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MODELLING AN EFFECTIVE SALES STRATEGY FOR ENTERPRISES SELLING PRODUCTS AT THE DOMESTIC MARKET AND ABROAD

The paper offers a methodological approach to the use of game theory in a situation that does not have an explicit interest's conflict and individual players. The procedure for constructing bimatrix game models is formulated. As the efficiency criteria, in addition to the usual profit maximization, the author applies Pareto optimum and Nash equilibrium.

Keywords: *game theory; Pareto optimum; Nash equilibrium; sales strategy.*
JEL classification: C67; C70.

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МОДЕЛЮВАННЯ ЕФЕКТИВНОЇ СТРАТЕГІЇ ЗБУТУ ДЛЯ ПІДПРИЄМСТВ, ЩО РЕАЛІЗУЮТЬ ПРОДУКЦІЮ НА ВНУТРІШНЬОМУ РИНКУ ТА ЗА КОРДОНОМ

У статті запропоновано методологічний підхід до використання теорії ігор в ситуації, що не має явного конфлікту інтересів та окремих гравців. Сформульовано процедуру побудови біматричної моделі гри. В якості критеріїв ефективності, крім звичної максимізації прибутку, використано поняття оптимальності за Парето та рівноваги Неша.

Ключові слова: *теорія ігор; оптимальність за Парето; рівновага Неша; стратегія збуту.*
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МОДЕЛИРОВАНИЕ ЭФФЕКТИВНОЙ СТРАТЕГИИ СБЫТА ДЛЯ ПРЕДПРИЯТИЙ, РЕАЛИЗУЮЩИХ ПРОДУКЦИЮ НА ВНУТРЕННЕМ РЫНКЕ И ЗА РУБЕЖОМ

В статье предложен методологический подход к использованию теории игр в ситуации, не имеющей явно выраженного конфликта интересов и отдельных игроков. Сформулирована процедура построения биматричной модели игры. В качестве критериев эффективности, кроме общепринятой максимизации прибыли, использованы понятия оптимальности по Парето и равновесия Неша.

Ключевые слова: *теория игр; оптимальность по Парето; равновесие Неша; стратегия сбыта.*

Problem setting. In the contemporary context of domestic production and market relations, there is an acute problem of finding new markets. A substantial part of raw materials and semi-finished products are export oriented, finding sales because of relatively low prices, which, however, remain unattainable for domestic consumers.

The economy has a proven law of "diminishing returns", according to which a significant increase in the scale of production does not always lead to a proportional increase in profits. Market prediction is that the simple increase production, especially exports, without solving the structural, quality and costs problems of supplies, will not correspond to the needs of both domestic and foreign markets.

This leads us to the issue of determining the optimal balance between exports and domestic sales in the sales structure of a company that would satisfy producer (in terms of profit maximization) and domestic consumers of its products.

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Recent research and publications analysis. At present the game theory is used to model economic problems of the industry (Gibbons, 1992), tenders and auctions (Bolton and Dewatripont, 2005), production behavior of firms (Kreps, 1990), competition and trade government's policy (Acemoglu, 2009), monetary policy (Gibbons, 1992), taxation (Busygin et al., 2000), the public good (Persson and Tabellini, 2000). Thus, game theory is now widely used as a powerful tool in socio-economic research.

The research objective is to determine the optimal distribution of sales between export and domestic sales in a way to maximize company's profits using the mathematical tools of the game theory.

Key research findings. The company, whose manufacturing facilities are known, sells its products at the global and domestic market at different prices. Let us denote P^{exp} – price for export and P^{dom} – price for domestic market, $P^{exp} \neq P^{dom}$. Known total capacity of consumption of the assortment for each of the markets: D^{exp}, D^{dom}. Exports cost of production and promotion C^{exp} are greater than the cost of production and promotion of products inside the country C^{dom}: $C^{exp} > C^{dom}$.

To determine the optimal distribution of sales between export and domestic sales, let's consider the application of more flexible economic and mathematical model for finding the optimal solution – game theory. Playing techniques make it possible to choose the best, under certain conditions, line of conduct, a set of rules under which it is possible to reach the highest possible average gain (von Neumann and Morgenstern, 1944; Vitlinskij et al., 2002).

Precondition for the use of game approach is the existence of a conflict, when a decision or outcome is determined by joint decision of several persons that pursue different goals. Such circumstances are so typical that rather difficult arbitrary situation of decision-making can be represented as game-theoretic; therefore, no coincidence games theory approaches take a leading position in the contemporary studies on optimal decision-making.

In our problem there is one side that makes decisions – a company that seeks to maximize its profit, and therefore there is no clearly defined conflict of interest. The author proposes to consider the sales share at the global and domestic markets as parts sales generated depending on various factors (price and non-price prevailing at each market) and seeking to increase their weight in the sales structure. Profits from the sale at both markets depend on: production costs and sales structure. This correlation has a conflicting nature and fully complies with the provisions of the game theory.

