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PERFORMANCE EVALUATION OF BLACK-SCHOLES AND GARCH MODELS ON USD/TRY AND EUR/TRY CALL OPTIONS

This paper empirically studies and compares the performances of the results of Black-Scholes and GARCH-M call option pricing models using the data on USD/TRY and EUR/TRY parities. Initial basic examination of the data set impresses the traces of heteroskedasticity effects and an ARCH model might give a better pricing model. Statistical tests are performed to reveal which model provides a closer result to actual prices. The estimated and simulated results indicate that the Black-Scholes model performs better than the GARCH model as the overall result.

Keywords: *option pricing; Black-Scholes model; GARCH model; Monte Carlo simulation.*
JEL Classification: G13.

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ОЦІНЮВАННЯ ЕФЕКТИВНОСТІ МОДЕЛЕЙ БЛЕКА-ШОУЛЗА ТА GARCH ДЛЯ КОЛ-ОПЦІОНІВ USD-TRY ТА EUR-TRY

У статті проведено порівняння результатів моделювання за Блеком-Шоулзом та GARCH для кол-опціонів за даними по парах валют USD-TRY та EUR-TRY. Аналіз виявив неоднорідність даних, при цьому модель ARCH демонструє кращі результати. Статистичні тести застосовано, щоб дізнатись, результати якого моделювання є найбільш близькими до реальних ринкових показників. Проведені розрахунки виявили, що модель Блека-Шоулза має результати кращі, ніж GARCH.

Ключові слова: *ціноутворення на опціони; модель Блека-Шоулза; модель GARCH; моделювання Монте-Карло.*
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ОЦЕНКА ЭФФЕКТИВНОСТИ МОДЕЛЕЙ БЛЕКА-ШОУЛЗА И GARCH ДЛЯ КОЛЛ-ОПЦИОНОВ USD-TRY И EUR-TRY

В статье проведено сравнение результатов моделирования по Блеку-Шоулзу и GARCH для колл-опционов по данным о парах валют USD-TRY и EUR-TRY. Анализ выявил неоднородность данных, при этом модель ARCH демонстрирует лучшие результаты. Стастистические тесты применены, чтобы узнать, результаты какого моделирования наиболее близки к реальным рыночным показателям. Проведенные расчеты показали, что модель Блека-Шоулза имеет лучшие результаты, чем GARCH.

Ключевые слова: *ценообразование на опционы; модель Блека-Шоулза; модель GARCH; моделирование Монте-Карло.*

Introduction. Black-Scholes and Merton (1973) introduced the shaping initial steps for option pricing. They derived a partial differential equation which governs the price of the option over time. Their main idea was that if options are correctly priced at a market, it should not be possible to make profits by arbitrage. Using this principle, a theoretical valuation formula for options is derived. The model has been used in pricing the currency options since the introduction.

Nevertheless, the results of empirical studies and evaluation of market observations exposed the biases in the Black-Scholes model. MacBeth and Mervile (1980) tested the Cox-Ross (1975) option model and for constant elasticity of variance dif-

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fusion process against the constant variance option model of Black-Scholes and found out that the Cox-Ross model outperformed the Black-Scholes one.

Introduction of stochastic interest rates by Amin and Jarrow (1991) and introduction of stochastic volatility model by Heston and Nandi (2000) were some improvements introduced to solve the biases. Amin and Jarrow built a general framework to price derivatives on FX using the Heath (1987) model of the term structure. They found closed form solutions for European options on currencies and currency futures with deterministic volatility functions. Heston (1993) used a new technique to derive a closed-form solution for the price of a European call option on an asset with stochastic volatility. His model allows arbitrary correlation between volatility and spot-asset returns. Heston introduced stochastic interest rates and showed the application for bond and foreign currency options. Hilliard, Madura and Tucker (1991) developed a currency option-pricing model under stochastic interest rates when interest rate parity holds. Their model assumes that domestic and foreign bond prices have local variances that depend only on time. The option model is tested empirically and the results exposed that the stochastic interest rate model revealed more accurate pricing than the model employing constant interest rates.

