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BUSINESS VALUE CHANGE FORECASTING WITHIN THE MIXED DISCRETE-CONTINUOUS MODEL OF JUMPS BASED ON BROWNING AND POISSON PROCESSES

The paper dwells on the fundamental principles of business value growth and stability of leveraged companies, studying the appropriate cases of stationary and non-stationary Markov models of growth. It provides their classification according to the known fundamental condition of the theory of stochastic processes, depending on which it turns out to be a continuous Wiener (in a certain sense — single) growth model, a discrete-continuous (as the most appropriate) growth model, and a piecewise constant Poisson (purely discrete) growth model.

Keywords: *forecasting; business value; stochastic model; Wiener growth model; Poisson growth model; Brownian motion.*

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ПРОГНОЗУВАННЯ ЗМІНИ ЦІНИ БІЗНЕСУ В РАМКАХ ЗМІШАНОЇ ДИСКРЕТНО-НЕПЕРЕРВНОЇ МОДЕЛІ НА БАЗІ СТРИБКІВ БРОУНІВСЬКОГО І ПУАССОНІВСЬКОГО ПРОЦЕСІВ

У статті розглянуто фундаментальні основи зростання і стійкості вартості бізнесу левериджових компаній. Вивчено низку випадків стаціонарних і нестаціонарних марківських моделей зростання. Наведено класифікацію останніх, за відомої фундаментальної умови теорії випадкових процесів, залежно від виконання якої отримуємо безперервну вінерівську (у певному сенсі — єдину) модель зростання, дискретно-неперервну (як найбільш адекватну) і кусково-постійну пуассонівську (суто дискретну).

Ключові слова: *прогнозування; вартість бізнесу; стохастична модель; вінерівська модель зростання; пуассонівська модель зростання; броунівський рух.*

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ПРОГНОЗИРОВАНИЕ ИЗМЕНЕНИЯ ЦЕНЫ БИЗНЕСА В РАМКАХ СМЕШАННОЙ ДИСКРЕТНО-НЕПРЕРЫВНОЙ МОДЕЛИ СКАЧКОВ НА БАЗЕ БРОУНОВСКОГО И ПУАССОНОВСКОГО ПРОЦЕССОВ*

В статье рассмотрены фундаментальные основы роста и устойчивости стоимости бизнеса левериджевых компаний. Изучены все подходящие случаи стационарных и нестационарных марковских моделей роста. Приведены классификация последних, по известному фундаментальному условию теории случайных процессов, в зависимости от выполнения которого получается непрерывная виннеровская (в определенном смысле — единственная) модель роста, дискретно-непрерывная (как наиболее адекватная) и кусочно-постоянная пуассоновская (чисто дискретная).

Ключевые слова: *прогнозирование; стоимость бизнеса; стохастическая модель; виннеровская модель роста; пуассоновская модель роста; броуновское движение.*

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Introduction. Evaluation of business value is a current subject of research in Russian and foreign science and practice. At present not only the system of interaction with stakeholders but also internal management of companies take into account the need to increase business value.

Value-based management (VBA) is widely studied in finance literature. At the same time a number of publications justify the use of stochastic processes as basic models.

The problem of company value changes in time were studied by V.V. Galasyuk et al. (2003). He created the CCF concept (conventionally-cash flow).

Recently there were proposed a number of stochastic models for obtaining business value: Ohlson's and Feltham-Ohlson's Linear Information Dynamics (LID) (Ohlson, 1995) within the EBO model (Edwards-Bell-Ohlson) (Edwards and Bell, 1961), and Bakshi and Chen's (Chen, 1976) model with its modifications.

Thus, the significance of the subject requires an indepth study of the issue and development of not only universal models, but also their particular cases that take into account the specifics of a particular business situation of a leveraged company and the level of stationarity processes.

In Section 1, we deal with the problem of forecasting the business selling value within continuous Brownian processes.

Considering that the model of Brownian processes is additive, the proposed method of forecasting is also additive.

Thus, it is suggested to compare the non-stationarity of the forecast period with the non-stationary business value growth model, which is understood as the cost of the totality of its voting shares.

Wiener process is an example of a continuous Markov process with independent increments, in a certain sense the only one. Therefore, it is interesting to turn to discrete processes that fulfil the independence increments condition but are only piecewise continuous. Poisson process gives the example of such process. Finally, there was an idea of the same distribution, uniting Wiener and Poisson processes. This forecasting model is developed in Section 2.

In Section 3, we prove that Poisson and Wiener processes are approximated by a discrete scheme of random walk. The proposed simple approximation of Poisson and Wiener processes can be useful for analysts at the stock market.

1. Business value change forecasting based on the continuous non-stationary Brownian process.

1.1. Problem setting and literature review on the problem. The problem of determining the selling value of X_n business at the end of the last n-forecast period in the revenue discounting method within the income approach to determine the market value of business.

