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STUDYING THE INTERRELATIONSHIP OF KEY MACROECONOMIC INDICATORS OF UKRAINE THROUGH SIMULTANEOUS EQUATIONS

A simplified model of the relationship between key macroeconomic indicators of Ukraine on the basis of the system of 3 simultaneous equations with lag variable was built in this paper. Calculations show that the estimates of the model parameters are stable.

Keywords: *simultaneous equations; least squares method; macroeconomic indicators.*

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ДОСЛІДЖЕННЯ ВЗАЄМОЗВ'ЯЗКУ ОСНОВНИХ МАКРОЕКОНОМІЧНИХ ПОКАЗНИКІВ УКРАЇНИ НА ОСНОВІ СИСТЕМИ ОДНОЧАСНИХ РІВНЯНЬ

У статті побудовано спрощену модель взаємозв'язку основних макроекономічних показників України на основі системи трьох одночасних рівнянь з лаговою змінною. Розрахунки показали, що оцінки параметрів моделі є стійкими.

Ключові слова: *система одночасних рівнянь; метод найменших квадратів; макроекономічні показники.*

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ИССЛЕДОВАНИЕ ВЗАИМОСВЯЗИ ОСНОВНЫХ МАКРОЭКОНОМИЧЕСКИХ ПОКАЗАТЕЛЕЙ УКРАИНЫ НА ОСНОВЕ СИСТЕМЫ ОДНОВРЕМЕННЫХ УРАВНЕНИЙ

В статье построена упрощенная модель взаимосвязи основных макроэкономических показателей Украины на основе системы трех одновременных уравнений с лаговой переменной. Расчеты показали, что полученные оценки параметров модели являются устойчивыми.

Ключевые слова: *система одновременных уравнений; метод наименьших квадратов; макроэкономические показатели.*

Problem setting. A simplified model of macroeconomics proposed by J. Johnston and J. DiNardo (1997), was based on the following assumptions: consumption is an increasing function of the available income; investments are an increasing function of national income; national income is the sum of consumer spending, investments and government procurement of goods and services. A mathematical model on the above mentioned macroeconomic indicators was built on the basis of these provisions.

Recent research and publications analysis. Theoretical basis of macroeconomic processes modelling was covered in the works of Ukrainian scientists: T. Klebanova et al. (2003), S. Kozlovskiy and V. Kozlovskiy (2005) and foreign scientists: D. Christopher (2011), J. Johnston and J. DiNardo (1997), Y. Magnus et al. (2004) and others.

However, multifaceted practical importance of this issue for the economy encourage further research on the problem of modelling economic growth and its components.

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The research objective is to construct a simplified model of the relationship of the key macroeconomic indicators on the basis of 3 simultaneous equations with variable lags based on the statistics data of Ukraine's economy in 2000–2012.

Key research findings. Based on this, a simplified model of the key macroeconomic indicators of Ukraine is proposed, which in contrast to J. Johnston and J. DiNardo (1997) has not considered state regulation of the economy; has accounted external borrowings and payments on foreign loans. The model represents the following system of 3 equations:

$$C_t = \alpha_0 + \alpha_1(Y_{t-1} - T_t) + \gamma_1 + \xi_{t1},\ t = 1,2,\ldots; \quad (1)$$

$$I_t = \beta_0 + \beta_1 Y_{t-1} + \xi_{t2},\quad \beta_1 > 0,\ t = 1,2,\ldots; \quad (2)$$

$$Y_t = C_t + I_t + G_t,\ t = 1,2,\ldots, \quad (3)$$

In the above equations C_t is consumption; I_t is investment; Y_t is national income; G_t is government purchases of goods and services; Tt is income tax in the t-th year. In (1)–(2) ξ_{t1} and ξ_{t2} are centered and uncorrelated random variables, (the standard assumption) i.e. for arbitrary $t_1,\ t_1,\ t_1 \neq t_2$ we have:

$$M\{\xi_{t1}\} = M\{\xi_{t2}\} = 0,\ t = 1,2,\ldots;$$
$$M\{\xi_{t_1 1}\xi_{t_2 1}\} = 0,\quad M\{\xi_{t_1 2}\xi_{t_2 2}\} = 0,\quad M\{\xi_{t_1 1}\xi_{t_2 2}\} = 0,\quad t_1 \neq t_2,\quad t_1, t_2 = 1,2,\ldots \quad (4)$$

Quantities ξ_{t1} and ξ_{t2} are introduced in order to take into account the effect of a set of random factors, on consumption and investment respectively.

