>6</sub>	0,1521	0,1188
Mazovia Province	O <sub>7</sub>	0,3181	0,3255
Opole Province	O <sub>8</sub>	0,3080	0,3750
Podkarpackie Province	O <sub>9</sub>	0,0233	0,0416
Podlasie Province	O <sub>10</sub>	0,4051	0,3954
Pomerania Province	O <sub>11</sub>	0,1565	0,1676
Silesia Province	O <sub>12</sub>	0,0339	0,1099
Świętokrzyskie Province	O <sub>13</sub>	0,1679	0,1101
Warmia-Masuria Province	O <sub>14</sub>	0,3190	0,2386
Wielkopolska Province	O <sub>15</sub>	0,3796	0,5020
West Pomerania Province	O <sub>16</sub>	0,0732	0,0796

Source: Authors' calculations.

The grouping of the objects with the model object changing each year was carried out according to the formula (13). The most interesting findings relate to the shifting of provinces in the extreme groups. In 2008 in the I group of the highest value of the SMR there were 3 provinces, in 2010 the additional fourth was Opole Province (Table 4). The shift of the Opole Province in the I group was influenced by the increase in the proportion of agricultural lands and agricultural employment. This group consists of typically agricultural provinces or those with developed agricultural culture. The IV group in 2010 compared to 2008 lost Silesia and West Pomerania, for which the SMR indicators increased, primarily as a result of improvements in the agricultural production results.

**Table 4. The classification of provinces by the SMR values, 2008 and 2010**

GROUP	2008	2010
I	O <sub>2</sub> , O <sub>10</sub> , O <sub>15</sub>	O <sub>2</sub> , O <sub>8</sub> , O <sub>10</sub> , O <sub>15</sub>
II	O <sub>3</sub> , O <sub>5</sub> , O <sub>7</sub> , O <sub>8</sub> , O <sub>14</sub>	O <sub>5</sub> , O <sub>7</sub> , O <sub>14</sub>
III	O <sub>1</sub> , O <sub>6</sub> , O <sub>11</sub> , O <sub>13</sub>	O <sub>1</sub> , O <sub>3</sub> , O <sub>6</sub> , O <sub>11</sub> , O <sub>12</sub> , O <sub>13</sub> , O <sub>16</sub>
IV	O <sub>4</sub> , O <sub>9</sub> , O <sub>12</sub> , O <sub>16</sub>	O <sub>4</sub> , O <sub>9</sub>

Source: Authors' calculations.

The linear ordering performed using diagnostic variables that determine the development of the agricultural property market, made it possible to rank the provinces (objects) on the basis of the development level of the market. One may notice some similarities in the classification of the objects in the following years, with the greatest similarity shown in the I group. The biggest shifts of the objects between the groups took place in the III group. In the II and IV groups there were a few shifts, and the most of them had the extreme SMR values in the group.

### Conclusions

The linear ordering performed using the diagnostic variables that determine the development of the agricultural property market made it possible to rank the



provinces (objects) on the basis of the market development level. The provinces in the I group have the highest level of development of the market, measured by both prices and real estate resources. The II and III groups cover mostly the provinces, where the dominant role at the real estate market play urban agglomerations (provincial capitals) setting the pace of the development of the real estate market, and the typically agricultural province but with an untapped potential. The last group consists of the provinces with the weakest state of agriculture (Podkarpacie, Silesia) or with good areas but with untapped potential (West Pomerania). The latter, in terms of investments, have the greatest potential of growth at the agricultural property market.

Determined for every test object, SMR values enable a detailed comparison of the created rankings over time and the assessment of the regional markets of agricultural properties considering the development phases of Polish market. In the article some of them are mentioned, indicating the spatial differentiation in the market development levels. Further research at the level of counties (or municipalities) may distinguish the leading regions in the province and assess, considering real estate resources, the direction of their development. The applied method can also be used to compare countries through the analysis of the level of development of various markets.

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Стаття надійшла до редакції 17.07.2013.