

Seit Kerimkulov<sup>1</sup>, Saida Teleuova<sup>2</sup>, Gulzhan Tazhbenova<sup>3</sup>  
**MEASURING CHAOTIC AND CYCLIC FLUCTUATIONS  
 OF CASS FREIGHT INDEX: EXPENDITURES**

*This article provides measuring and analysis of the chaotic dynamics of the Cass Freight Index: Expenditure for the period 2001–2014. Chaotic attractors for the position, force and phase portrait are developed, as well as long and medium cycles for econometric analysis of chaotic and cyclic fluctuations of the index. The constructed attractors are used to measure conjunctural and cyclic characteristics of the transport index for the stochastic and economic dynamics.*

*Keywords:* Cass Freight Index; chaotic attractor; phase portrait; chaotic and cyclic fluctuations; econometric analysis; stochastic dynamics.

*JEL:* C43; D91; L91.

Сейт Керімкулов, Саїда Телеуова, Гульжан Тажбенова  
**ВИМІР ХАОТИЧНИХ І ЦИКЛІЧНИХ КОЛИВАНЬ  
 ВАНТАЖНОГО ІНДЕКСУ КАСС: «ВИТРАТИ»**

*У статті проведено економетричний вимір та аналіз хаотичної динаміки вантажного індексу Касс: «Витрати» за період 2001–2014 років. Побудовано хаотичні аттрактори для положення, сили і фазового портрету, а також економетрично виміряно всі періоди і фази для одного довгого і двох середніх циклів руху досліджуваного часового ряду. Побудована структура аттракторів може використовуватися для вирішення прикладних завдань прогнозування, зокрема, при визначенні кон'юнктурних характеристик циклів стохастичної економічної динаміки транспортного сектору.*

*Ключові слова:* вантажний індекс Касс; хаотичний аттрактор; фазовий портрет; хаотичні і циклічні коливання; економетричний аналіз; стохастична динаміка.

*Форм. 6. Рис. 3. Табл. 3. Літ. 17.*

Сейт Керимкулов, Саида Телеуова, Гульжан Тажбенова  
**ИЗМЕРЕНИЕ ХАОТИЧЕСКИХ И ЦИКЛИЧЕСКИХ КОЛЕБАНИЙ  
 ГРУЗОВОГО ИНДЕКСА КАСС: «РАСХОДЫ»**

*В статье проанализирована и измерена хаотическая динамика грузового индекса Касс: «Расходы» за период 2001–2014 годов. Построены хаотические аттракторы для положения, силы и фазового портрета, а также длинный и средние циклы для эконометрического анализа хаотических и циклических движений индекса. Построенные аттракторы могут быть использованы для измерения конъюнктурных и циклических характеристик стохастической экономической динамики транспортного сектора.*

*Ключевые слова:* грузовой индекс Касс; хаотический аттрактор; фазовый портрет; хаотические и циклические колебания; эконометрический анализ; стохастическая динамика.

**Introduction.** Transport plays crucial role in the economy of any country. The logistics and transportation industry in developed countries, in particular, in the United States is highly competitive. In order to keep such a competitive environment it is important to regulate this sector. Indices play a major role in maintaining competitive environment. There are 5 major transport indices today: Cass Freight Index: Expenditures; Cass Freight Index: Shipments; Total Transportation Services Index; Passenger Transportation Services Index; Freight Transportation Services Index. The

<sup>1</sup> Gumilyov Eurasian National University; Kazakh Humanitarian Law University, Astana, Kazakhstan.

<sup>2</sup> Prague Economic University, Czech Republic.

<sup>3</sup> Kazakh Humanitarian Law University, Astana, Kazakhstan.

main feature of transport indices are high uncertainty and also high stochastic and cyclic dynamics. Hence, the traditional research methods need to be complimented by such models as the theory of deterministic chaos, parametric statistics and econometric modelling that reflects cyclical processes and stochastic dynamics, to which the current work is dedicated.

The abovementioned methods were investigated by many authors. Similar research has been carried out by Lee, Lun and Yan (2013). They studied a drop in profitability for transport. Angelidis and Skiadopoulos (2008), Drobetz, Richter and Wambach (2013) argue that transport indices can be seen as a universal mechanism for risks hedging. There is a number of contemporary publications on the issue of predicting transport indices as a basic indicator for transport. The work by Kavussanos and Dimitrakopoulos (2011) is noteworthy for using the GARCH model. Similar methodology has been used by Erdogan, Tata, Karahasan and Sengoz (2013) that studied interactions between transport and stock indices and found evidence of correlation.

Literature distinguishes two main issues within the framework of the theory of chaos. Goulielmos and Psifia (2011) argue that the presence of a large number of events in the system creates uncertainties that have tendencies to center around the trend or diverge, the so-called "fat tails" anomaly. Scott (1993) states that the tendency and development of chaotic process significantly depend on the initial state of chaos, in literature this process is also known as a "butterfly effect".

**Literature review.** During the first half of the last century, much research was devoted to empirical characterization of business cycles. Literature distinguishes the two main features in the definition of business cycles. One is the comovement or concurrence among individual economic indicators, and the other is a switching process between different regimes, i.e. expansions and recessions. Burns and Mitchell (1946: 373) considered business cycles as the division into separate phases or regimes, which basically means that certain series are classified as leading or lagging indicators of the cycle, depending on the general state of business conditions.