In (1)–(3) $\alpha_0, \alpha_1, \beta_0, \beta_1$ are the unknown model coefficients estimated from the statistical data (Statistical Yearbook of Ukraine, 2006; 2012). Constant β_0 is introduced to account for the difference between the amount received and the payment of previous loans. This quantity is assumed to be independent of time, as information on loans and their payments is only available for the recent years 2008–2012.

The equations (1)–(3) are a structural form of econometric system consisting of 3 equations. We assume that C_t, I_t, Y_t are endogenous (dependent) variables; variables T_t, G_t are exogenous (independent) variables.

Equations of the structural form are divided into behavioral equations (describe the interactions between variables, equations (1) and (2)) and equation-identities (the ratio that must be met in all cases is equation (3)). To determine the coefficient estimates of the structural form model, we turn to the reduced form model in which all equations contain dependences of endogenous variables on the independent variables. These are exogenous variables and lagged values of endogenous variables (Magnus, Katyshev, Peresetskiy, 2004).

We substitute equations (2) and (3) in (1) and obtain the reduced form:

$$C_t = \alpha_0 + \alpha_1(C_t + \beta_1 Y_{t-1} + \beta_0 + G_t - T_t + \xi_{t2}) + \xi_{t1},\ t = 1,2,\ldots$$
$$or \quad (5)$$
$$C_t = \delta_{10} + \delta_{11}Y_{t-1} + \delta_{12}(G_t - T_t) + \varepsilon_{t1},\ t = 1,2,\ldots$$

Investments already have the reduced form:

$$I_t = \delta_{20} + \delta_{21}Y_{t-1} + \varepsilon_{t2},\ t = 1,2,\ldots \quad (6)$$

Using formulas (3), (4) and (5),

$$Y_t = \delta_{30} + \delta_{31} Y_{t-1} + \delta_{32} G_t + \delta_{33} T_t + \varepsilon_{t3},\ t = 1,2,\ldots \tag{7}$$

Formulae (5)–(7) are the reduced form of econometric equations system (1)–(3). In (5)–(7) coefficients of the reduced form are:

$$\delta_{10} = \frac{\alpha_0 + \alpha_1\beta_0}{1-\alpha_1},\ \delta_{11} = \frac{\alpha_1\beta_1}{1-\alpha_1},\ \delta_{12} = \frac{\alpha_1}{1-\alpha_1};$$
$$\delta_{20} = \beta_0,\ \delta_{21} = \beta_1; \tag{8}$$
$$\delta_{30} = \frac{\alpha_0 + \beta_0}{1-\alpha_1},\ \delta_{31} = \frac{\beta_1}{1-\alpha_1},\ \delta_{32} = \frac{1}{1-\alpha_1},\ \delta_{33} = -\frac{\alpha_1}{1-\alpha_1}.$$

Random values are:

$$\varepsilon_{t1} = \frac{\xi_{t1} + \alpha_1\xi_{t2}}{1-\alpha_1} = f_1(\xi_t),\ \varepsilon_{t2} = \xi_{t2} = f_2(\xi_t),\ \varepsilon_{t3} = \frac{\xi_{t1} + \xi_{t2}}{1-\alpha_1} = f_3(\xi_t), \tag{9}$$

where the vector $\xi_t = \begin{bmatrix} \xi_{t1} \\ \xi_{t2} \end{bmatrix}$, $f_i(\xi_t),\ i = 1, 2, 3$ is the linear functions of the components ξ_t (according to (9)).

From (4) and (9) it follows:

$$M\{\varepsilon_{t1}\} = M\{\varepsilon_{t2}\} = M\{\varepsilon_{t3}\} = 0,\ t = 1,2,\ldots;$$
$$M\{\varepsilon_{t_1 1}\varepsilon_{t_2 1}\} = 0,\quad M\{\varepsilon_{t_1 2}\varepsilon_{t_2 2}\} = 0,\quad M\{\varepsilon_{t_1 3}\varepsilon_{t_2 3}\} = 0; \tag{10}$$
$$M\{\varepsilon_{t_1 1}\varepsilon_{t_2 2}\} = 0,\quad M\{\varepsilon_{t_1 1}\varepsilon_{t_2 3}\} = 0,\quad M\{\varepsilon_{t_1 2}\varepsilon_{t_2 3}\} = 0,\quad t_1 \neq t_2,\quad t_1, t_2 = 1,2,\ldots$$

According to (10), the errors in linear regressions (5)–(7) ε_{ti}, i = 1, 2, 3 are centered, without autocorrelation. Moreover, they are not correlated with each other in different time periods. Therefore, the coefficients of each regression can be estimated separately. In this case they will be optimal and consistent.

