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FORMATION OF NEW TYPES OF BONUS CERTIFICATES

This paper is dealing with the bonus certificates formation. The authors found an alternative investment to the purchase of bonus certificate without a cap, i.e. the combination of a buying underlying asset and a purchase of down and knock-out put options, and to the purchase of bonus certificate with a cap, i.e. a combination of a buying underlying asset, a purchase of down and knock-out put barrier options, at the same time, a sale of call vanilla options. Bonus certificates provide protection in a case of stagnation, respectively growth in the market. The main aim is to present new types of bonus certificates with a cap and without a cap formed by down and knock-in put options. New types of bonus certificates would be useful for the investors who expect a significant market decrease or increase.

Keywords: investment certificate; bonus certificate; down and knock-out put option; down and knock-in put option.

JEL Classification: G15, G11.

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ФОРМУВАННЯ НОВИХ ТИПІВ БОНУСНИХ СЕРТИФІКАТІВ

У статті описано процес формування нових типів бонусних сертифікатів. Автори знайшли альтернативний спосіб інвестування з придбанням бонусних сертифікатів без кепи, зокрема, комбінуючи придбання базисного активу та пут-опціонів з нижньою межею. Також розглянуто придбання бонусного сертифікату з кепом, тобто комбінація придбання базисного активу та бар'єрних опціонів з нижньою межею, продаючи одночасно ванільні опціони. Бонусні сертифікати можуть захистити інвесторів як від стагнації на ринку, так і від раптового його зростання. Представлено нові типи бонусних сертифікатів з кепами та без них, сформовані на основі пут-опціонів з нижньою межею та опціонів пок-ін. Нові типи бонусних сертифікатів можуть бути корисні інвесторам, які очікують на суттєве зниження або навпаки стрибок на ринку.

Ключові слова: інвестиційний сертифікат; бонусний сертифікат; пут-опціон з нижньою межею; пут-опціон пок-ін.

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ФОРМИРОВАНИЕ НОВЫХ ТИПОВ БОНУСНЫХ СЕРТИФИКАТОВ

В статье описан процесс формирования новых типов бонусных сертификатов. Авторы нашли альтернативный способ инвестирования с приобретением бонусных сертификатов без кэпа, в частности, комбинируя приобретение базисного актива и пут-опционов с нижней границей. Также рассмотрена покупка бонусного сертификата с кэпом, т. е. комбинация покупки базисного актива и барьерных опционов с нижней границей, одновременно продавая ванильные опционы. Бонусные сертификаты могут защитить инвесторов как от стагнации на рынке, так и от внезапного его роста. Представлены новые типы бонусных сертификатов с кэпами и без них, сформированные на основе пут-опционов с нижней границей и опционов пок-ин. Новые типы бонусных сертификатов могут быть полезны инвесторам, ожидающим существенного понижения или наоборот скачка на рынке.

Ключевые слова: инвестиционный сертификат; бонусный сертификат; пут-опцион с нижней границей; пут-опцион пок-ин.

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1. Introduction

Investment certificate is a security, the value of which is derived from the value of an underlying asset. The underlying asset is usually a share in a company, a basket of shares, or an entire index etc. In order to be accessible to small investors, it represents a proportional sample, typically 1:100 or 1:1000. Since an optional number of such certificates may be purchased, this instrument is also suitable for all investors.

Bonus certificate is a certificate with a partial risk protection. It is suitable mainly when stagnation is estimated. As we will observe later, it is formed using barrier options, specifically buying down and knock-out put barrier options. Contrary to airbag, discount and speed certificates (see Soltes M., Soltes V., 2007; Soltes M., 2010), how the underlying asset price has changed till that date is significant for determining a profit profile. These theoretical results are robust to our analysis. A key difference from the previous studies, for example (Soltes V., 2011), (Soltes M., 2012) is that this paper is concentrated on proposing new types of bonus certificates which would be suitable for conservative investors. Further, the alternative investment with the same profit profile as a certificate is found. In the proposal of bonus certificate construction barrier options are used.