Tinbergen (1939) started to use the linear difference equation as a tool in business cycle analysis. His work was mainly focused on time-series properties of a few macroeconomic aggregates and ignored the pervasive movement.

According to Diebold and Rudebusch (1996) the linear structure imposed eliminated consideration of any nonlinearity of business cycles that would require separate analyses of expansions and contractions.

According to Mintz (1972), for a classical cycle, turning points in real GDP are described as peaks and troughs, with the periods between peaks and troughs denoted as contraction (expansion) phase.

Cycles, in shipping, last not only 20 years – as argued by Kavussanos and Alizadeh (2002) but also 4 years, as identified in (Goulielmos and Psifia, 2011).

Comin and Gertler (2003) presented evidence based on times series data that medium term cycle differ from the conventional measured cycles: "medium frequency movements appear to beat some relation to the high frequency volatility of output".

In other words, periods of stagnation are often associated with significant recessions, while persistent booms typically are either the idea of medium term cycles, which they define as reflecting the sum of high or medium frequency variation in

data. One of the major advantages of studying business cycles in the transport industry is that the size of the transport sector is much smaller than the economy as a whole. Jing et al. (2008) investigated the volatility in dry freight rates. He found that shocks are stronger per day and there are external shocks too that have different influence on volatility. Overall, academics agreed on high volatility of the transport industry to the changes of other macroeconomic indicators and therefore it has leading characteristics in relation to economic growth.

Efficiency of transport services is largely dependent on the presence of transport indices in economy, which create preconditions to make objective decisions within economic policy, rather than make them due to loyalty, administrative and territorial resources and subjective factors. They are important for attracting investment in the transport sector as well as the implementation of national programs, especially for highly productive economy with a large volume of transport services.

**Research objective.** The purpose of the current research is to measure and analyze the chaotic and cyclical movements of Cass Freight Index: Expenditure, based on the theory of deterministic chaos and parametric statistics and econometric modelling for various applications of stochastic dynamics of the transport industry. The study has the following objectives:

- to construct the chaotic attractors of the Cass Freight Index: Expenditures and to estimate its parameters for econometric measuring and analysis of chaotic and cyclical movements;
- to construct the chaotic attractors of the Cass Freight Index: Expenditures, change from a year ago and to estimate its parameters for econometric measuring and analysis of chaotic and cyclical movements;
- to construct the phase portrait of the Index and to estimate its parameters for econometric measuring and analysis of chaotic and cyclical movements.

**Indicators, data, models and methodology.** For econometric modelling and measuring the chaotic and cyclic movement of indices of the transport industry, in particular, for the Cass Freight Index: Expenditures the following well-known indicators will be used:

Moving Average with Time Periods  $T = 1, 2, \dots$  of the time series  $y = \{y_t\}_{t \in Z}$ ,  $Z = \{0, \pm 1, \pm 2, \dots\}$  is:

$$MA(T)y_t = \frac{1}{2T+1} \sum_{j=t-T}^{t+T} y_j, \quad t \in Z, \quad T = 1, 2, \dots \quad (1)$$

A normal distribution (ND) of the distribution is:

$$N(\mu, \sigma, 0)y = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(y-\mu)^2}{2\sigma^2}}. \quad (2)$$

Cumulative distribution function (CDF) of the distribution is:

$$N(\mu, \sigma, 1)y = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^y e^{-\frac{(\xi-\mu)^2}{2\sigma^2}} d\xi. \quad (3)$$

Indicator of chaotic attractor of the time series  $y_t$  is:

$$ChAy = \begin{cases} \theta N(\mu, \sigma, 0)y + \eta, \\ \theta N(\mu, \sigma, 1)y + \eta, \end{cases} \quad (4)$$

where the parameter  $\mu$  in this definition is the mean or expectation of the distribution and the parameter  $\sigma$  is its standard deviation;  $\theta$  – the value of the interval to provides the total probability;  $\eta$  is the value of the initial interval.

For econometric modelling and measuring the chaotic and cyclic movements of the Cass Freight Index: Expenditures the following information data were used:

- Cass Freight Index: Expenditures, January 1990 = 1.00, 1999–2014, Monthly, FRGEXPUSM649NCIS Series of FRED (2014).
- Cass Freight Index: Expenditures, January 1990 = 1.00, Change from Year Ago, 1999–2014, Monthly, FRGEXPUSM649NCIS Series of FRED (2014).
- Cass Freight Index: Expenditures, January 1990 = 1.00, Change from Year Ago, 1999–2014, Monthly, FRGEXPUSM649NCIS Series of FRED (2014).
- US real gross domestic product, bln of chained 2009 USD, 1947–2014, Quarterly, GDPC96 Series of FRED (2014).
- US NBER based recession indicators series, +1 or 0, 1854–2014, Monthly, USRECQ Series of FRED (2014).