One way of determining X_n is that this value is taken as the selling value forecast at the end of the last n-period (Baturina et al., 2007; Basangov et al., 2009):

$$X_n = X_0 \prod_{t=1}^{n} (1 + j_t). \tag{1}$$

However, this way of determining the selling value faces the lack of relevant statistics for forecasting.

Reviews of the market can show only the capitalization growth rate j_t in the industry over the previous years t = -1, -2, ..., -T, however, the extrapolation of these data on the trend for n = 5–7 years ahead has low precision.

Some authors (Basangov and Perevozchikov, 2012; Perevozchikov and Lesik, 2011), used the multiplicative forecasting method based on the stationary log-normal model of valuation changes.

But it was applicable only in the case when the change of some economic index is the base, for example, the RTS index, besides it assumes the stationarity of the forecast period that doesn't exist in the forecast period determination.

For example, it doesn't work on historical data on valued business when assessing the changes of its balance cost because in practice they differ in spread in values.

Therefore, the log-normal stationary model is suitable for economic indices forecasting, but it can't be actually applied to forecast changes in business selling value in the future on the basis of retrospective changes in the value of its shares.

Taking the abovementioned into account, the authors propose a different forecasting method of business selling price based on the non-stationary Brownian motion model of share price.

1.2. Problem formalization. We identify business value with the value of all its voting shares.

Assume that these shares are listed at the stock exchange.

Let $X(t)$ be share price at time $t \in [0,+\infty)$ and suppose that at time t = 0 it takes the value $u \in [0,+\infty)$ almost surely (a.s.):

$$X(0) = u. \tag{2}$$

Suppose the stochastic process c $X = X(t)$ a is adequately described by non-stationary Brownian motion which is set by a transition function (Gryaznova and Fedotova, 2002):

$$p_t(u,E) = \frac{1}{\sqrt{2\pi ct}} \int_E \left(e^{-(v-u)^2/2ct} + e^{-(v+u)^2/2ct} \right) dv, \tag{3}$$

where $u \geq 0, E \subset [0,+\infty); c$ is the constant that defines the time scale and is estimated statistically. In theoretical studies it is possible to consider that c = 1 by changing respectively the time scale presented in further computation. The value of (1) is the probability that the random value is $X(t) \in E.$ The sub-integral function (3) is the conditional probability density $f_t(u,v)$ of random value $X(t)$ at the moment of time t, provided that it takes the value (2) a.s.:

$$f_t(u,v) = \frac{1}{\sqrt{2\pi ct}} \left(e^{-(v-u)^2/2t} + e^{-(v+u)^2/2t} \right). \tag{4}$$

1.3. Formula for the conditional expectation of the value. Since $m_u = m_u(t) = M(x(t) | x(0) = u, п.н.)$ is a conditional mathematical expectation of share price $X(t)$ providing that at the initial time its price takes on a $u \in [0,+\infty)$ value comprises almost surely (a.s.):

$$m_u = \frac{1}{\sqrt{2\pi t}} \int_0^\infty v \left(e^{-(v-u)^2/2t} + e^{-(v+u)^2/2t} \right) dv = I_1 + I_2. \tag{5}$$

It is proposed to divide it into two integrals I_1, I_2:

$$I_1 = \frac{1}{\sqrt{2\pi t}} \int_0^\infty (v-u+u)\left(e^{-(v-u)^2/2ct}\right) d(v-u) =$$

$$\sqrt{\frac{t}{2\pi}} \int_0^\infty e^{-(v-u)^2/2t} d(v-u)^2/2t + \frac{u}{\sqrt{2\pi}} \int_0^\infty e^{-(v-u)^2/2t} d(v-u)/\sqrt{t} = \tag{6}$$

$$-\sqrt{\frac{t}{2\pi}} e^{-(v-u)^2/2t} \Big|_{v=0}^{\infty} + u(1-\Phi(-\frac{u}{\sqrt{t}})) = \sqrt{\frac{t}{2\pi}} e^{-u^2/2t} + u(1-\Phi(-\frac{u}{\sqrt{t}}));$$

$$I_2 = I_1(-u) = \sqrt{\frac{t}{2\pi}} e^{-u^2/2t} - u(1-\Phi(\frac{u}{\sqrt{t}})) = \sqrt{\frac{t}{2\pi}} e^{-u^2/2t} - u\Phi(-\frac{u}{\sqrt{t}}). \tag{7}$$

The Φ function is the cumulative distribution function of the normal distribution law $m_x = m = 0$, $\sigma_x = \sigma = 1$:

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{x^2}{2}} dx, \tag{8}$$

which is tabulated in special tables (Gryaznova and Fedotova, 2002).