The issue of volatility term structure and skewness has been examined by Heynen, Kemna and Vorst (1994). Their study has shown that the term structure of the implied volatilities can be explained by the GARCH option pricing model. Duan (1999) succeeded in fitting the volatility smile and the term structure of implied volatilities by implementing the GARCH option pricing.

Noh, Engle and Kane (1994) predicted the future mean volatility using a GARCH model (Bollerslev, 1986) on past returns and used it in the Black-Scholes model. Bollerslev, Chou and Kroner (1992) made a review of some developments in the formulation of ARCH models and gave empirical applications using financial data and discussed the pricing of derivative assets. Duan (1995) proposed a new model for pricing options based on the exchange rate as underlying which follows a GARCH process. Duan's article develops an option pricing model and its corresponding delta formula in the context of the GARCH asset return process, which utilizes the locally risk-neutral valuation relationship.

The GARCH option pricing model denotes changes in the conditional volatility of the underlying asset. Duan and Wei's paper (1999) generalizes the GARCH option pricing methodology of Duan (1995) in the two-country settings by assuming a bivariate nonlinear GARCH system for exchange rate and foreign asset price. Heston and Nandi's paper (2000) revealed a closed-form option valuation formula for a spot asset whose variance follows a GARCH process that can be correlated with the returns of spot assets.

Harikumar; Boyrie and Pak (2004) empirically examined the performance of Black-Scholes and GARCH-M call option pricing models using call options data for several currencies. They found that Black-Scholes model outperforms GARCH models.

Apart from all theoretical studies, there has been not enough comprehensive empirical study on foreign exchange returns. Eventually, we compared the performance of Black-Scholes and GARCH option pricing models in estimating call option prices on USD/TRY and EUR/TRY, which have not been examined by using GARCH pricing perspective.

Currency and Volatility data. Daily exchange rates were obtained from the Reuters and monthly implied and realized volatility were acquired from Bloomberg. EUR/TRY and USD/TRY exchange rates are taken as currency units per EUR and USD. Data sample is taken for the period starting from 2 March, 2006 and ending 10 November, 2011 (1461 observations per currency) for USD/TRY and EUR/TRY. The historical volatilities were derived from Bloomberg but it could be calculated by the following procedure: $n + 1$ observed prices $S_0, S_1, \ldots, S_n$ at the end of the ith time interval, $i = 0, 1, \ldots, n$ are taken for calculation and τ is the length of time interval in years.The continuously compounded return in each time interval is calculated as:

$$u_i = \ln\left(\frac{S_i}{S_{i-1}}\right) \quad i = 1, 2, \ldots, n+1. \tag{1}$$

The standard deviation is calculated by using the average of the u_i's:

$$S = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left[u_i - avr(u)\right]^2} \tag{2}$$

and the estimate of historical volatility is:

$$\hat{\sigma} = \frac{s}{\sqrt{\tau}}. \tag{3}$$

There is no exchange traded market for currency options in Turkey. The values of options with at-the-money strike can be calculated by using the Black-Scholes pricing model.

Valuation of Options. The Black-Scholes formulas for pricing European currency call and put options are:

$$c = se^{-r_f t}N(d_1) - xe^{-rt}N(d_2) \tag{4}$$

Eq. (4) is the closed from solution for pricing European currency call options, s denotes the spot price of the currency pair and x denotes the strike price. The time value which is taken in years until the expiration of the option is given by t.

$$p = -xe^{-rt}N(-d_2) - se^{-r_f t}N(-d_1) \tag{5}$$

Eq. (5) is the related closed from solution for pricing European currency put options. The $N(.)$ stands for the cumulative normal distribution, where

$$d_1 = \frac{\ln\left(\frac{s}{x}\right) + \left(r - r_f + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{t}} \quad d_2 = \frac{\ln\left(\frac{s}{x}\right) + \left(r - r_f - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t}.$$

σ denotes the volatility estimate of the underlying currency pair.