For measuring the chaotic and cyclic movements of the Cass Freight Index: Expenditures we use the econometric models with the following specification of the regression for the time series  $y_t$  and  $\Delta y_t$  will be used:

$$\begin{bmatrix} y_t \\ \Delta y_t \\ \Delta y_t \end{bmatrix} = \beta_0 + \beta_1 \times \begin{bmatrix} ChAI_t \\ ChAIII_t \\ ChAV_t \end{bmatrix} + \beta_2 \times \begin{bmatrix} ChAII_t \\ ChAIV_t \\ ChAVI_t \end{bmatrix} + \beta_3 \times \begin{bmatrix} Date_t \\ Date_t \\ y_t \end{bmatrix} + \varepsilon_t, \quad (5)$$

where  $\Delta y_t = y_t - y_{t-1}$ ;  $ChA(\cdot)_t$  – independent variables of the model, i.e. Indicator Chaotic Attractor of the time series  $y_t$ ;  $Date_t$  – date, 2001–2014, monthly;  $\beta_0, \beta_1, \beta_2, \beta_3$  – the unknown parameters;  $\varepsilon_t$  – a random errors such that for all  $t, s \in Z$  and  $t \in s$  satisfy the following conditions:

$$\begin{aligned} E[\varepsilon_t | X] &= 0, \\ Var[\varepsilon_t | X] &= \sigma^2, \\ Cov[\varepsilon_t, \varepsilon_s | X] &= 0, \\ \varepsilon_t | X &\approx N[0, \sigma^2 I], \end{aligned} \quad (6)$$

where  $X$  – a matrix of the observations compiled by of  $ChA(\cdot)_t, Date_t$  and  $y_t$ ;  $E[\cdot]$  – a expectation;  $Cov[\cdot]$  – covariance;  $Var[\cdot]$  – variance;  $I$  – identity matrix.

The expenditures of the above listed indicators, data, methods and models will be used in the following sequence:

- indicator moving average with time periods  $T = 1, 2, \dots$  of the time series  $y = \{y_t\}_{t \in Z}$  (1)–(3) to identify the parameters  $\mu$  and  $\sigma$  for distribution  $N(\mu, \sigma, 1)y$  and for the density of distribution  $N(\mu, \sigma, 0)y$ ;
- indicator chaotic attractor  $ChA(\cdot)_t$  of the time series  $y_t$  (4) to choose covariates for the regression equation (5);
- econometric measuring and analysis (5)–(6) of chaotic and cyclic movements of the Cass Freight Index: Expenditures by using the method of least squares.

**Key research findings.**

**1. Chaotic Attractors of the Cass Freight Index: Expenditures.** For measuring of chaotic and cyclic movements of the Cass Freight Index: Expenditures Figure 1 is constructed:

- Cass Freight Index: Expenditures, based on the observation of the process during 1999–2014 with the monthly frequency measurement;
- moving averages with the period of 89 months, which serve to identify the parameters and characteristics of chaotic and cyclic movements of the Cass Freight Index: Expenditures;
- Chaotic Attractor  $ChAI_t$  of CDF as  $\theta N(\mu, \sigma, 1)y + \eta$  and the following parameters  $\mu = 01.01.2008$ ;  $\sigma = 610$ ;  $\theta = 1.294$ ;  $\eta = 1.206$ ;
- Chaotic Attractor  $ChAll_t$  of CDF as  $(\theta_1 N(\mu_1, \sigma_1, 1)y + \eta_1) \cup (\theta_2 N(\mu_2, \sigma_2, 1)y + \eta_2)$  and the following parameters  $\mu_1 = 01.07.2004$ ;  $\sigma_1 = 377$ ;  $\theta_1 = 0.647$ ;  $\eta_1 = 1.206$  and  $\mu_2 = 01.07.2011$ ;  $\sigma_2 = 377$ ;  $\theta_2 = 0.647$ ;  $\eta_2 = 1.853$ ;
- US real gross domestic product, bln of chained 2009 USD, 2001–2014, Quarterly;
- US NBER based recession indicators series, +1 or 0, 2001–2014, monthly is used to analyze comparability of the abovementioned indicators of the US transportation industry and its national economy within the measurements of chaotic and cyclic of their movements.

The identified chaotic attractors can be characterized by one long cycle (2001–2014) and two medium cycles (2001–2007 and 2008–2014). Applying these attractors as covariates for the specification of the regression equation, more indepth econometric studies can be carried out, as discussed in the following sections.

**2. Econometric Measuring of Chaotic Attractors of the Cass Freight Index: Expenditures.** Parameters of the constructed attractors using OLS estimates are presented for econometric measuring and analysis of chaotic and cyclical movements Cass Freight Index: Expenditures (Table 1 and Figure 1). Applying OLS estimates provides various scenario specification of the regression equation, however, this article presents only some of them, in particular:

- Chaotic attractor  $ChAI_t:LS$  is constructed based on the OLS estimates by Scenario (1) of Table 1 (Figure 2).
- Chaotic attractor  $ChAll_t:LS$  is constructed based on the OLS estimates by Scenario (2) of Table 1 (Figure 1).
- Date, 2001–2014, monthly:  $Data:LS$  is constructed based on the OLS estimates by Scenario (3) of Table 1 (Figure 1).
- Chaotic attractor  $ChAYObs:LS$  is constructed based on the OLS estimates (Table 1, Scenario (7); Figure 1).