We have a formal conflict situation: distribution of scarce possibilities output at different prices to maximize the total return, where the role of the conflict sides perform the sales at foreign and domestic markets, which the company aims to increase for gross profit. Demand at both markets has restrictions that must be considered. Using the game theory in this case is unconventional because the sides of the conflict (sales at two markets) are subordinated to the same system, but may be considered as separate subsystems, each tending to grow and increase its share.

The main purpose of this study is to build the general game model to find the optimal sales structure that would ensure the best satisfaction of producer. The share of sales of other companies with similar products at the same markets is not considered, neither are internal state regulation of economic activity (fiscal policy, subsidies et al.), export restrictions by importers (quotas and so on). It is believed that production volume for the optimal solution is fully implemented.

Determination of the optimal sales structure and the corresponding total theoretical game model is below.

Let us denote by Y_i, $i = (1,n)$ and Y_j, $j = (1,m)$ the set or space of all possible strategies that games players can use. Due to previous assumptions, strategies of the first player will be all possible export share in sales structure and strategies of the other players – all possible share of sales at the domestic market. Total sales volume is not greater than the production capacity of the enterprise Y_0:

$$Y_i + Y_j \le Y_0. \tag{1}$$

Volumes at the global and domestic markets cannot exceed demand, formed at each market for this price:

$$0 \le Y_i \le D^{dom}(p_i); \tag{2}$$

$$0 \le Y_j \le D^{\exp}(p_j). \tag{3}$$

Since the number of players is 2, and the results can be evaluated numerically (by calculating income), game can be represented in a wide array of strategy, the so-called game's bi-matrix. It is convenient to set (Figure 1) the first line set strategy issued by the first player, and the first column – the set strategy of the second player.

	Y_1	...	Y_j	...	Y_m
Y_1	(a_{11}, b_{11})	...	(a_{1j}, b_{1j})	...	(a_{1m}, b_{1m})
…	…		…		…
Y_i	(a_{i1}, b_{i1})	...	(a_{ij}, b_{ij})	...	(a_{im}, b_{im})
…	…		…		…
Y_n	(a_{n1}, b_{n1})	...	(a_{nj}, b_{nj})	...	(a_{nm}, b_{nm})

Figure 1. **Game's bi-matrix for the optimal sales structure's determination,** *authors' development*

The limit dimension of the matrix is defined by the limited abilities of manufacturer and demand. Considering the previous limitations (formula 1), we have two marginal situations that determine the maximum matrix index by i and j:

$$Y_n = Y_m = Y_o. \tag{4}$$

The total number of possible solutions is n x m, where n and m are determined by the minimum deliveries. With the maximum dimension n and m can construct a matrix model of the game.

Each matrix element is composed of two parts: the profit received by the producer realizing its products at domestic and global markets. Total return I_0 the company is receiving consists of two parts: I_j^{exp} and I_i^{dom} – income from sales domestically and globally, respectively. The optimality criterion of this model will simultaneously maximize both indicators that is the essence of the conflict: $I_i^{dom}, I_j^{exp} \to \max$.

Incomes from domestic sales depends on the marginal cost (MC_i) of production and sales structure:

$$I_i^{dom} = f(Y_i, Y_j, MC_i). \quad (5)$$

Similarly, for the export trade:

$$I_j^{\exp} = f(Y_i, Y_j, MC_j). \quad (6)$$

At the same time, Y_i / Y_j is the function of demand $D_i^{dom}(p_i)$ / $D_j^{exp}(p_j)$ and production capabilities Y_0:

$$Y_i = f(D_i^{dom}(p_i), Y_0); \quad (7)$$

$$Y_j = f(D_j^{\exp}(p_j), Y_0). \quad (8)$$

Suppose the demand function $D_j^{exp}(p_j)$ is linearly dependent on domestic prices in such way – $D_i^{dom}(p_i) = D_i^{dom} - b_i \times p_i$ and decreases with rising prices: $\frac{\partial D^{dom}}{\partial p} < 0.$

Similar considerations are for demand at the international market: $D_j^{exp}(p_j) = D_j^{exp} - b_j \times p_j$ for $\frac{\partial D^{\exp}}{\partial p} < 0,$ where b_i, b_j are the coefficient of elasticity for the relevant markets.

Marginal cost functions $MC_i(Y_i)$, $MC_j(Y_j)$ in the longer term period reflect the well-known characteristics of most production patterns and have the U-shaped form (Yastremskyi and Grycenko, 1998).