Moving back to (5), we get

$$m_u = I_1 + I_2 = \sqrt{\frac{2t}{\pi}} e^{-u^2/2t} + u[1-2\Phi(-\frac{u}{\sqrt{t}})]. \tag{9}$$

Note that

$$2\Phi(-\frac{u}{\sqrt{t}}) = P(|x(t)| \ge x = \frac{u}{\sqrt{t}}) = \sqrt{\frac{2}{\pi}} \int_x^\infty e^{-y^2/2} dy. \tag{10}$$

Integrating by parts, therefore we get the idea (Gryaznova and Fedotova, 2002):

$$2\Phi(-\frac{u}{\sqrt{t}}) = \sqrt{\frac{2}{\pi}} (\frac{e^{-x^2/2}}{x} - \int_x^\infty e^{-y^2/2}/y^2 dy) = \sqrt{\frac{2}{\pi}} (\frac{\sqrt{t} e^{-u^2/2t}}{u} - \int_{u/\sqrt{t}}^\infty \frac{e^{-y^2/2}}{y^2} dy). \tag{11}$$

Using this formula in (9) we come to the formula:

$$m_u(t) = u(1 + \sqrt{\frac{2}{\pi}} \int_{u/\sqrt{t}}^\infty \frac{e^{-y^2/2}}{y^2} dy). \tag{12}$$

Note that the first integral in (11) is improper integral if $x = 0$. When $x > 0$ it is measured by the next formula:

$$0 < \sqrt{\frac{2}{\pi}} \int_x^\infty e^{-y^2/2}/y^2 dy < \sqrt{\frac{2}{\pi}} e^{-x^2/2} \int_x^\infty \frac{dy}{y^2} = \sqrt{\frac{2}{\pi}} e^{-x^2/2} (-1/y)\Big|_{y=x}^{\infty} = \sqrt{\frac{2}{\pi}} e^{-x^2/2}/x. \tag{13}$$

Let us show that the order of the first integral in (11) when $x \to +0$ coincides with the received assessment (13). So let $x_0 > x > 0$ be at any fixed sufficiently small $x_0 > 0$, where in $e^{-x^2/2} \approx e^{-x_0^2/2} \approx 1$. Then:

$$\sqrt{\frac{2}{\pi}} \int_x^\infty e^{-y^2/2}/y^2 dy = \sqrt{\frac{2}{\pi}} (\int_x^{x_0} + \int_{x_0}^\infty) \approx \sqrt{\frac{2}{\pi}} (e^{-x^2/2}(-1/y)\Big|_{y=x}^{x_0} + \int_{x_0}^\infty e^{-y^2/2}/y^2 dy) =$$
$$= \sqrt{\frac{2}{\pi}} (e^{-x^2/2}/x - e^{-x_0^2/2}/x_0 + \int_{x_0}^\infty e^{-y^2/2}/y^2 dy) = \sqrt{\frac{2}{\pi}} (e^{-x^2/2}/x + C), \tag{14}$$

where $c = c(x_0)$ is some constant, whence it follows that the order of the first integral in (11) when $x \to +0$ coincides with the received assessment (13). The first integral non-property in (11) shows the non-property of the integral in (12) when $t = \infty$. In particular, we get the following when $x = u/\sqrt{t}$:

$$0 \le \sqrt{\frac{2}{\pi}} \int_{u/\sqrt{t}}^{\infty} e^{-y^2/2} / y^2 dy < \sqrt{\frac{2t}{\pi}} e^{-u^2/2t} / u. \quad (15)$$

Substituting it in (12) we get the equation:

$$u \le m_u(t) = u + \sqrt{\frac{2t}{\pi}} e^{-u^2/2t}, \quad (16)$$

where the integral order in (12) at $t \to \infty$ coincides with the received assessment (15).

Now it comes out from the formula for the conditional mean (12) that $m_u(0) = u$, a.s. and from the estimation of its order when $t \to +\infty$ we have the asymptote $\sqrt{2t/\pi}$ that doesn't depend on the initial state u.

1.4. Statistical characterization of non-stationary process. By defining the single parameter c of the Brownian process for any $l,t \ge 0$ the increment $\Delta x = x(t + l) - x(t)$ is normally distributed with the average $M_{\Delta x}(l) = 0$ and variance $D_{\Delta x}(l) = cl$. To estimate the unknown parameter c it is necessary to receive a statistical variance estimate $D_{\Delta x}(l)$ when $l = 1, 2, ..., L$.

Let $x_1, ..., x_N$ be the value obtained during the random value observation $x(1), ..., x(N)$.