Econometric measurements with high statistical significance of chaotic attractor of regressors  $ChAI_t:LS$ ,  $ChAll_t:LS$  and  $Data:LS$ , can be characterized by one long cycle (2001–2014) and two medium cycles (2001–2007 and 2008–2014), respectively.

**3. Chaotic Attractors of the Cass Freight Index: Expenditures, Change from Year Ago.** To measure chaotic and cyclic movements of the Cass Freight Index: Expenditures, Change from a Year Ago the following attractors are constructed (Figure 2):

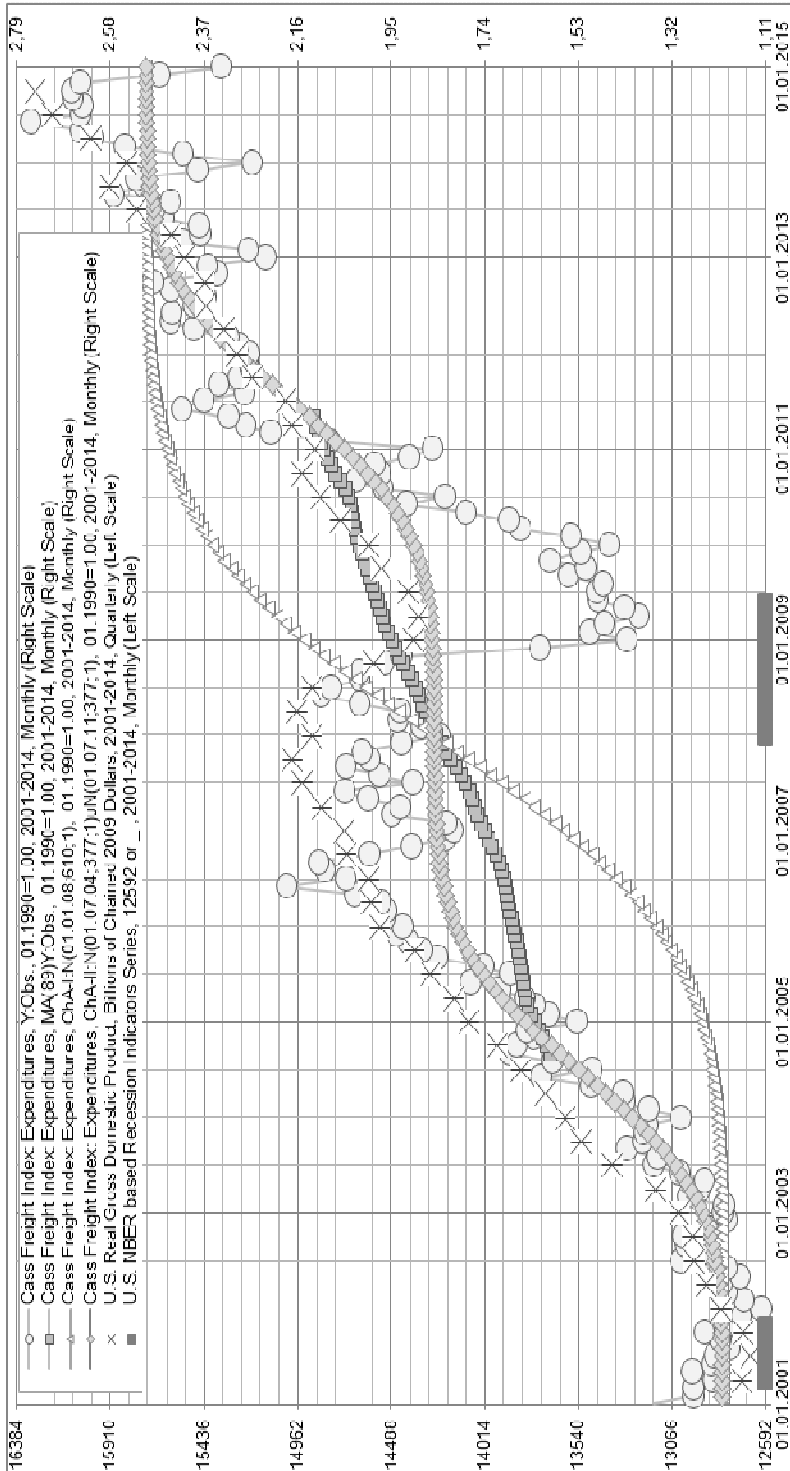


Figure 1. Chaotic Attractors of the Cass Freight Index: Expenditures, January 1990 = 1.00, 2001–2014, time series: monthly, authors'

- Chaotic Attractor  $ChAll_t$  of ND as  $\theta N(\mu, \sigma, 0)y + \eta$  and the range of values of the following parameters  $\mu = 01.01.2008$ ;  $\sigma = 610$ ;  $\theta = 377$ ;  $\eta = 0.000$ .
- Chaotic Attractor  $ChAIV_t$  of ND as  $(\theta_1 N(\mu_1, \sigma_1, 0)y + \eta_1) \cup (\theta_2 N(\mu_2, \sigma_2, 0)y + \eta_2)$  and the values of the following parameters  $\mu_1 = 01.10.2004$ ;  $\sigma_1 = 377$ ;  $\theta_1 = 233$ ;  $\eta_1 = 0.000$  and  $\mu_2 = 01.04.2011$ ;  $\sigma_2 = 377$ ;  $\theta_2 = 233$ ;  $\eta_2 = 0.000$ .
- US Real Gross Domestic Product, Change from a Year Ago, bln of chained 2009 USD, 2001–2014, Quarterly is used to analyze comparability of the abovementioned indicators of the US transportation industry and its national economy within the measurements of their chaotic and cyclic movements.