The assumptions of Black-Scholes model are restrictive and open to violation in practice. There are 5 assumptions of the model:

- All arbitrage opportunities are eliminated.
- There are no taxes and transaction costs.
- Trading is continuous, no dividend is paid.
- Investors borrow and lend unlimited amounts at the risk free rate constant over the life of the option.
- Prices follow lognormal distribution with a constant expected return and variance.

From the previous assumption, the model is derived for a geometric Brownian motion with a constant volatility. Stochastic moves in volatility may cause deviations from realistic estimations, which has been the motivation of our study. There have been many extensions to the Black-Scholes model, yet in this paper the extensions are not taken into consideration.

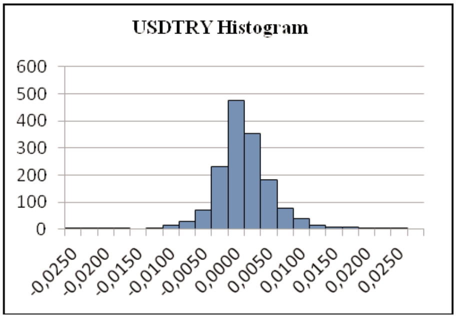
The daily interest rates used for risk-free rates were the British Bankers Association (BBA) settlement rates with maturity of 3 months taken from the Reuters. Thecorrectness of the Black-Scholes model depends vitally on whether the assumptions specified above are sensible if real data is used.

Data Analysis and GARCH-M model. Jarque-Bera is test statistics for testing whether series is normally distributed. The test measures the difference of skewness and kurtosis of the series with those from normal distribution. Under the null hypothesis of a normal distribution, the Jarque-Bera statistic is distributed as X^2 with 2 degrees of freedom. The small probability value of Jarque-Bera leads to the rejection of the null hypothesis of a normal distribution. The kurtosis of the normal distribution is 3 and the skewness of symmetric distribution, such as normal distribution, is zero. Table 1 presents the distribution analysis results.

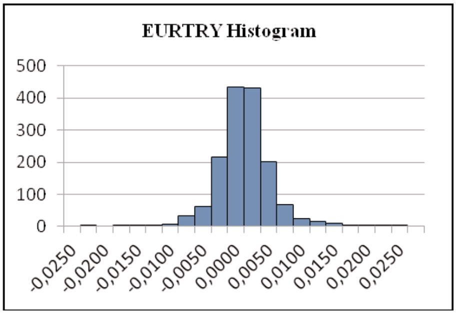
Table 1. **The results of USD/TRY and EUR/*TRY distribution analysis,** *authors' calculations*

	Mean	Median	Skewness	Kurtosis	Jarque-Bera	Probability
USD/TRY	1.35 e-19	-9.88 e-05	0.5196	7.2040	1191.618	0.0000
EUR/TRY	-1.67e-19	-.000292	0.6847	9.5214	2821.523	0.0000

Table 1 and Figure 1 demonstrate that Jarque-Bera test, skewness and kurtosis of USD/TRY and EUR/TRY return data are not normally distributed. The Black-Scholes model assumes log-normal distribution and constant expected return and variance, yet as it can be seen, these tables violate the assumptions.



a) USD/TRY



b) EUR/TRY

Figure 1. **Histogram,** *authors'*

On the contrary to the Black-Scholes assumptions, in order to reveal whether the volatility structure needs to change, the Portmanteau Q-test and the Lagrange multiplier LM-test for ARCH disturbances are carried out. Table 2 presents the results of the ARCH test for USD/TRY and EUR/TRY. The results of the tests show statistically significant presence of ARCH disturbances in the sample data. The ARCH LM and Q-tests are conducted using the equation for the underlying following a Brownian motion given as $\ln(\frac{S_{t+1}}{S_t}) = \mu + \varepsilon_{t+1}$. The autoregressive conditional heteroskedasticity classifies non-constant volatility and future periods of volatility fluctuations cannot be identified.