A necessary precondition of consistency of regression' coefficient estimates is the lack of correlation of the independent variables with the regression error. Let's prove that this condition is satisfied for regressions (5)–(7).

We will consider regression (5) first. The independent variables are a unit (a dummy variable with δ_{10}), Y_{t-1}, $G_t - T_t$ in it. All these variables, except Y_{t-1}, are deterministic values which therefore do not correlate with error ε_{t1}. According to (7) Y_t and, consequently, Y_{t-1} are random variables. Using the result of (Demidenko, 1981), it may be shown that Y_t is the function of the random variables ε_{t3}, $\varepsilon_{t-1,3}$, $\varepsilon_{t-2,3}$, ..., ε_{13}. This has to be involved according to designation in (9), that $Y_{t-1} = F(f_3(\xi_{t-1}), f_3(\xi_{t-2}), \ldots, f_3(\xi_1))$, where $F()$ is some function. But according to (10) vectors ξ_{t-1}, ξ_{t-2}, ..., ξ_1 do not correlate with ε_{t1}. Therefore, the variable Y_{t-1} is not correlated with ε_{t3}.

Similar reasoning leads to the statement that in (6) Y_{t-1} does not correlate with ε_{t2}, and in (7) is it not correlated with ε_{t3}.

Thus, the random independent variable Y_{t-1}, as well as all deterministic independent variables are not correlated with errors in the relevant regressions. For consistency of estimates of the reduced form coefficients, it is also necessary for the asymptotic properties of the independent variables to meet certain requirements, see for example (Demidenko, 1981; Johnston and DiNardo, 1997; Korkhin, 2005). As it is generally accepted in econometrics, let's assume that all the independent variables in (5)–(7) should meet these requirements. This allows to conclude that the reduced form estimates of the coefficients (5)–(7) will be consistent.

When they are estimated for each regression separately for final observation interval, the problem of non-unique transition from the reduced form factor estimates to the estimates of the structural coefficients appears. Indeed, the system of nonlinear equations (8) consists of 9 equations for the 4 unknown coefficients. For example, all structural coefficients can be clearly assessed by the 4 equations for δ_{21}, δ_{30}, δ_{31} while estimating regression coefficients (6) and (7). However, these estimates do not necessarily meet the other 5 equations in (8).

To avoid such a case, the regressions (5)–(7) with the constraints (8) should be estimated at the same time for example by the method (Korkhin 1999). Moreover, the decision variables will be coefficients of the reduced and structural forms. To simplify the solution of the problem of estimating, let's substitute constraints (8) to (5)–(7). Then the system of equations will be:

$$C_t = \frac{\alpha_0 + \alpha_1\beta_0}{1-\alpha_1} + \frac{\alpha_1\beta_1}{1-\alpha_1}Y_{t-1} + \frac{\alpha_1}{1-\alpha_1}(G_t - T_t) + \varepsilon_{t1},\ t = 1,2,\ldots; \quad (11)$$

$$I_t = \beta_0 + \beta_1 Y_{t-1} + \varepsilon_{t2},\ t = 1,2,\ldots; \quad (12)$$

$$Y_t = \frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1}Y_{t-1} + \frac{1}{1-\alpha_1}G_t - \frac{\alpha_1}{1-\alpha_1}T_t + \varepsilon_{t3},\ t = 1,2,\ldots \quad (13)$$

Regressions (11)–(13) are non-linear by the sought structural coefficients. They emphasize the coefficients of the exogenous variables, these coefficients show the reaction in the current period of each endogenous variable to change of the current values of exogenous one. Such quantities are called impulse multipliers.

The reduced form (11)–(13) is linear by the independent variables, which are exogenous variables and lagged values of the endogenous variable Y_{t-1}, the effect of simultaneous changes in several independent variables (they are in the right sides of the equations) will be equal to the sum of partial effects.