Table 1. **Econometric Measuring of the Cass Freight Index: Expenditures, January 1990 = 1.00, 2001–2014, time series: monthly, authors**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ChA-I	0.614***			-0.302***	-0.753***		-0.177**
	(0.04)			(0.04)	(0.08)		(0.09)
ChA-II		1.000***		1.356***		1.820***	1.605***
		(0.03)		(0.06)		(0.12)	(0.16)
Date			0.000***		0.001***	0.000***	0.000*
			(0.00)		(0.00)	(0.00)	(0.00)
Constant	0.716***	0.001	-8.615***	-0.100**	-17.792***	7.826***	3.726
	(0.08)	(0.03)	(0.44)	(0.05)	(1.06)	(1.13)	(2.29)
Numb of obs.	169	169	169	169	169	169	169
R-square	0.5749	0.8767	0.7735	0.9053	0.8495	0.9045	0.9069

Note: The dependent variable – Cass Freight Index: Expenditures, January 1990 = 1.00, 2001–2014, Monthly. In parentheses are standard errors. \*, \*\*, \*\*\* – significance at 10%, 5%, 1% levels.

The identified chaotic attractors  $ChAll_t$  and  $ChAIV_t$  consist of one long cycle (2001–2014) and two medium cycles (2001–2007 and 2008–2014). Applying these attractors as covariates for the specification of the regression equation, more indepth econometric studies can be carried out, as discussed in the following sections.

**4. Econometric Measuring of Chaotic Attractors of the Cass Freight Index: Expenditures, Change from a Year Ago.** Parameters of the constructed attractors using OLS estimates are presented for econometric measuring and analysis of chaotic and cyclical movements Cass Freight Index: Expenditures (Table 2 and Figure 2). Applying OLS estimates provides various scenario specification of the regression equation, however, this article presents only some of them, in particular:

- Chaotic attractor  $ChAll_t:LS$  constructed is based on the OLS estimates by Scenario (1) of Table 2 (Figure 2).
- Chaotic attractor  $ChAIV_t:LS$  is constructed based on the OLS estimates (Table 2, Scenario (7); Figure 2).

Econometric measurements with high statistical significance of chaotic attractor of regressors  $ChAll_t:LS$ ,  $ChAIV_t:LS$  and  $Date:LS$ , can be characterized by one long cycle (2001–2014) and two medium cycles (2001–2007 and 2008–2014), respectively.

**5. Phase Portrait of the Cass Freight Index: Expenditures.** To measure the chaotic and cyclic movements of the Cass Freight Index: Expenditures, Change from a Year Ago the following attractors are constructed (Figure 3):



