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FORMATION OF NOMINAL VALUES OF THE PROCESS INDICATORS UNDER FUZZY-STOCHASTIC UNCERTAINTY

The paper investigates the problems of nominal values calculation of the process indicators under information uncertainty. On the basis of the characteristic values of representation, which combines several types of uncertainty the methods for solving optimization problems of socioeconomic system management under information uncertainty by aggregating different types of uncertain parameters are offered.

Keywords: *stochastic uncertainty; fuzzy uncertainty; information support; higher education institusion.*

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ФОРМУВАННЯ НОМІНАЛЬНИХ ЗНАЧЕНЬ ПОКАЗНИКІВ ПРОЦЕСУ В УМОВАХ НЕЧІТКО-СТОХАСТИЧНОЇ НЕВИЗНАЧЕНОСТІ

У статті досліджено проблеми розрахунку номінальних значень показників процесу в умовах інформаційної невизначеності. На основі характеристичного представлення величини, яка поєднує в собі декілька типів невизначеності, запропоновано методи розв'язування оптимізаційних задач управління соціально-економічною системою в умовах інформаційної невизначеності шляхом агрегування різних типів невизначених параметрів.

Ключові слова: *стохастична невизначеність; нечітка невизначеність; інформаційне забезпечення; вищий навчальний заклад.*

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ФОРМИРОВАНИЕ НОМИНАЛЬНЫХ ЗНАЧЕНИЙ ПОКАЗАТЕЛЕЙ ПРОЦЕССА НЕЧЕТКО-СТОХАСТИЧЕСКОЙ НЕОПРЕДЕЛЕННОСТИ

В статье исследованы проблемы расчета номинальных значений показателей процесса в условиях информационной неопределенности. На основе характеристического представления величины, которая сочетает в себе несколько типов неопределенности, предложены методы решения оптимизационных задач управления социально-экономической системой в условиях информационной неопределенности путем агрегирования различных типов неопределенных параметров.

Ключевые слова: *стохастическая неопределенность; нечеткая неопределенность; информационное обеспечение; высшее учебное заведение.*

Problem setting. Processes in higher education differ substantially from those in production. The main characteristics of processes within higher education institutions is the lack of normative values for most processes, uncertainty of indicators, a large quantity of quality indicators and considerable influence of the human factor (Bogolib, 2012). In this case, the most effective method of finding nominal values of the process characteristics is modelling the problem of utility function optimization of the key process indicators under stochastic and fuzzy uncertainty.

Latest research and publications analysis. Most scientific publications on the methodology of process management under uncertainty are related to production

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processes, with key performance indicators which have quantitative characteristics. Methods of statistical process control were widely considered by D. Chambers (2009), W. Deming (2007), J. Shottmiller (2004), D. Wheeler (1991; 2009) and others. However, the characteristics of processes in higher education institutions make it impossible to use the classic principles of monitoring used in the production sector.

General settings of stochastic (Yastremskiy, 1983; Yudin, 1979) and fuzzy (Altunin and Semuhin, 2000; Borisov, Alekseev and Merkureva, 1989) optimization are well known, but examples of solving specific economic problems with stochastic and fuzzy parameters are rather seldom.

The goal of the article is to develop the principles of formation of nominal values of key performance indicators based on process control optimization models under stochastic and fuzzy uncertainty.

Key research findings. We introduce the characteristic representation of the value, which combines several types of uncertainty and can be represented as a stochastic, fuzzy and deterministic value (Yemets and Roskladka, 2008):

$$\hat{a}(\omega)=\{[(m^{(1)},\sigma^{(1)}),\mu^{(1)}],[(m^{(2)},\sigma^{(2)}),\mu^{(2)}],\ldots,[(m^{(\eta)},\sigma^{(\eta)}),\mu^{(\eta)}]\}, \quad (1)$$

where $a^i=[(m^{(i)},\sigma^{(i)}),\mu^{(i)}]$ – normally distributed random variable with mean $m^{(i)}$, the mean square deviation $\sigma^{(i)}$ and membership function $\mu^{(i)}$; η – the quantity of elements of a fuzzy set; ω – stochastic parameter.

If the resource restrictions are linear and the target function contains a deterministic values, the model can be written as

$$F=\sum_{j=1}^{n} w_j x_j \to \max; \quad (2)$$

$$\sum_{j=1}^{n} w_j = 1; \quad (3)$$

$$\sum_{j=1}^{n} \hat{a}_{ij}(\omega) x_j \le b_i,\ i=1,2,\ldots,m; \quad (4)$$

$$x'_j \le x_j \le x''_j,\ i=1,2,\ldots,n. \quad (5)$$

In this model F is the utility function of the set of indicators $x = \{x_1, x_2, ..., x_n\}$ with the corresponding weighting coefficients $w = \{w_1, w_2, ..., w_n\}$, whose elements satisfy the condition of normalization (3). The system (4) may contain human, financial, informational, logistical and other resource linear restrictions with fuzzy-stochastic coefficients of the form (1). Double inequalities of the form (5) include top and lower limits of normative values of indicators.