Table 2. **USD/TRY and EUR/TRY ARCH Test Results on ARCH/GARCH effects,** *authors' calculations*

Panel A: Q-Test					Panel B: LM-Test				
	USDTRY		EURTRY			USDTRY		EURTRY	
Lag (Days)	Q	p-value	Q	p-value	Lag (Days)	Coefficient (â)	p-value	Coefficient (â)	p-value
1	48.85	0.03	0.03	0.86	1	0.21	0.00	0.18	0.00
2	50.28	0.08	0.04	0.98	2	-0.01	0.68	0.00	0.90
3	71.42	0.07	49.68	0.17	3	0.07	0.01	0.15	0.00
4	82.96	0.08	49.69	0.29	4	0.08	0.00	0.13	0.00
5	10.06	0.07	64.30	0.27	5	0.21	0.00	0.10	0.00
6	10.88	0.09	67.99	0.34	6	-0.09	0.00	-0.03	0.26
7	11.58	0.12	68.21	0.45	7	0.07	0.01	0.06	0.04
8	11.58	0.17	73.09	0.50	8	-0.06	0.01	-0.05	0.08
9	11.62	0.24	85.70	0.48	9	0.17	0.00	0.03	0.31
10	11.81	0.30	10.14	0.43	10	-0.08	0.00	0.05	0.08
11	11.91	0.37	10.14	0.52	11	0.12	0.00	0.02	0.55
12	14.77	0.26	11.74	0.47	12	0.08	0.00	0.08	0.00

Table 2 presents the Q and LM tests with 12 days lags and p-values. The ARCH LM test statistic is computed from the auxiliary test regression. To test the null hypothesis there is no ARCH up to order q in the residuals, the regression is run as $\varepsilon_t^2 = \beta_0 + \beta_1\varepsilon_{t-2}^2 + \ldots + \beta_q\varepsilon_{t-q}^2 + v_t$.This is a regression of the squared residuals on constant and lagged squared residuals up to order q. The F-statistic is an omitted variable test for the joint significance of all lagged squared residuals. Engle's LM test statistic (LM) is computed as the number of observations times the R^2 from the test regression. F-statistic values of the tests are 36.9265 and 26.6715, where the LM test statistics are 345.0318 and 266.0619 for USD/TRY and EUR/TRY data, all with the p-value of zero.

The LM-test report statistically significant ARCH effects for most of the 12 days prior to the day of the observation.

These tests suggest that pricing of options by using the Black-Scholes model might be inappropriate due to the assumptions made. As a result, a GARCH-M process is used similar to that of Duan (1995) given as:

$$\ln(\frac{S_{t+1}}{S_t}) = r - r_f - \frac{1}{2}h_t + \sigma_{t+1}\varepsilon_{t+1}^*. \qquad (6)$$

(6) gives the return of logarithm for successive time steps with conditional volatility formula as in (7). r and r_f are the risk free rates of foreign exchange currency pairs, so to say domestic and foreign interest rates.

$$h_t = \alpha_0 + \sum_{i=1}^{q}\alpha_i\left(\varepsilon_{t-i}^* - \lambda\sqrt{h_{t-i}}\right) + \sum_{i=1}^{p}\beta_1 h_{t-i}. \qquad (7)$$

α_0, α_1 and β_1 are the coefficients of the GARCH in the mean model.

$$\varepsilon_{t+1}^* \sim N(0,1)$$

The epsilon is a random number drawn from a normal distribution to simulate the next step for the conditional volatility.

Tables 3 and 4 show the coefficients of the GARCH-M model and that the unit risk-premium (λ) is not statistically significant but has been taken into consideration since the significance of the unit risk premium is not being taken into consideration in Duan's GARCH option pricing model (1995).