Initial information for assessment of structural coefficient was: investing data, consumer spending information, Gross Domestic Product (GDP), national income, income tax in Ukraine of 2000–2012 (Statistical Yearbook of Ukraine, 2006; 2012). To eliminate the incompatibility of statistics in time, they were brought to a comparable form by comparing levels of GDP growth in the percentage to 2000 (Table 1).

To do this, the indicators of the base period in 2000 should be multiplied by the conversion factors (GDP by 2000).

Estimation of structural coefficients has occurred in two stages. At the first stage, due to lack of information about the error variance in the regressions (11)–(13) it was assumed they were the same, which led to the problem of estimating

$$S_1 + S_2 + S_3 \to \min, \quad (14)$$

where the minimization was performed by α_0, α_1, β_0 and β_1,

$$S_1 = \sum_{t=2}^{T}\left[C_t - \left(\frac{\alpha_0 + \alpha_1\beta_0}{1-\alpha_1} + \frac{\alpha_1\beta_1}{1-\alpha_1}Y_{t-1} + \frac{\alpha_1}{1-\alpha_1}(G_t - T_t)\right)\right]^2; \quad (15)$$

$$S_2 = \sum_{t=2}^{T} \left[I_t - \left(\beta_0 + \beta_1 Y_{t-1} \right) \right]^2 ; \tag{16}$$

$$S_3 = \sum_{t=2}^{T} \left[Y_t - \left(\frac{\alpha_0 + \beta_0}{1-\alpha_1} + \frac{\beta_1}{1-\alpha_1} Y_{t-1} + \frac{1}{1-\alpha_1} G_t - \frac{\alpha_1}{1-\alpha_1} T_t \right) \right]^2 . \tag{17}$$

Table 1. **Initial Data Brought to a Comparable Form**

Years	*t*	GDP at current prices, *bln UAH*	GDP by 2000, %	In the prices of 2000, *bln UAH*				
				National Income	Investing	Consumer Spending	Government Spending	Income Tax
2000	1	170.07	1	196.08	45.33	127.98	22.77	6.42
2001	2	204.19	1.09	214.24	49.53	139.83	24.87	7.01
2002	3	225.81	1.15	225.58	52.15	147.24	26.19	7.39
2003	4	267.34	1.26	246.92	57.09	161.16	28.67	8.08
2004	5	345.11	1.41	276.87	64.01	180.72	32.15	9.07
2006	7	544.15	1.56	305.47	70.62	199.38	35.46	10.00
2007	8	720.73	1.68	329.52	76.18	215.08	38.26	10.79
2008	9	948.06	1.72	336.79	77.86	219.82	39.10	11.03
2009	10	913.35	1.47	287.31	66.43	187.53	33.36	9.41
2010	11	1082.57	1.52	298.66	69.05	194.94	34.67	9.78
2011	12	1302.08	1.60	314.55	72.72	205.30	36.52	10.30
2012	13	1408.89	1.61	315.00	72.83	205.60	36.57	10.31

Source: Statistical Yearbook of Ukraine, 2006; 2012.

In (15)—(17) T is the duration of the observation period, in our case T = 13 years.

As the result of solving the problem of estimating (14)—(17), estimates of structural coefficients $\hat{\alpha}_0 = 0.02,\ \hat{\alpha}_1 = 0.68,\ \hat{\beta}_0 = 5.55,\ \hat{\beta}_1 = 0.21$ have been obtained. They satisfy a priori the restrictions on the coefficients of the model in structural form $0 < \alpha_1 < 1$, $\beta_1 > 0$. Moreover, relative errors for the 3 regression equations are e_1 = 5%, e_2 = 8%, e_3 = 3%. These errors have been defined as the ratio of the average values C_t, I_t, Y_t for the range [1,T] to the standard deviation of the residuals in the corresponding regressions (14)—(17).