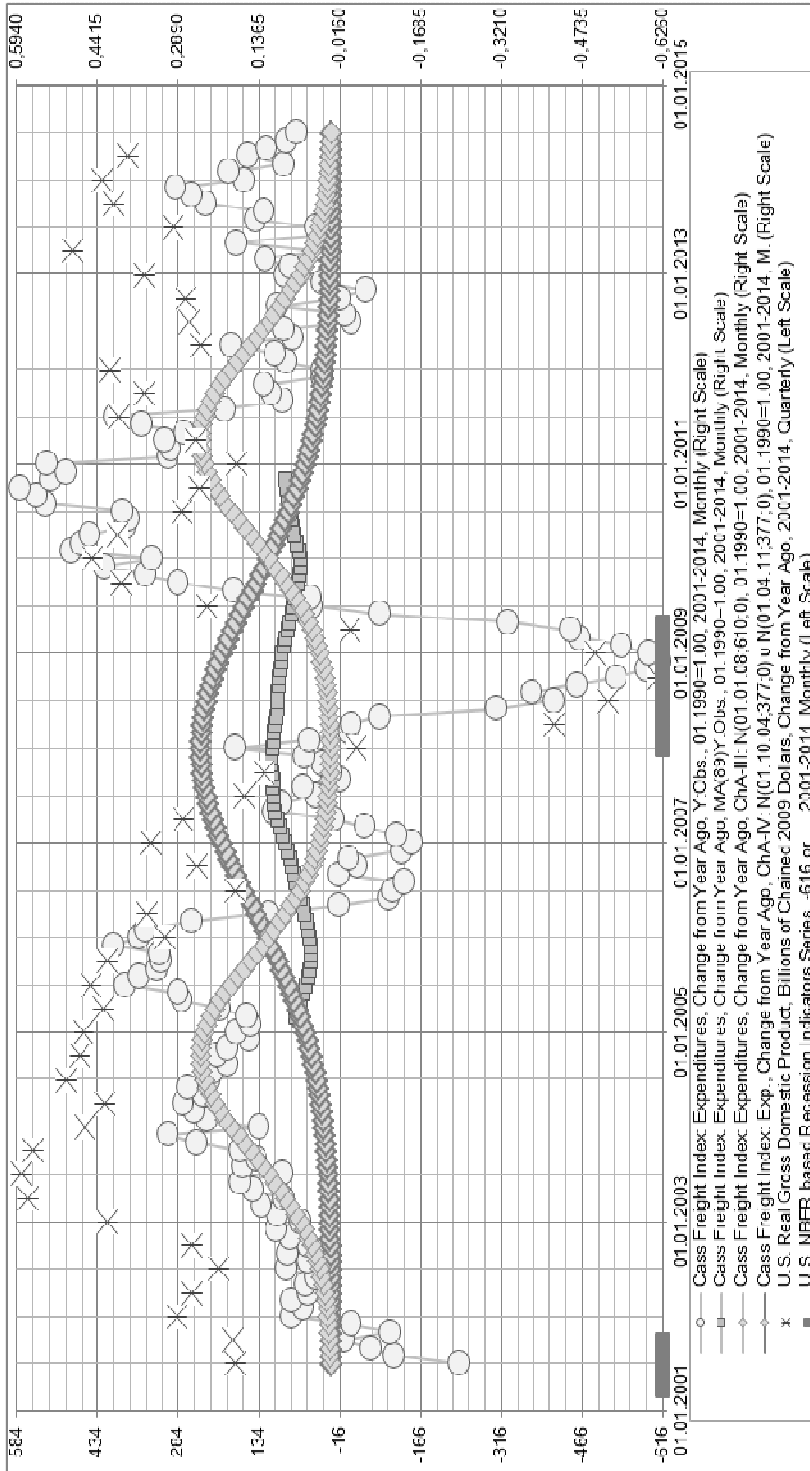


Figure 2. Chaotic Attractors of the Cass Freight Index: Expenditures, Change from a Year Ago, January 1990 = 1.00, 2001–2014, time series: monthly, authors'



- Chaotic Attractor  $ChAV_t$  of Phase Portrait as  $\theta N(\mu, \sigma, 0)y + \eta$  with value of the following parameters  $\mu = 01.01.2008$ ;  $\sigma = 610$ ;  $\theta = 377$ ;  $\eta = 0.000$ .
- Chaotic Attractor  $ChAVI_t$  of Phase Portrait as  $(\theta_1 N(\mu_1, \sigma_1, 0)y + \eta_1) \cup (\theta_2 N(\mu_2, \sigma_2, 0)y + \eta_2)$  with value of the following parameters  $\mu_1 = 01.10.2004$ ;  $\sigma_1 = 377$ ;  $\theta_1 = 233$ ;  $\eta_1 = 0.000$  and  $\mu_2 = 01.04.2011$ ;  $\sigma_2 = 377$ ;  $\theta_2 = 233$ ;  $\eta_2 = 0.000$ .
- Moving Averages with the period of 89 months, which serve to identify the parameters and characteristics of chaotic and cyclic movements of the Cass Freight Index: Expenditures, Phase Portrait.
- US Real Gross Domestic Product, Phase Portrait, bln of chained 2009 USD, 2001–2014, Quarterly is used to analyze comparability of the abovementioned indicators of the US transportation industry and its national economy within the measurements of chaotic and cyclic of their movements.

**Table 2. Econometric Measuring of the Cass Freight Index: Expenditures, Change from a Year Ago, January 1990 = 1.00, 2001–2014, time series: monthly, authors'**

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ChA-III	-1.087***			-0.614***	-1.087***		-0.614***
	(0.20)			(0.18)	(0.20)		(0.18)
ChA-IV		1.629***		1.424***		1.629***	1.424***
		(0.03)		(0.18)		(0.17)	(0.18)
Date			0.000		0.000	0.000	0.000
			(0.00)		(0.00)	(0.00)	(0.00)
Constant	0.189***	-0.055**	-0.408	0.013	-0.322	-0.566	-0.498
	(0.02)	(0.02)	(0.54)	(0.03)	(0.49)	(0.43)	(0.42)
Numb of obs.	157	157	157	157	157	157	157
R-square	0.1616	0.3623	0.0058	0.4081	0.1674	0.3681	0.4139

Note: The dependent variable – Cass Freight Index: Expenditures, Change from Year Ago, January 1990 = 1.00, 2001–2014, Monthly. In parentheses are standard errors. \*, \*\*, \*\*\* – significance at 10%, 5%, 1% levels.

The identified chaotic attractors  $ChAV_t$  and  $ChAVI_t$  consist of one long cycle (2001–2014) and two medium cycles (2001–2007 and 2008–2014). Applying these attractors as covariates for regression equation specification, more indepth econometric studies can be carried out, as discussed in the following sections of the work.

**6. Econometric Measuring of the Phase Portrait of the Cass Freight Index: Expenditures.** Parameters of the constructed attractors using OLS estimates are presented for econometric measuring and analysis of chaotic and cyclical movements Cass Freight Index: Expenditures, Phase Portrait (Table 3 and Figure 3). Applying OLS estimates provides various scenario specification of the regression equation, however, this article presents only some of them, in particular:

- Chaotic attractor  $ChAV_t;LS$  is constructed based on the OLS estimates by Scenario (1) of Table 3 (Figure 3).
- Chaotic attractor  $ChAVI_t;LS$  is constructed based on the OLS estimates by Scenario (2) of Table 3 (Figure 3).