Let us assume that

$$m_N(l,x) = \frac{1}{N-l} \sum_{k=1}^{N-l} (x_{l+k} - x_k). \quad (17)$$

Then it is unbiassed estimator of the average increase $\Delta x = x(t + l) - x(t)$, i.e.

$$M(m_N(l,x)) = M\left(\frac{1}{N-l} \sum_{k=1}^{N-l} (x_{l+k} - x_k\right) = \frac{1}{N-l} \sum_{k=1}^{N-l} M(x_{l+k} - x_k) = 0. \quad (18)$$

Since

$$D_{\Delta x}(l) = M(x(t+l) - x(t) - M(x(t+l) - x(t))^2 = M[x(t+l) - x(t)]^2, \quad (19)$$

as a value estimation according to the N observation results $x_1, \ldots, x_N (N > l \ge 0)$ it is naturally taking the value:

$$d_N(l,x) = \frac{1}{N-l} \sum_{k=1}^{N-l} (x_{l+k} - x_k - m_N(l,x))^2. \quad (20)$$

The following variant is possible:

$$d_N(l,x) = \frac{1}{N-l} \sum_{k=1}^{N-l} (x_{l+k} - x_k)^2. \quad (21)$$

The last function is unbiased, that is:

$$M(d_N(l,x)) = D_{\Delta x}(l), 0 \le l < N - K = L. \quad (22)$$

K is the minimum volume of the representative sample, which helps to find the average in (17), (20), (21). After a statistical estimate of the variance $y_i = d_N(l,x)$, $0 \le l < N - K = L$, the unknown parameter value c in $D_{\Delta x}(l) = cl$ regression can be estimated by the least-squares method using the following formula:

$$c = \frac{\sum_{l=1}^{L} l y_l}{\sum_{l=1}^{L} l^2}. \tag{23}$$

After this the forecasting of the conditional mean of random value $x = x(t)$ can be computed by the formula (9).

2. Business value change forecasting based on the mixed discrete-continuous non-stationary process.

2.1. Problem setting, the literature review on the problem. In Section 1, we proposed the method of business selling value forecasting based on the non-stationary Brownian motion model of the share price. Since the Brownian motion model is an additive (Perevozchikov and Lesik, 2010a), the proposed method of forecasting is also additive, i.e. postulates the independence of increments. We studied the additive way of modelling the business value change in another paper (Deloitte&Touche, 2005) and also (Perevozchikov and Lesik, 2010b):

$$X_n = X_0(1 + \sum_{e=1}^{T} i_t), \tag{24}$$

where i_t is the appropriate growth rates.

Therefore, it was proposed to compare the non-stationarity of the forecast period, understood as the value of its total voting shares, with the non-stationary business value growth model.

Wiener process is an example of the continuous additive Markov process with independent increments, in a certain sense the only one, as shown in (Gryaznova and Fedotova, 2002).

The authors refer to discrete processes which satisfy the condition of independence of increments but they are only piecewise continuous.

Poisson process gives the example of such piecewise constant process, as described in (Deloitte&Touche, 2005).

In Poisson process the average increment value in time t linearly depends on t with the factor that can be evaluated statistically similar just as it was done for a single parameter of Wiener process (Gryaznova and Fedotova, 2002).

Moreover, Poisson process in contrast to stationary process with independent realizations, as we studied before (Baturina et al., 2007) is a non-stationary one; and the forecasting model developed on its basis is a discrete non-stationary constant-growth model.

Having constructed a transition function (Baturina et al., 2007), we got the mixed conditional discrete-continuous distribution and there was a thought about the same distribution uniting Wiener and Poisson processes.

This forecasting model developed in the present paper, adequately simulates the changing process of shares value at the stock exchange, which is a piecewise continuous and where individual jumps are interchanged by segment of continuous value change.

Both normal and Poisson laws are infinitely divisible (Gryaznova and Fedotova, 2002), therefore, the transfer function constructed on the basis of their integration will be spatially uniform (Gryaznova and Fedotova, 2002), consequently will lead to the process that fulfils the independence of increments condition (Gryaznova and Fedotova, 2002).

2.2. Problem formalization. It is proposed to identify the business value with the value of all voting shares quoted at the exchange. Let $X(t)$ be a share price at time $t \in [0,+\infty)$ and assume that at time $t = 0$ it takes the value $u \in [0,+\infty)$ a.s.:

$$X(0) = u. \tag{25}$$

Suppose that stochastic process $X = X(t)$ is adequately described by the discrete non-stationary Poisson process that is defined by the transition function (Gryaznova and Fedotova, 2002):

$$p_t(u,E) = e^{-\lambda t}\frac{1}{\sqrt{2\pi ct}}\int_E e^{-(v-u)^2/2ct}dv + (1-e^{-\lambda t})\sum_{m=0}^{\infty}\frac{e^{-\lambda t}(\lambda t)^m}{m!}\aleph(u+ma,E), \tag{26}$$

where $a \neq 0$ is the spatial increment, for example, the required calculation accuracy; c, $\lambda > 0$ are constants evaluated statistically.