The residuals have been determined by the formulas:

$$\hat{\varepsilon}_{t1} = C_t - \hat{C}_t,\ \hat{\varepsilon}_{t2} = I_t - \hat{I}_t,\ \hat{\varepsilon}_{t3} = Y_t - \hat{Y}_t,\ t = \overline{2,T}. \tag{18}$$

Here

$$\hat{C}_t = \frac{\hat{\alpha}_0 + \hat{\alpha}_1\hat{\beta}_0}{1-\hat{\alpha}_1} + \frac{\hat{\alpha}_1\hat{\beta}_1}{1-\hat{\alpha}_1} Y_{t-1} + \frac{\hat{\alpha}_1}{1-\hat{\alpha}_1}(G_t - T_t),\ t = \overline{2,T}; \tag{19}$$

$$\hat{I}_t = \hat{\beta}_0 + \hat{\beta}_1 Y_{t-1},\ t = \overline{2,T}; \tag{20}$$

$$\hat{Y}_t = \frac{\hat{\alpha}_0 + \hat{\beta}_0}{1-\hat{\alpha}_1} + \frac{\hat{\beta}_1}{1-\hat{\alpha}_1} Y_{t-1} + \frac{1}{1-\hat{\alpha}_1} G_t - \frac{\hat{\alpha}_1}{1-\hat{\alpha}_1} T_t,\ t = \overline{2,T}, \tag{21}$$

where C_t, I_t, Y_t, $t = \overline{1,T}$ are the actual values.

After analyzing the results we reveal high correlation between the residues (Table 2).

Table 2. **Correlation coefficients of residuals**

	$\hat{\varepsilon}_{t1}$	$\hat{\varepsilon}_{t2}$	ε_{t3}
$\hat{\varepsilon}_{t1}$	1		
$\hat{\varepsilon}_{t2}$	0.90	1	
ε_{t3}	0.84	0.52	1

At the second stage to improve the estimation accuracy, we divide S_1 by $\hat{\sigma}_1^2$, S_2 by $\hat{\sigma}_2^2$, S_3 by $\hat{\sigma}_3^2$, where $\hat{\sigma}_i^2$ is the variance of residuals of i-th regression, i = 1, 2, 3 defined by (18). Estimation problem takes the form

$$\frac{S_1}{\hat{\sigma}_1^2}+\frac{S_2}{\hat{\sigma}_2^2}+\frac{S_3}{\hat{\sigma}_3^2}\to \min, \qquad (22)$$

where S_1, S_2, S_3 are defined by (15)–(17).

Its solution is the estimate of the structural coefficients of the model $\hat{\alpha}_0=0.02$, $\hat{\alpha}_1=0.67$, $\beta_0=6.67$, $\beta_1=0.21$. The relative errors for the 3 regression equations are e_1 = 5%, e_2 = 7%, e_3 = 3%. Thus, the accuracy of the model has increased slightly and is satisfactory. The obtained estimates of the model parameters were substituted in the structural form of the model (1)–(3), and the calculated values of endogenous variables were found as:

$$\hat{C}_t=0{,}02+0{,}67(Y_{t-1}-T_t),\ t=\overline{2,T}; \qquad (23)$$

$$\hat{I}_t=6{,}72+0{,}21Y_{t-1},\ t=\overline{2,T}; \qquad (24)$$

$$\hat{Y}_t=\hat{C}_t+\hat{I}_t+G_t,\ t=\overline{2,T}. \qquad (25)$$

For these values relative errors of the structural form equations were determined for the following endogenous variables: $\hat{C}_t$ is 0.3%, $\hat{I}_t$ is 0.2%, $\hat{Y}_t$ is 0.0%. This accuracy is satisfactory.

It can be deducted from the analysis of the original data (Table 1) that a sharp decline in the dynamics of the quantities is due to the economic crisis of 2008. Therefore, the parameter estimates were calculated for the data model for 2000–2008: $\hat{\alpha}_0=0.02$, $\alpha_1=0.67$, $\beta_0=5.75$, $\beta_1=0.22$; wherein for the errors e_1 = 4%, e_2 = 3%, e_3 = 2%.

Comparing the estimates of the model parameters obtained by excluding the impact of the economic crisis in 2008 and taking into account the impact of the economic crisis in 2008, we can conclude about the stability of estimates. The present study has not identified changes in the structural coefficients.

Conclusions. In the paper, the estimates parameters of the model for determining the national income as the sum of consumer spending, investment and government procurement of goods and services are determined.

The impact of the economic crisis in 2008 on the values of the estimates of the model parameters is analyzed.

The direction of future research is to test the quality of the resulting model, to make the statistical analysis of the accuracy of the constructed model and to identify the ways to improve it.

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