Table 3. **The results of the USD/TRY and EUR/TRY GARCH-M Model,** *authors' calculations*

	USD/TRY			EUR/TRY		
	Coefficient	z-statistics	p-value	Coefficient	z-statistics	p-value
(λ)	-0.06	-0.68	0.49	-0.07	-0.72	0.47
(α_0)	0.00	6.57	0.00	0.00	6.14	0.00
(α_1)	0.12	9.56	0.00	0.13	9.10	0.00
(β_1)	0.84	55.30	0.00	0.83	44.40	0.00

The process is given as $\ln(\frac{S_{t+1}}{S_t}) = r - r_f - \frac{1}{2}h_t + \sigma_{t+1}\varepsilon^*_{t+1}$. Only the lag structure (1,1) is taken into consideration where variance for the GARCH (1,1) process is given as $h_t = \alpha_0 + \sum_{i=1}^{q}\alpha_i\left(\varepsilon^*_{t-i} - \lambda\sqrt{h_{t-i}}\right) + \sum_{i=1}^{p}\beta_1 h_{t-i}$.

Methodology. Using the equations of the GARCH model and the acquired coefficients, Monte-Carlo simulation is used to simulate various paths for exchange rate progress under the GARCH-M model. The purpose of simulating the underlying exchange rate is to get a distribution of spot parities at the maturity of options. The simulation of foreign exchange paths entails the estimation of conditional volatilities on the daily basis and the resultant estimates of daily rates by means of the GARCH-M model. An arbitrary number of simulation paths is chosen as Nsims = 3000 where each option expires in 90 days. For each of 3000 iterations, we make sure that each time step corresponds to one day.

The strikes of the options are 90 day at the money forward rates calculated with given interest rates. At the money forward rates are not influenced by skewness of the volatility curve, out of money options have been designated to be better priced by the GARCH model. Since skewness of the volatility curve is provided by market quotations, it has not been considered as an advantage to the Black-Scholes model and has not been taken into the scope of the paper. Please, see "Pricing Foreign Currency and Cross-Currency Options under GARCH" by Duan (1999) and "Analysis of Term Structure of Implied Volatilites" by Heynen Kemna and Vorst. To calculate a forward FX rate, the formula used is:

$$forward\ \ rate = spot\ \ rate \times \frac{(1+r_1)^n}{(1+r_2)^n}, \tag{8}$$

where r_1 is the interest rate for the first currency; r_2 is the interest rate for the second currency; n is the time period. Since 3-month Libor is the interest rate used for calculations, the formula can be taken as:

$$at\,the\,money\ forward\,(ATMF) = spot\,rate \times \left[1 + \left(rdif/4\right)\right], \tag{9}$$

where $rdif$ is the difference of the interest rates of both currencies.

At maturities 3000 various spot prices appear as the results of simulations. The payoff resulting from every single path is the difference between the strike and the spot price. Since these options are call options, the payoff function is Max(*Strike* – *Spot*;0). Summation of these option payoffs is divided by the number of simulations and this average number is the option price at maturity. This option price is discounted with the risk free rate to get the option price at the start date of the option as seen in (10), where rf is the risk free rate, ATMF is the strike and z is the number of value date of the option z = 1, 2, ..., 1461.

$$C_{Garch(1,1)} = \frac{1}{3000}\sum_{y=1}^{3000} e^{-r_f} Max\,(ATMF(z) - Spot(z,y);0). \tag{10}$$

As mentioned, the 3-month (90 days) historical volatility that is actually the volatility observed at the option expiry for the past 3 months is acquired from the Reuters and put into the Black-Scholes option pricing formula, and this value is taken as the real option value and used for comparison.