We will consider formation of the target function and the resource restrictions for determining the nominal values of the process indicators of information support in higher education institutions.

Let x_i be the required nominal value of the i-th indicator process; w_i – weight coefficient of indicator x_i.

We will consider the 4 key indicators of the information support process in higher education institutions: x_1 – speed Internet access; x_2 – the quality level of the site; x_3 – providing students with computers; x_4 – workload of the information center employees.

According to expression (2), the target function of model for determination of nominal values of the indicators will look like:

$$F = w_1x_1 + w_2x_2 + w_3x_3 - w_4x_4 \to \max. \quad (6)$$

Sign «-» in front of the fourth term indicates a negative role of this component (workload of the information center employees) in the overall usefulness of the process.

We will consider the examples of restrictions of human and financial support of the process. For the formation of human restrictions, we introduce the following notation: $\hat{a}_{11}$ – quantity of network technology specialists, controlling hardware and software to access the Internet; $\hat{a}_{12}$ – the quantity of specialists required for a functional site at a given level; $\hat{a}_{13}$ – the quantity of personnel involved in servicing computer equipment for the education process; $a_{14}(\omega)$ – the quantity of applications received by the information center; b_1 – the total quantity of employees in the information center.

Coefficients $\hat{a}_{11}$, $\hat{a}_{12}$, $\hat{a}_{13}$ are fuzzy parameters, which depend on the qualifications of the expert; $a_{14}(\omega)$ – stochastic parameter.

The restriction of staffing process will look like:

$$\hat{a}_{11}x_1 + \hat{a}_{12}x_2 + \hat{a}_{13}x_3 + \frac{a_{14}(\omega)}{x_4} \le b_1. \quad (7)$$

For the formation of financial restriction, we introduce the following notations:

$\hat{a}_{21}$ – tariff of the Internet service provider;

$\hat{a}_{22}$ – the budget for the normal operation of the site;

$\hat{a}_{23}$ – the costs of operating per one computer;

$\hat{a}_{24}$ – hourly pay of information support specialist;

b_2 – budget of Information Center.

Facto $\hat{a}_{21}$ is the fuzzy value that depends on the Internet service provider; $\hat{a}_{22}$ – fuzzy value, which has membership function, is proportional to the estimate of the site quality; $\hat{a}_{23}$ is a fuzzy value that depends on hardware and software configuration of a computer; the coefficient $\hat{a}_{24}$ – fuzzy value that depends on the qualification of an expert.

Restrictions on the financial support of the process will look like

$$\hat{a}_{21}x_1 + \hat{a}_{22}x_2 + \hat{a}_{23}x_3 + \frac{\hat{a}_{24}}{x_4} \le b_2. \quad (8)$$

Among the 4 key indicators only indicator x_3 has a certain normative limit: $x_3 \ge 12$. Other indicators should satisfy the condition of non-negativity only. Thus, the restriction of the form (5) take the form

$$x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 12,\ x_4 \ge 0. \quad (9)$$

Model (6)–(9) is the model of nonlinear fuzzy-stochastic optimization. For example, we will consider the task with two parameters x_1 and x_3, which enables the graphical interpretation of the optimal solution. The model takes the form:

$$F = w_1x_1 + w_3x_3 \to \max; \quad (10)$$

$$\hat{a}_{11}x_1 + \hat{a}_{13}x_3 \le b_1; \quad (11)$$

$$\hat{a}_{21}x_1 + \hat{a}_{23}x_3 \le b_2; \quad (12)$$

$$x_1 \ge 0,\ x_3 \ge 12. \quad (13)$$

Values w_1 and w_3 in the target function (10) are obtained on the basis of expert estimates and their subsequent valuation: $w_1 = 0.436$, $w_3 = 0.564$.

Values $\hat{a}_{ij}$ in the restrictions (11) and (12) are defined by the following empirical research of the author. Factor $\hat{a}_{11}$ determines the quantity of network technology specialists required for hardware and software support access to the global network when ensuring the speed of 1 Mbit/s. For increasing the speed of Internet connection 5 Mbit/s for controlling the hardware and software requires 1 or 2 specialists depending on qualification. Having made the recalculation of 1 Mbit/s required quantity of specialists can be represented by the fuzzy number {(0.2|0.65)(0.4|0.01)}.

Factor $\hat{a}_{13}$ determines the quantity of service technics of hardware and software per 1 computer. Depending on the qualification, one specialist of information center can serve 5, 8 or 10 computers. Having made recalculation per 1 computer, the required quantity of specialists can be represented by the fuzzy number {(0.2|0.85)(0.125|0.8)(0.1|0.15)}.

Let the total quantity of service employees in information center be 12.

Factor $\hat{a}_{21}$ represents the tariff of Internet service provider for 1 Mbit/s, that fluctuates within 40 UAH/(Mbit/s), and can be represented by the fuzzy number {(37|0.15)(42|0.97)(43|0.46)}.