Here $\aleph(u,E)$ is the function of the incidence point u to the set $E \subset R_1$ that will be equal to unity if the point belongs to the set or otherwise – zero. Recall that transition function is to determine the accessory probability of random value $X(t)$ measurable by Lebesgue to the set E on the real line R_1. Density $p_t(u,v)$ of the conditional distribution $X(t)$ under condition (25) is expressed by the formula:

$$p_t(u,v) = e^{-\lambda t}\frac{1}{\sqrt{2\pi ct}}e^{-(u-v)^2/2ct} + (1-e^{-\lambda t})\sum_{m=0}^{\infty}\frac{e^{-\lambda t}(\lambda t)^m}{m!}\delta(u+ma-v), \tag{27}$$

where $\delta(u - v) - \delta -$ is the function, i.e. the generalized function that satisfies for any continuous function $\varphi(v)$ the condition:

$$\varphi(u) = \int_E \varphi(v)\delta(u-v)dv. \tag{28}$$

The function $\delta(u - v)$ is a generalized probability density of the discrete random value being in u point. In particular its mathematical expectation is also equal to u:

$$u = \int_E v\delta(u-v)dv. \tag{29}$$

2.3. Formula for the conditional expectation of the value. The limiting density of the conditional distribution (27) is the distribution density of the mixed discrete-continuous random value. Going to the average, using the received density of the conditional distribution (27), we get a formula for the conditional average:

$$m_u(t) = e^{-\lambda t}u + (1-e^{-\lambda t})\times(u+\lambda t) = u + (1-e^{-\lambda t})\lambda t. \tag{30}$$

From this formula it follows in particular that

$$m_u(0) = u; \tag{31}$$

$$m_u(t) \to \infty, t \to \infty. \tag{32}$$

and it has the asymptote $u + \lambda t$.

Thus, the relative process average:

$$\overline{m}_u(t) = [m_u(t) - u]/t = (1 - e^{-\lambda t})\lambda. \quad (33)$$

is the value going to λ when $t \to \infty$, it allows to approximate this process from the above by non-stationary constant-growth model (Gryaznova and Fedotova, 2002) when $a > 0$ or decrease when $a < 0$.

2.4. Statistical determination of non-stationary process parameters. Constants $c, \lambda > 0$ should be statistically estimated separately from each other. Constant $c > 0$ should be estimated on the continuous parts of the process, and constant $\lambda > 0$ should be estimated on the points of jumps t_1, t_2, ... and their values $\Delta x_k = \Delta x(t_k)$.

2.4.1. Continuous part of the process. By defining the only parameter c of the Brownian process for $l, t \geq 0$ and increment $\Delta x_k = x(t + l) - x(t)$, it is normally distributed with the average $M_{\Delta x}(l) = 0$ and variance $D_{\Delta x}(l) = cl$. Therefore, to estimate the unknown parameter c it is necessary to get the statistical estimate of variance $D_{\Delta x}(l)$ when $l = 1, 2, ..., L$.

Let $x_1, ..., x_N$ be the values of random value $x(1), ..., x(N)$ obtained during the observation on any continuous part of the process. Suppose

$$m_N(l, x) = \frac{1}{N - l} \sum_{k=1}^{N-l} (x_{n+k} - x_k). \quad (34)$$

Then it is a mean unbiased estimate of the increment $\Delta x_k = x(t + l) - x(t)$, i.e.

$$M(m_N(l, x)) = M\left(\frac{1}{N - l} \sum_{k=1}^{N-l} (x_{n+k} - x_k\right) = \frac{1}{N - l} \sum_{k=1}^{N-l} M(x_{n+k} - x_k) = 0. \quad (35)$$

Considering that

$$D_{\Delta x}(l) = M(x(t + l) - x(t) - M(x(t + l) - x(t))^2 = M[x(t + l) - x(t)], \quad (36)$$

as a value estimation according to the N observation results $x_1, ..., x_N$ $(N > l \geq 0)$ it is naturally to take the value:

$$d_N(l, x) = \frac{1}{N - l} \sum_{k=1}^{N-l} (x_{n+k} - x_k - m_N(l, x))^2. \quad (37)$$

or

$$d_N(l, x) = \frac{1}{N - l} \sum_{k=1}^{N-l} (x_{n+k} - x_k)^2. \quad (38)$$

The latter is an unbiased in that

$$M(d_N(l, x)) = D_{\Delta x}(l), 0 \leq l < N - K = L. \quad (39)$$

K is the minimum volume of the representative sample, which helps find the average in (34), (37), (38). After a statistical estimate of the variance $y_l = d_N(l,x)$, $0 \leq l < N - K = L$, the unknown parameter value c in $D_{\Delta x}(l) = cl$ regression can be estimated by the least-squares method using the following formula:

$$c = \frac{\sum_{l=1}^{L} l y_l}{\sum_{l=1}^{L} l^2}. \quad (40)$$

2.4.2. Jumps in a process. The constant $\lambda > 0$ in the moments of jumps t_1, t_2, ... and their values $\Delta x_k = x(t_k)$, $k = 1, 2, \ldots, K$, can be estimated as the unknown value of the parameter λ at regression $y_k = \Delta x_k = \lambda t_k$ by the least-squares method using the following formula:

$$c = \frac{\sum_{k=1}^{K} t_k y_k}{\sum_{k=1}^{K} (t_k)^2}. \tag{41}$$

After this the forecasting of the conditional average of random value $X = X(t)$ can be computed by the formula (34).

2.5. Process classification. Transition function of the formed mixed discrete-continuous process for any $\varepsilon > 0$ fulfils the condition:

$$p_t(x, E_1 - [x - \varepsilon, x + \varepsilon]) = o(1), \tag{42}$$

uniformly in x. For breaking points this is because Poisson transition function fulfils this condition (Gryaznova and Fedotova, 2002). For continuity points Wiener transition function fulfils the strengthened condition (40), where the right part can be replaced (Gryaznova and Fedotova, 2002). Considering that the normal and Poisson distribution are infinitely divisible, the transfer function (26) built on its basis leads to spatially uniform Markov transition function (Perevozchikov and Lesik, 2010a), i.e. such that

$$p_t(u, E) = p_t(u + v, v + E) \tag{43}$$

for all substantial v. This along with (42) provides by Kinney theorem (Gryaznova and Fedotova, 2002) that there is a piecewise continuous, the right continuous and the left limited process that has independent increments (the basic property of additive processes, which is postulated antecedently). In particular, paths of the constructed mixed discrete-continuous process can be considered to be the right continuous and having the left limit.

In earlier papers (Perevozchikov and Lesik, 2010a) we suggested that it's necessary to pass from the process with independent realizations to the processes with independent increments satisfying the conditions of (42) types. Hence, the main value of turning to the classical theory of spatially uniform transfer function, leading to processes with independent increments (Perevozchikov and Lesik, 2010a), lies in confirmation and clarification of this hypothesis. So, if you enhance the condition (42) that the right part behaved like $O(t)$, then it would be continuous processes specifically for the Wiener transition function studied in our previous work (Perevozchikov and Lesik, 2010a) – the Brownian process. If we weakened condition in (42) so that the right part behaved like $O(1)$ (i.e. there's no need to impose additional conditions because the probability is limited by unit), the stationary process would be included with independent rejections of random value, studied by us earlier in (Perevozchikov and Lesik, 2010a).

This process classification occurs depending on the right side in the condition (42), which has resulted in the present paper in mixed discrete-continuous model of the non-stationary business value growth model, making it possible to transfer most adequately process properties of the real share value change at the stock exchange.

3. Discrete models of share price change approximating Poisson and Wiener processes.

3.1. Problem setting. Wiener process is an example of a continuous, additive process with independent increments. The authors refer to discrete processes, which satisfy the condition of independence of the increments, but they are only piecewise continuous. In the Poisson process the average value of increment in time Δt linearly depends on Δt with coefficient λ, that can be estimated statistically just as it was done for a single parameter of Wiener process in our earlier work (Perevozchikov and Lesik, 2010a). The corresponding conditional Poisson distribution is approximated binomial and the difference of two equally distributed binomial has a distribution that is obtained from a binomial convolution with negative binomial and approximates the normal conditional distribution section of a continuous Wiener process, as the sum of equally distributed discrete random values. The third part of this paper shows that the Poisson process is approximated by the scheme of discrete random walk with transition probability in the following (in ascending order) discrete state with probability $p = \lambda\Delta t > 0$ and Wiener process is approximated by the scheme of discrete random walk with probability of transition to the following discrete state with probability p, return to the previous — also p, and finally remain in the same state $1 - 2p$. The parameters of both limited conditional distributions are the corresponding transition probabilities, which are estimated statistically, like the parameters of the limiting processes in our earlier works. It is important that these simple models show why in addition to the Poisson and Wiener processes nothing can get in limit for processes with independent increments, which is the main point of the non-stationary processes theory.

3.2. Approximation of the Poisson process. It is proposed to identify business value with the value of all its voting shares. Assume that these shares are listed at the stock exchange. Let $X(t)$ be share price at time $t = n\Delta t$, $n = 0, 1, \ldots,$ of discrete time, where $\Delta t > 0$ is a temporal discrete, for example the given frequency monitoring data on the change in share price at the stock exchange. And suppose that in time $t = 0$ it takes the value:

$$u = k\Delta x, k = 0,1,\ldots; \tag{44}$$

$$X(0) = u. \tag{45}$$

Here $\Delta x > 0$ is a spatial increment, for example, the fixed precision of computations.