2. Barrier options

European vanilla option yields depend only on the underlying asset value at the maturity (Haug, 1998; Hull, 2008; Chorafas, 2008; Kolb, 2007). Less known, but much more sophisticated are barrier options. Barrier options are characterized by the fact that except for the strike price they have another important price, the so-called "barrier". It is a very important limit, as if the underlying asset value, at any time, touches the barrier, or even crosses it, the option is deactivated (in which case it becomes a knock-out option) or activated (in which case it is a knock-in option). Barrier can be placed below the spot price of the underlying asset at time of the option issue (in which case it becomes a down and knock-out or a down and knock-in option) or above the spot price (in which case it is an up and knock-out or up and knock-in option). Therefore, there are altogether 16 basic option positions of barrier options with one barrier. These barrier options have been studied by several authors (Kundrikova, 2008; Nelken, 1996; Rubinstein, 1991; Weert, 2008; Zhang, 1998). Ritchken (1995), Rubinstein and Reiner (1991), Ye (2009) dealing with barrier option pricing. In terms of our analysis, profit functions of all the basic positions of barrier options with one barrier are important. In the construction of a bonus certificate we are going to use a buying of down and knock-out/in put options.

Buying of down and knock-out put option gives a right to sell a particular underlying asset at a strike price E at time T if the option is not deactivated, that is, the price of an underlying asset does not exceed the predetermined barrier B during the life of the option which the following condition represent:

$$\min_{0 \leq t \leq T} (S_t) > B. \quad (2.1)$$

The buyer of a down and knock-out put option have the obligation to pay an option premium p_{BDO} per option at the time of contract conclusion.

Let us denote the underlying spot price at expiration time S_T , the profit function from buying n down and knock-out put option is:

$$P(S_T) = \begin{cases} -n(S_T - E + \overline{p_{BDO}}) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < E, \\ -np_{BDO} & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < E, \\ -np_{BDO} & \text{if } S_T \geq E. \end{cases} \quad (2.2)$$

A down and knock-in put option is activated if the price of an underlying asset exceeds predetermines the barrier level during the life of the option.

The profit function from buying n down and knock-in put option with premium p_{BDI} is expressed by as (2.3):

$$P(S_T) = \begin{cases} -n(S_T - E + \overline{p_{BDI}}) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < E, \\ -np_{BDI} & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < E, \\ -np_{BDI} & \text{if } S_T \geq E. \end{cases} \quad (2.3)$$

3. Analysis of bonus certificates

Bonus certificates are appropriate for retail and institutional investors who expect stagnation or growth at the market. At maturity time of a bonus certificate there may be 3 alternatives of underlying price development. If the barrier level is not reached by the underlying asset price during the time to maturity and the underlying asset price at time to maturity is below a bonus level, then the investor will obtain fixed profit in the amount of a bonus level. If the barrier level is not reached by the underlying asset price during time to maturity and the underlying asset price at time to maturity is under the a bonus level, then the investor participates in price increase. In the case of reaching barrier, the bonus certificate is linear, i.e. its profit respectively loss replicates underlying price development.

Every bonus certificate at its issue has defined basic parameters, i.e. subscription ratio, maturity date, barrier level, bonus level and cap level. The first parameter is the subscription ratio (ρ). Most often it is $\rho = 0.01$ or $\rho = 0.001$. This parameter is selected in a way which enables the price of one bonus certificate to be in a low order position to the price or value of the underlying asset. If an index is, for example, of a value of 11 000 points, then most probably $\rho = 0.001$ and the bonus certificate price is 11 units. The certificate price is thus available even for small investors identified as retail clients.

The maturity date (T) of a bonus certificate is another parameter. It is usually between 2 to 6 years. It is the date on which trading with the given bonus certificate terminates and there is a settlement between an issuer and an investor.

Barrier level (B) is the limit which the underlying asset price must not reach or fall below from the issue date to maturity. In this case, at maturity the investor will be paid at minimum the bonus level (B_L). If the underlying asset value falls below the barrier level, then this protection is cancelled and the investor participates in the loss in full, just as with the linear certificate. As bonus certificate is almost always more expensive than the linear one, the loss is even greater. The barrier level is generally determined below the bonus level.

The investor should select a bonus certificate with another parameter, the so-called cap level (C). Cap is the highest possible participation in the growth of the

underlying asset value. If the underlying asset value at maturity date of the bonus certificate is higher than cap, then the investor receives the cap level from the issuer. It is virtually the maximum value which the investor can get from the bonus certificate at maturity.