Factor $\hat{a}_{23}$ is a monthly cost of hardware and software. According to experts estimates, such costs fluctuate between 40–60 UAH per 1 computer and can be represented by the fuzzy number {(40|0.11)(50|0.61)(60|0.85)}.

Let the monthly budget for computer support not exceed 2800 UAH.

Then the restrictions in the problem to define the nominal values of speed access to the Internet and providing students with computers take the form:

$$\{(0.2|0.65),(0.4|0.01)\}x_1 + \{(0.2|0.85),(0.125|0.8),(0.1|0.15)\}x_3 \le 12; \quad (14)$$

$$\{(37|0.15),(42|0.97),(43|0.46)\}x_1 + \\ + \{(40|0.11),(50|0.61),(60|0.85)\}x_3 \le 2800. \quad (15)$$

For solving the problem we need would the defuzzification of fuzzy variables. We apply defuzzification by the gravity center method:

$$a = \frac{\sum_{i=1}^{\eta} a_i \mu(a_i)}{\sum_{i=1}^{\eta} \mu(a_i)}, \quad (16)$$

where a_i – the element of the carrier of a fuzzy set; $\mu(a_i)$ – the value of the membership function for the element a_i.

We apply the defuzzification coefficients $\hat{a}_{ij}$ in restrictions (14), (15) by formula (16). The problem (10)–(13) takes the form

$$F = 0.436x_1 + 0.564x_3 \to \max; \quad (17)$$

$$0.203x_1 + 0.158x_3 \le 12; \quad (18)$$

$$41.816x_1 + 54.714x_3 \le 2800; \quad (19)$$

$$x_1 \ge 0,\ x_3 \ge 12. \quad (20)$$

We give the graphical interpretation of restrictions (18)–(20), using the function *inequal* in the system of computing algebra "Maple" (Figure 1).

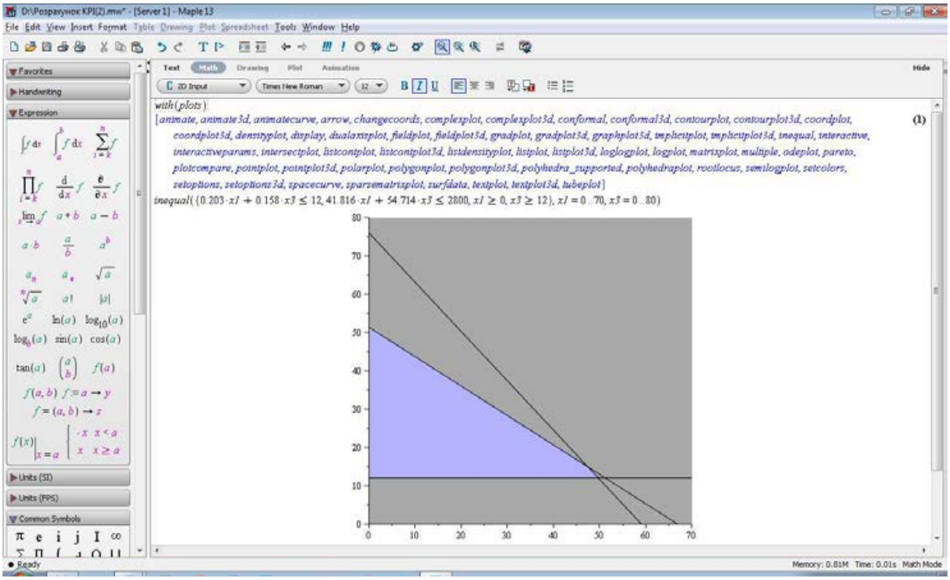

Figure 1. **Graphical interpretation of restrictions of the problem (18)–(20),** *author's development*

Solution of the problem is obtained by applying the function *maximize* from library simplex in "Maple" (Figure 2).

with(*simplex*)

[*basis, convexhull, cterm, define_zero, display, dual, feasible, maximize, minimize, pivot, pivoteqn, pivotvar, ratio, setup, standardize*] **(2)**

maximize(0.436·*x1* + 0.564·*x3*, {0.203·*x1* + 0.158·*x3* ≤ 12, 41.816·*x1* + 54.714·*x3* ≤ 2800, *x3* ≥ 12}, *NONNEGATIVE*)

{*x1* = 47.59274082, *x3* = 14.80173173} **(3)**

Figure 2. **The solution of problem (17)–(20) in the system "Maple",** *author's development*

Thus, the nominal value of access speed to the global network is 47.593 Mbit/s; the nominal value of the index of providing students with computers is 14.802 ≈ 15 computers per 100 students.

Conclusions. Using fuzzy and stochastic parameters increases the level of model adequacy to real processes. The approach presented in this article allows establishing the nominal values of key performance indicators and use them successfully in the monitoring systems for diagnosis and control of processes at higher education institutions.

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