Suppose that the stochastic process $X = X(t)$ is adequately described by a discrete non-stationary Poisson process, given by the transition function (Gryaznova and Fedotova, 2002):

$$p_t(u,E) = \sum_{m=0}^{n} p^m (1-p)^{n-m} C_n^m \aleph(u + m\Delta x, E), \tag{46}$$

where $p = \lambda\Delta t$; λ is the constant estimated statistically.

Here $\aleph(u,E)$ is the function of incidence point u to the set $E \subset R_1$, that will be equal to unity if the point belongs to the set or otherwise zero. Recall that transition function is to determine the accessory probability of random value $X(t)$ measurable by Lebesgue to the set E on the real line R_1. Density $p_t(u,v)$ of the conditional discontinuous distribution X(t) under condition (44) is expressed by the formula:

$$p_t(u,v) = \sum_{m=0}^{n} p^m (1-p)^{n-m} C_n^m \delta(u + m\Delta x - v) \quad (47)$$

it is a density of binomial distribution with the average value

$$m_u(t) = u + np\Delta x \quad (48)$$

and variance

$$D_u(t) = npq\Delta x. \quad (49)$$

Here $q = 1 - p$, and $\delta(u - v) - \delta -$ is a function, i.e. the generalized function that satisfies for any continuous function $\varphi(v)$ the condition:

$$\varphi(u) = \int_E \varphi(v)\delta(u - v)dv. \quad (50)$$

The function $\delta(u - v)$ is a generalized probability density of the discrete random value being in u point. In particular, its mathematical expectation is also equal to u:

$$u = \int_E v\delta(u - v)dv. \quad (51)$$

The process with discrete conditional distribution defined by the density (47) can be formed as a process with independent increments possessing on each step of the discrete time values 0 and Δx with probability $p = \lambda\Delta t > 0$ Because each $t > 0$

$$n \to \infty, \quad p \to 0, \quad np = \lambda t, \quad (52)$$

when $\Delta t \to +0$ and conditional binomial distribution (47) converges to the Poisson distribution with density:

$$p_t(u,v) = \sum_{m=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^m}{m!} \delta(u + m\Delta x - v), \quad (53)$$

centered on:

$$m_u(t) = u + \lambda t\Delta x \quad (54)$$

and variance

$$D_u(t) = \lambda t\Delta x. \quad (55)$$

The corresponding transition function is defined by the formula:

$$p_t(u,E) = \sum_{m=0}^{\infty} \frac{e^{-\lambda t}(\lambda t)^m}{m!} \aleph(u + m\Delta x, E), \quad (56)$$

defining Poisson process (Perevozchikov and Lesik, 2010a). In this regard, described random walk leading to the binomial conditional distribution (47) approximates the Poisson process.

3.3. Approximation of Wiener process. Along with a conditional distribution with density $p_t(u,v) = p_t^+(u,v)$ given by the formula (44) let's consider its plane reflection with respect to u point on the axis of ordinates, given:

$$p_t^-(u,v) = \sum_{m=0}^{n} p^m (1-p)^{n-m} C_n^m \delta(u - m\Delta x - v) \quad (57)$$

and it represents the density of the binomial distribution of random value with the average value

$$m_u(t) = u - np\Delta x \quad (58)$$

and variance

$$D_u(t) = npq\Delta x. \tag{59}$$

Let's consider the distribution obtained by convolution of the conditional distribution (47) and (57):

$$p_t(u,v) = p_t^-(u,v) * p_t^+(u,v) \tag{60}$$

Here * is a convolution of two functions (Gryaznova and Fedotova, 2002).

The distribution (60) is the sum distribution of two independent random values: a random value subordinated to binomial distribution (47), with its plane reflection with respect to u point on the axis of ordinates and approximates the normal conditional distribution of $X(t)$ section as a sum of identically distributed discrete increments, taking each step of the discrete time values $-\Delta x$, 0, and Δx with probability, respectively qp, $1-2pq$, pq. With mathematical expectation $m = 0$ and variance $d = 2pq\Delta x$. Since

$$n \to \infty,\ p \to 0,\ D_u(t) = 2npq\Delta x = 2\lambda t(1-\lambda\Delta t)\Delta x \to 2\lambda t\Delta x, \tag{61}$$

when $\Delta t \to +0$ conditional binomial distribution (60) converges to a normal distribution with density:

$$p_t(u,v) = e^{-(u-v)^2/2ct}, \tag{62}$$

when

$$c = 2\lambda\Delta x \tag{63}$$

centered on:

$$m_u(t) = u \tag{64}$$

and variance

$$D_u(t) = ct. \tag{65}$$

The corresponding transition function is defined by the formula

$$p_t(u,E) = \frac{1}{\sqrt{2\pi ct}}\int_E e^{-(v-u)^2/2ct}dv \tag{66}$$

defining Poisson process (Perevozchikov and Lesik, 2010a) corresponding to continuous Brownian motion. In this regard, described random walk leading to the conditional distribution (60) approximates the Brownian motion. At the same time the meaning of a single parameter c, defining Wiener transitional function (66) is clarified. Namely, $c = 2\lambda\Delta x$, where λ is the single parameter of the approximating Poisson distribution (53).

Having the distribution of random value $X(t)$ subordinated to the law (60), we can form random value distribution $|X(t)|$. Then there will be the distribution corresponding to the described walk with reflecting boundary at zero and for it the average will not be the same as the initial value. There is the possibility of constructing nontrivial formulas for forecasting the conditional average share price in the future and the related brokerage criteria as it was done in the continuous case in our previous work (Perevozchikov and Lesik, 2010a). The parameters of both approximated distributions are the corresponding transition probabilities, which are estimated statistically, like the parameters of the limiting processes in our earlier works (Perevozchikov and Lesik, 2010a; 2011a).

3.4. Formula for the conditional expectation of the value within the model of discrete walks with reflecting boundary. Let us set for computational convenience

$$p_m^+ = P(X_u^+(t) = u + m\Delta x), m = 0,1,\dots,n; \quad (67)$$

$$p_m^- = P(X_u^-(t) = u - k\Delta x), k = 0,1,\dots,n. \quad (68)$$

Here $X_u^+(n)$ is the random value subordinated to the discrete distribution law (47) and $X_u^-(n)$ – to the law (57) accordingly. Then

$$p_l = P(X_u(t) = u + l\Delta x), l = -n,\dots,-1,0,1,\dots,n, \quad (69)$$

where $X_u(t)$ is the random value subordinated to the discrete distribution law (60). Then by convolution definition

$$p_l = \sum_{\max(-l,0)}^{\min(n,n-l)} p_{l+k}^+ p_k^-, l = -n,\dots,-1,0,1,\dots,n. \quad (70)$$

Formula (70) gives the discrete distribution law of random value $X_u(n)$, which is a discrete analog of Brownian process. In order to construct a discrete analogue of the Brownian process reflecting boundary at zero (Gryaznova and Fedotova, 2002) it is sufficient to construct the distribution law of random value $|X_u(t)|$ (Gryaznova and Fedotova, 2002). When

$$u - n\Delta x \ge 0, \quad (71)$$

then $X_u(n) \ge 0$ and the distribution $|X_u(t)|$ coincides with the distribution (70). Otherwise, let us denote

$$l_u = u / \Delta x; \quad (72)$$

then

$$\overline{p}_l = P(|X_u(n)| = u + l\Delta x) = \begin{cases} p_l, l \ge n - 2l_u \\ 2p_l, l < n - 2l_u \end{cases}, l = -l_u,\dots,-1,0,1,\dots,n. \quad (73)$$

The corresponding conditional average is obtained by the usual formula for the mathematical expectation of discrete random value:

$$m_u(t) = \sum_{l=-l_u}^{n} (u + l\Delta x)\overline{p}_l = 2\sum_{l=-l_u}^{n-2l_u-1} (u + l\Delta x)p_l + \sum_{l=n-2l_u}^{n} (u + l\Delta x)p_l. \quad (74)$$

Formula (70), (73) completely solve the problem of constructing a conditional discrete distribution approximating Brownian process with reflecting boundary at zero. Formula (74) can be used to forecast changes in the share price in the future. The offered simple approximation of Poisson and Wiener processes can be useful for analysts of the stock market and also form a basis for further theoretical researches in the field of modeling changes in the share price on the stock exchange and the corresponding business value, understood as the totality of all its voting shares.

Conclusions.

1. The paper considers the problem of determining the business selling value at the end of the forecast period in the revenue discounting method within the income approach to determine its market value.

We proposed the forecasting model based on the non-stationary Brownian motion model of the share price.

The model is based on the Wiener process, which is an example of a continuous Markov additive process with independent increments.

2. For evaluation purposes of business value under different conditions we study discrete processes (Poissons), which satisfy the condition of independence of increments, being piecewise continuous.

On their basis the forecasting model of company value was developed, which is a discrete non-stationary model of constant growth.

3. The problem of creating a conditional discrete distribution approximating the Brownian process with the reflecting boundary at zero is solved.

We proposed a simple approximation of Poisson and Wiener processes, which can serve as a basis for modelling changes in share price and the corresponding business value.

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