The profit function of n bonus certificate without a cap at expiration time is

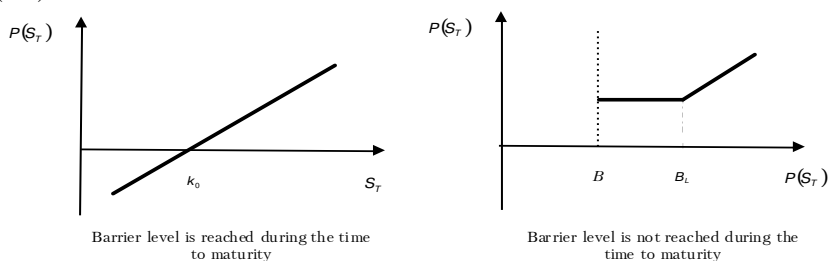
$$P(S_T) = \begin{cases} n(\rho B_L - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ n(\rho S_T - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ n(\rho S_T - k_0) & \text{if } S_T \geq B_L, \end{cases} \quad (3.1)$$

where k_0 is the purchase price of a certificate;

S_T is the spot price of the underlying asset at expiration time;

n is the number of certificates.

Figure 1 depicts the profit function of a bonus certificate without a cap, i.e. function (3.1).



Source: Constructed by the authors.

Figure 1. Profit function of a bonus certificate without a cap

We can see that the profit profile depends on an underlying asset price at expiration time and an underlying price development during the time to maturity, i.e. barrier level reaching during the time to maturity. Based on this, the following alternative investment strategy using barrier options is found.

The alternative investment is created by the combination of buying n pieces of an underlying asset and a purchase of n down and knock-out put options on the same asset with a strike price equal to the bonus level, a same barrier level and an underlying asset as the bonus certificate. Down and knock-out put option is a right to sell the underlying asset. The barrier is set below the current price of the underlying asset. If the underlying asset price over the term of the option touches or breaks the barrier, the option expires. We can observe an analogy with the bonus certificate, where investor in reaching or breaking the barrier loses entitlement to the payment of bonus.

By the sum of the profit function of n pieces of underlying asset

$$P(S_T) = n(S_T - S_0) \quad (3.2)$$

where S_0 is the spot price of an underlying asset at the time of a purchase, and the profit function from buying n down and knock-out put options (2.2) we get

$$P(S_T) = \begin{cases} n(B_L - S_0 - \overline{p_{BDO}}) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ n(S_T - S_0 - \overline{p_{BDO}}) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ n(S_T - S_0 - \overline{p_{BDO}}) & \text{if } S_T \geq B_L. \end{cases} \quad (3.3)$$

In this case, only assets with yield dividends are the underlying assets. The present value of dividends PV_D is used to pay the option premium p_{BDO} for the purchase of a down and knock-out put option.

The profit function of the alternative investment with the subscription ratio ρ and $PV_D = p_{BDO}$ has the following form

$$P(S_T) = \begin{cases} n\rho(B_L - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ n\rho(S_T - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ n\rho(S_T - S_0) & \text{if } S_T \geq B_L. \end{cases} \quad (3.4)$$

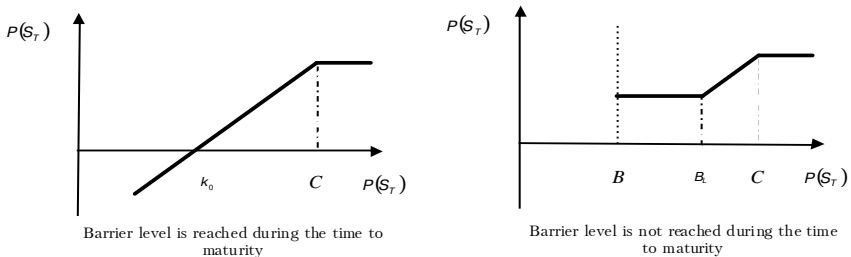
Let us suppose $\rho S_0 = k_0$, then the profit function of the alternative investment opportunity (3.4) is identical with the profit profile of the bonus certificate without a cap (3.1). There are significantly fewer bonus certificates without a cap than there are bonus certificates with a cap. The reason lies in the fact that it is necessary to pay an option premium in the time of investment from underlying assets dividends.

The characteristics of a cap provide an investor with information on the maximum price of a bonus certificate at maturity, which the investor can get even in the case $S_T > B_L$. Cap level is generally higher than the bonus level, i.e. $C > B_L$. There are significantly more bonus certificates which also have a cap. Such bonus certificates are issued also to underlying assets without dividends.