Now we have a multiobjective optimization problem with 3 objective functions:

$$I_i^{dom} = f(Y_i, Y_j, MC_i, D_i^{dom}(p_i)) \to \max; \quad (9)$$

$$I_j^{\exp} = f(Y_i, Y_j, MC_j, D_j^{\exp}(p_j)) \to \max; \quad (10)$$

$$I^{total} = I_i^{dom} + I_j^{\exp} = I_i^{dom} = f(Y_i, Y_j, MC_i, D_i^{dom}(p_i)) + \\ + f(Y_i, Y_j, MC_j, D_j^{\exp}(p_j)) \to \max, \quad (11)$$

where I^{total} is the total income.

Let us consider the approaches to choosing the optimal solution using the game theory. As for the problems of decision-making under clarity with numerical estimation results, the aim is to find the extremum of objective function. For such problems, there is actually only the concept of optimum solution: the best solution would be the one, which provides objective function extreme value. We have a problem of decision making under certainty, but the result of it is measured by two indicators, and the aim is to increase both indicators at the same time.

Consider two approaches to determine the optimal solution in gaming models:

1. Pareto efficiency, or Pareto optimality, is the state of allocation of resources in which it is impossible to make anyone individual better off without making at least one individual worse off (Pareto, 1897). This means that the strategy (a_{ij}, b_{ij}) is Pareto optimal if $a_{ij} \geq a_{ij}'$ and $b_{ij} \geq b_{ij}'$ and at least one of these inequalities is strict. Point (a_{ij}', b_{ij}') is dominated. Any Pareto-optimal state that can not be improved immediately for all players is therefore the most advantageous for the coalition, which includes all players, but it may be disadvantageous for one (or more) of these players. Selecting

players Pareto-optimum state involves their cooperation (information exchange between them about decisions taken), in result "collective" coalition's interests of all players are put above the interests of individuals. If players choose their own strategy without cooperation, they are guided by only personal interests; in this case, we expect them to choose the state optimal by Nash.

2. Nash equilibrium is a solution concept of a non-cooperative game involving two or more players, in which each player is assumed to know the equilibrium strategies of other players, and no player has anything to gain by changing only their own strategy (Osborne and Rubinstein, 1994). The best decision in Nash equilibrium for game, set in a matrix (Figure 1) – meaning equability state (a_{i0}, b_{j0}), where all i = 1, ..., n, j = 1, ..., m, $a_{ij0} \le a_{i0j0}$, $b_{i0j} \le b_{i0j0}$.

Nash equilibrium situation is manifested in relation to stability. Volatility of the situation can happen due to the collapse caused by one of the players with opportunities to get the best result by an unilateral change of strategy.

Possible divergence between Pareto optimality and Nash equilibrium has no paradox because of different optimality "ideological foundations". Pareto optimality is based on an advantage for the whole system, understood as the advantage for all its subsystems at once. And Nash equilibrium is the stability of the system, caused by the interests and abilities of individual subsystems. Thus, the difference between Pareto optimality and Nash equilibrium is the difference between cooperation and stability.

Using both approaches to determine the optimum decision we define two sets of optimal solutions P for Pareto and N for Nash:

$$P(a^*, b^*) : a_i^* \ge a_i, b_j^* \ge b_j; \tag{12}$$

$$N(a_{i^*}, b_{j^*}) : a_{i^* j^*} \ge a_{ij^*}, b_{i^* j^*} \ge b_{i^* j}. \tag{13}$$

Following the principle of maximizing total return for each set of P and N, we choose the best solution:

$$I^{total} = \max\{I^{dom} + I^{\exp}\}, \tag{14}$$

which corresponds to the optimal sales structure: $I^{total}(Y_i^*, Y_i^*)$.

Taking into account that the decision is made by the company, both subsystems of which are subordinated, we can expect that more correct is the use of Pareto optimality, but at the same time there is a chance that the best solutions by Pareto and Nash will coincide.

Conclusions. In this article the methodology and application of game theory to determine the optimal sales structure is suggested. Using the game theory in this case is unconventional because the conflict parties here (sales at two different markets) as subordinated to the same system, but may be considered as individual players, each committed to growth and rising own share. This interdependence and limited possibilities of production make up the essence of the conflict situation, where the role of conflicting sides belongs to separate sales parts. We have a large number of solutions that correspond to different ratios of its parts in the sales structure.

The procedure of determining the optimal solution is based on building game's bi-matrix, each cell describing a market structure.

When finding the set of efficient solutions in addition to profit maximization criterion the use Nash equilibrium and Pareto optimality is considered. The specificity of the conditions under which a given problem using game theory is suggested that despite some differences of both views the best solutions coincide.

Using this approach in practice makes it possible to develop scientifically grounded policy for large companies that sell their products at both domestic and foreign markets.

Further research would be are aimed at developing the general model for a number of game player equal to 3. This extends the range of economic problems that can be formally described using this model, including the ability to take into account the impact of regulatory state.

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