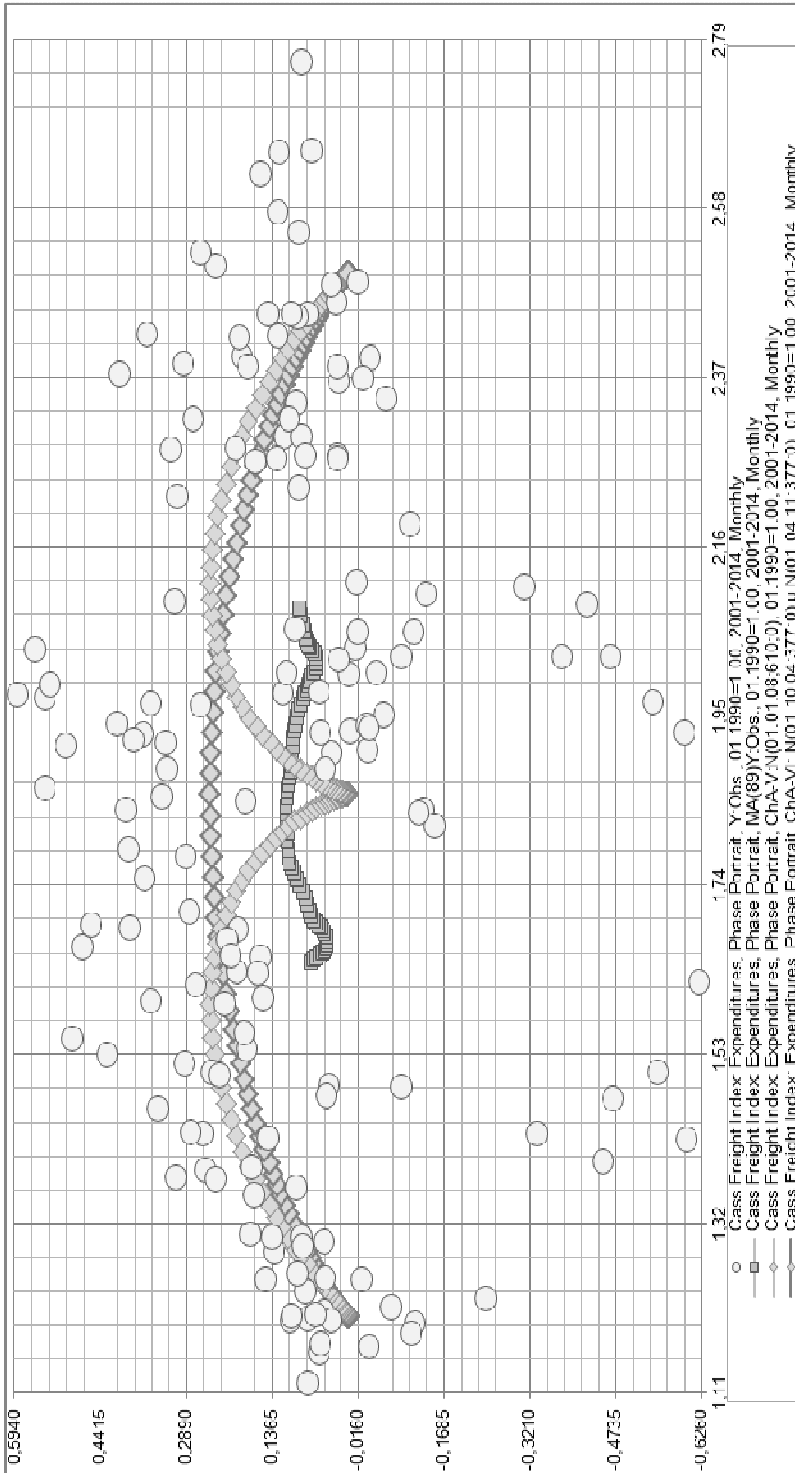


Figure 3. Chaotic Attractors of the Phase Portrait of Cass Freight Index: Expenditures, January 1990 = 1.00, 2001–2014, monthly, authors'

Table 3. Econometric Measuring of the Phase Portrait of Cass Freight Index: Expenditures, January 1990 = 1.00, 2001–2014, monthly, authors'

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ChA-V	-1.087*** (0.20)			-0.614*** (0.18)	-1.089*** (0.20)		-0.616*** (0.18)
ChA-VI		1.629*** (0.03)		1.424*** (0.18)		1.629*** (0.17)	1.424*** (0.18)
ChAYObs			0.037 (0.04)		0.039 (0.04)	0.037 (0.04)	0.039 (0.03)
Constant	0.189*** (0.02)	-0.055** (0.02)	0.035 (0.08)	0.013 (0.03)	0.116 (0.08)	-0.125* (0.07)	-0.059 (0.07)
Numb of obs.	157	157	157	157	157	157	157
R-square	0.1616	0.3623	0.0058	0.4081	0.1668	0.3669	0.4131

Note: The dependent variable – Cass Freight Index: Expenditures, Change from Year Ago, January 1990 = 1.00, 2001–2014, Monthly. In parentheses are standard errors. \*, \*\*, \*\*\* – significance at 10%, 5%, 1% levels.