Performance analysis and results. Market prices calculated by putting market quoted volatiles into the Black-Scholes option pricing formula and GARCH option prices resulting from simulations are all put in a matrix. Differences of these values with the Black-Scholes option prices acquired with historical prices are squared and summed up to find out the average squared error in order to find the correctness of estimations. The explained procedure can be summarized as:

$$average\ squared\ errors(a) = \frac{1}{n}\sum_{y=1}^{n}\left(C_y - C_y\,(a)\right)^2. \tag{11}$$

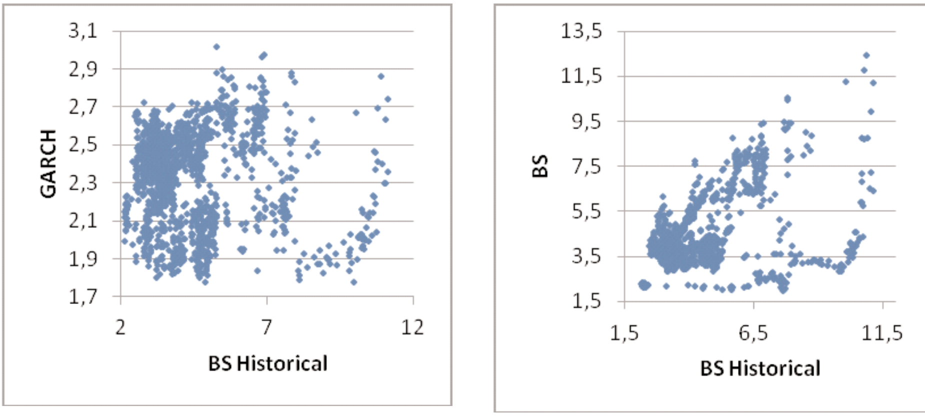
C_y denotes the Black-Scholes option price calculated with realized volatility, where the C_y (a) denotes the option price estimated by model a (GARCH-M or Black-Scholes) and n is the number of sample data.

That comparison of the error terms implies that the Black-Scholes model outperforms the GARCH one. Comparison of the single values of residuals showed that for USD/TRY data out of 1461 sample days, 1151 estimates by Black-Scholes have been closer to realized results against 310 estimates of GARCH model, and for EUR/TRY data out of 1461 sample days, 1090 estimates by Black-Scholes have been closer to the realized results against 371 estimates of the GARCH model. Libor 3 month rates have been used for pricing the Black-Scholes model; however, daily interest rates have been used for the GARCH model pricing. One of the biases of the Black-Scholes model is put forward as the non-stochastic interest rates. Constant volatility and interest rate seem to be the drawbacks of Black-Scholes (Heston, 1993).

The scatter graph of the GARCH and Black-Scholes against the realized option prices is given in Figure 2.

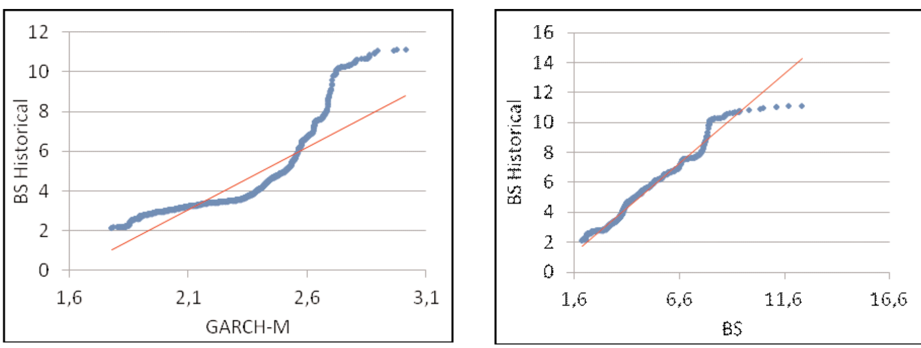
For performance evaluation another approach is to use the Q-Q plots against the empirical Quantile-Quantile. Q-Q plots are applied to form a group of simulated series from the Black-Scholes and GARCH models and plot them against the quantiles of the series of actual values. As it appears in Figure 3, the Q-Q plots suggest a better approximation of Black-Scholes calculated by using market quoted volatility.