The profit function of a bonus certificate with a cap at expiration time is as follows

$$P(S_T) = \begin{cases} n(\rho B_L - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ n(\rho S_T - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ n(\rho S_T - k_0) & \text{if } B_L \leq S_T < C, \\ n(\rho C - k_0) & \text{if } S_T \geq C. \end{cases} \quad (3.5)$$

Figure 2 shows the graphs of a bonus certificate with a cap.



Source: Constructed by the authors

Figure 2. Profit function of a bonus certificate with a cap

Let us construct an alternative investment position by buying n pieces of underlying asset at an actual price S_0 per piece, further by a purchase of n down and knock-out put barrier options with a strike price B_L , a barrier B , premium p_{BDO} and, at the same time, by a sale of n call vanilla options with a higher strike price C , premium p_S .

The profit function from the sale of n call vanilla options with a strike price C (Soltes, 2002) has the following form:

$$P(S_T) = \begin{cases} np_s & \text{if } S_T < C, \\ -n(S_T - C - p_s) & \text{if } S_T \geq C. \end{cases} \quad (3.6)$$

It can be proven that the profit function from the alternative investment expressed as the sum of individual functions (3.2), (2.2) and (3.6) at the subscription ratio ρ using the conditions $\rho S_0 = k_0$ and $np_{BDO} = np_s$ is

$$P(S_T) = \begin{cases} np(B_L - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ np(S_T - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ np(S_T - S_0) & \text{if } B_L \leq S_T < C, \\ np(C - S_0) & \text{if } S_T \geq C. \end{cases} \quad (3.7)$$

Bonus certificates are suitable for investors at moderately declining or increasing markets. If the underlying asset value stays above the barrier, then the investor is guaranteed a bonus expressed by the bonus level, even though the underlying asset value can be even significantly lower at maturity. Investment risk lies in crossing the barrier. In such a case, bonus is no longer guaranteed (barrier option is deactivated).

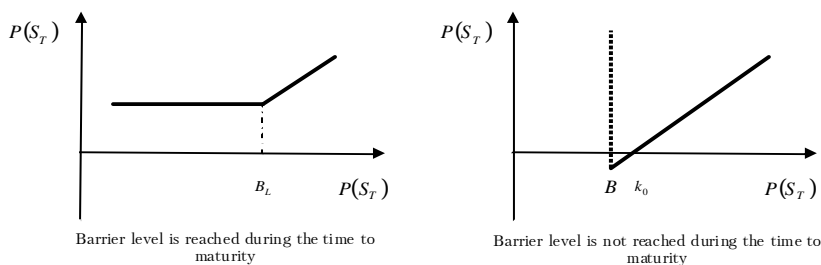
4. New types of bonus certificates

Let us construct new types of bonus certificates with the barrier level setting below the current price of the underlying asset. In this case, an investor will obtain the bonus payment if the condition of barrier reaching is fulfilled.

The profit function of a new type of the bonus certificate without a cap at expiration time is

$$P(S_T) = \begin{cases} n(\rho B_L - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ n(\rho S_T - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ n(\rho S_T - k_0) & \text{if } S_T \geq B_L. \end{cases} \quad (4.1)$$

The graphs of a new type of bonus certificate without a cap are shown in Figure 3.



Source: Constructed by the authors.

Figure 3. Profit function of a new type of bonus certificate without a cap

If the underlying asset price during the time to maturity touches or breaks the barrier and the spot price at the time to maturity is below the bonus level, the investor obtains a bonus. If the underlying asset price during the time to maturity touches or breaks the barrier and the spot price at the time to maturity is under a bonus level, the investor participates in price increase. If the underlying asset price during the time to maturity does not break the barrier, the bonus certificate copies underlying asset yields linearly.

The alternative investment is created by the combination of buying n pieces of an underlying asset with a spot price at the time of investment S_0 and the purchase of down and knock-in put options on the same underlying asset with barrier B , bonus level B_L , premium p_{BDI} .

We easily get the profit function of the alternative investment possibility (4.2) as the sum of functions (3.2) and (2.3). After modification we have the following function:

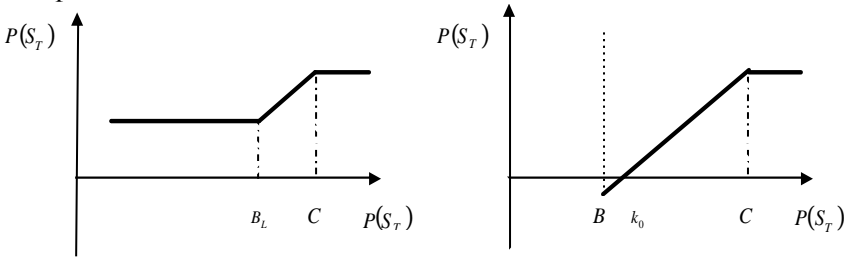
$$P(S_T) = \begin{cases} np(B_L - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ np(S_T - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ np(S_T - S_0) & \text{if } S_T \geq B, \end{cases} \quad (4.2)$$

where p is the subscription ratio, $PV_D = p_{BDI}$, $pS_0 = k_0$.

The profit function of a new type of bonus certificate with a cap at expiration time is:

$$P(S_T) = \begin{cases} n(pB_L - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ n(pS_T - k_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ n(pS_T - k_0) & \text{if } B_L \leq S_T < C, \\ n(pC - k_0) & \text{if } S_T \geq C. \end{cases} \quad (4.3)$$

By analogy, Figure 4 depicts the profit function of a new type of bonus certificate with a cap.



Barrier level is reached during the time to maturity Barrier level is not reached during the time to maturity
Source: Constructed by the authors.

Figure 4. Profit function of a new type of a bonus certificate with a cap

It can be proven, that the profit function from the alternative investment expressed as the sum of individual functions (3.2), (2.3) and (3.6) has the following form:

$$P(S_T) = \begin{cases} np(B_L - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) \leq B \wedge S_T < B_L, \\ np(S_T - S_0) & \text{if } \min_{0 \leq t \leq T}(S_t) > B \wedge S_T < B_L, \\ np(S_T - S_0) & \text{if } B_L \leq S_T < C, \\ np(C - S_0) & \text{if } S_T \geq C, \end{cases} \quad (4.4)$$

$np_{BDI} = np_S$.

New types of bonus certificates are suitable for the investors who expect a decline or growth at the market. If the underlying asset value falls below the barrier, then the investor is guaranteed a bonus. Due to this feature characteristic, in most cases these

new types of bonus certificates proposed by us would be more expensive than the classical bonus certificates. On the other hand, the maximum loss is limited by the barrier level. In the case of crossing the barrier, bonus is guaranteed (barrier option is activated).

4. Conclusion

The development at financial markets, especially in the past decade, has reminded investors of the fact that stock markets do rise in the long term, but trading is very risky. Investors will long remember the stock indices crash in 2008 reaching 50–70%, just as they will remember the preceding crash in 2000. Many investors have begun, more than ever, looking for less risky instruments. On the contrary, some investors continue to develop investment certificates.

It can be stated that for every estimated development of an asset (growth, fall, stagnation), or for every attitude to risk (conservative or aggressive) there is a suitable kind of certificates. At moderately declining market, bonus certificates may be used for investment. Bonus certificates are appropriate for conservative investors who expect stagnation or growth at the market. The bonus certificate has established barrier and bonus level at the time of buying. If the barrier level is not reached by the underlying asset price during the time to maturity, then the investor will obtain fixed profit in the amount of bonus level or he will participate in price increase (only in the case of bonus certificates without a cap). In the case of reaching barrier, the bonus certificate is linear, i.e. its profit respectively loss replicates the underlying price development.

In the context of bonus certificates formation these are the sophisticated products which use financial derivatives, specifically barrier or vanilla options. Investors need to know the nature of such formation. In this paper, we used the interesting method based on the derivation of profit profile depending on the underlying asset development in the construction analysis. We specified the alternative investment opportunity to the purchase of bonus certificate without cap, i.e. a combination of a buying underlying asset and a purchase of down and knock-out put options, and to the purchase of bonus certificate with cap, i.e. a combination of a buying underlying asset, a purchase of down and knock-out put barrier options, at the same time, a sale of call vanilla options. The greatest contribution of this paper is the proposal of the new types of bonus certificates without cap and with cap formed by down and knock-in put options. These new types of bonus certificates are suitable for the investors who expect a significant decrease, or an increase at the market (in the case of bonus certificate without a cap).

The performed analysis should help to understand the construction of bonus certificates in detail, as well as the influence of individual factors. The following propose of new kinds of this certificate could be useful for the issuers of investment certificates from the methodological as well as practical perspective.

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