Q-Q plots reveal that the Black-Scholes results are relatively close to the realized results however at high volatilities, shown with high premium values, the Black-Scholes has a relative higher deviation from the realized results.



a) historical and GARCH b) historical and BS

Figure 2. **Scatter Graph of Option Price,** *aurhors'*



a) b)

Figure 3. **Q-Q Plots of (a) GARCH against Black-Scholes with historical volatility; (b) Black-Scholes against Black-Scholes with historical volatility,** *authors'*

For performance evaluation the last method used is the Kolmogorov-Smirnov test. *F(z)* is the cumulative distribution of absolute errors generated by the model *z*, where *z* stands in this case for Black-Scholes or GARCH-M. The KS 2 tail test shows if the data samples are from the same population and the KS 1 tail test evaluates which of the two distributions is stochastically superior. The tests are set up to compare the pricing errors from the Black-Scholes and GARCH-M models for options on both USD/TRY and EUR/TRY.

KS 1 tail test		KS 2 tail test	
H0: F(BS) = F(G11)	H1: F(BS) > F(G11)	H0: F(BS) = F(G11)	H1: F(BS) ? F(G11)

In this framework, if the null hypothesis is rejected, it implies that the Black-Scholes model has a higher valued cumulative distribution, denoting smaller magnitude of absolute error compared to the GARCH model.

The Kolmogorov-Smirnov 2-tail and 1-tail test is stated as: KS-1-tail: H0: F (BS) = F (G11); H1: F (BS) > F(G11); KS-2-tail: H0: F (BS) = F (G11); H1: F (BS) ≠ F(G11).

Table 4. **The results of the Kolmogorov-Smirnov Test,** *authors*

Black-Scholes and GARCH-M	KS 1 Tail	KS 1 Tail	Superior Model
USD/TRY	Reject H0	Reject H0	Black-Scholes
EUR/TRY	Reject H0	Reject H0	Black-Scholes

The two-sample Kolmogorov-Smirnov test is one of the most useful and general nonparametric methods for comparing two samples in both location and shape of empirical cumulative distribution functions. The null hypothesis of the Kolmogorov-Smirnov statistics is calculated testing if the samples are of the same distribution. The results of the Kolmogorov-Smirnov test imply that the cumulative distribution of the errors resulting from the Black-Scholes model is greater than the distribution for GARCH model for both currency pairs. This designates that the Black-Scholes model produces smaller errors.

Conclusion. The foreign exchange currency options are used for hedging purposes and speculation; however, assumptions of the Black-Scholes model cause uncertainties about the pricing of options. The Black-Scholes model assumes a diffusion process with a constant volatility. Analytic methods are being developed and extensions are delivered to enhance pricing performances, nevertheless new empirical solutions are required to cover the need for more accurate calculations.

The motivation of the paper stems from the Duan's GARCH model, and it analyzes a data set for Turkish Lira options against major currencies – USD and EUR. Spot movements are simulated and the realized volatilities at the maturities are taken for comparison and correctness of the estimations. The results showed that Black-Scholes outperformed and provided more appropriate prices for the options. Considering the USD/TRY data out of 1461 sample days, 1151 estimates by Black Scholes have been closer to the realized results against 310 estimates of the GARCH model. Considering the EUR/TRY data out of 1461 sample days, 1090 estimates by Black-Scholes have been closer to the realized results against 371 estimates of the GARCH model. The scatter graphs, QQ plots and the Kolmogorov-Smirnov test confirm these results.

The study shows that the Black-Scholes model outperforms the GARCH. The implication of these results is that the quoted implied volatilities at the counter market are a good indication of the future realized movements in foreign exchange underlying.

The next step would be to study the performances of the GARCH and Black-Scholes models by applying stochastic interest rates for the models and examine the results by using the money strike rates and examine the term structure of the implied volatilities with skewness and smile effects.

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