Econometric measure with high statistical significance of chaotic attractor of the regressors  $ChAV_t:LS$  and  $ChAVI_t:LS$  can be characterized by one long cycle (2001–2014) and two medium cycles (2001–2007 and 2008–2014).

**Discussion and directions for further investigation.** The achieved results add to the body of studies on the interactions of Cass Freight Index: Expenditure and economic growth. In this article chaotic attractors were generated for the period 2001–2014, a long cycle with duration of 2001–2014 was identified as well as two secondary cycle: first cycle lasted from 01.01.2001 to 01.01.2008, the second cycle – 01.01.2008–01.01.2015. Time segments which intersect all the diagrams show that these cycles are complete. The integral function of normal distribution was applied for visual examination, which has "S" like shape, also known in literature as logistic curve. Presented chaotic attractors are invariant to the least squares estimator, which also confirms the adequacy of the selected attractors within the analysis. Also other indicators were presented in the analysis, such as real GDP in order to compare and study whether the phases and trends remain and uncertainty of the Cass Freight Index: Expenditure, GDP and recession. This method of investigation can be used in various applications, in particular for diagnosis, prediction or analysis. It should be noted that the measurement of real GDP is carried out quarterly, but it requires several months of treatment, whereas the Cass Freight Index: Expenditure is monthly. This implies that the index is showing signs of future recessions in one and a half and two years before they start. A weakness of this model is its amplitude inconsistency due to average current changes, which reflect only change trends. However, it should be noted that the moving average is an effective tool to identify trends. The analysis shows that the structure of attractors constructed on the basis of econometric estimates are the leading indicators to identify cyclical volatility of economic growth. It follows that the economic model can serve as an effective tool for forecasting and analysis of GDP growth and other important indicators of national economies.

**Conclusion.** Using the theory and methodology of deterministic chaos parametric statistics and econometric modelling we built a chaotic and cyclical attractor for the period of 2001 to 2014, which consist of one long length cycle and two medium

length cycle: the first cycle starts is 01.01.2001 to 01.01.2008, the second one is from 01.01.2008 to 01.01.2015. This study confirms the theory of cycles according to which not every economic activity is carried out in accordance with the classical economic theory.

The structure of the attractor of the Cass Freight Index developed through the method of moving average from a year ago was ahead to the main indicators of macroeconomic developments to 1.5 to 2 years. Whereas the model of index Cass Freight Index Expenditure: change from a year ago, observation considered as a times series was ahead from 2 to 2.5 years.

The analysis shows that the structure of attractors constructed on the basis of econometric estimates are the leading indicators to identify cyclical instability of economic growth. Cass Freight Index Expenditure provides a possibility to identify not only the recession, but also the GDP growth. Moreover, the conducted econometric measurements had parameters with a high statistical significance indicating that the analyzed model can be used for analysis, diagnostics and forecasts.

#### References:

- Angelidis, T., Skiadopoulos, G.* (2008). Measuring the Market Risk of Freight Rates: A Value-at-Risk Approach. *International Journal of Theoretical and Applied Finance*, 11(5): 447–469.
- Burns, A.F., Mitchell, W.C.* (1946). *Measuring Business Cycles*. New York: National Bureau of Economic Research.
- Comin, D., Gertler, M.* (2003). *Medium Term Business Cycles*. NBER Working Paper No. 10003. 1050 Massachusetts Avenue, Cambridge, MA 02138, September.
- Diebold, F.X., Rudebusch, G.D.* (1996). Measuring Business Cycles: A Modern Perspective. *Review of Economics and Statistics*, 78: 67–77.
- Drobetz, W., Richter, T., Wambach, M.* (2012). Dynamics of time-varying volatility in the dry bulk and tanker freight markets. *Applied Financial Economics*, 22(16): 1367–1384.
- Erdogan, O., Tata, K., Karahasan, B., Sengoz, M.* (2013). Dynamics of the co-movement between stock and maritime markets. *International Review of Economics and Finance*, 25(C): 282–290.
- FRED (2014). Federal Reserve Economic Data. Federal Reserve Bank of St. Louis // [research.stlouisfed.org](http://research.stlouisfed.org).
- Goulielmos, A.M. Psifia, M.-E.* (2009). Is history repeated? Cycles and recessions in shipping markets, 1929 and 2008, *International Journal Shipping and Transport Logistics*, 1(4): 329–360.
- Goulielmos, A.M. Psifia, M.-E.* (2011). Forecasting short-term freight rate cycles: Do we have a more appropriate method than normal distribution? *Maritime Policy and Management*, 38(6): 645–672.
- Jing, L., Hui, W., Marlow, P.B.* (2008). An analysis of freight rate volatility in dry bulk shipping markets. *Maritime Policy and Management*, 35(3): 237–251.
- Kavussanos, M.G., Alizadeh, A.* (2002). Efficient Pricing of Ships in the Dry Bulk Sector of the Shipping Industry. *Maritime Policy and Management*, 29(3): 303–330.
- Kavussanos, M.G., Dimitrakopoulos, D.* (2011). Market risk model selection and medium-term risk with limited data: Application to ocean tanker freight markets. *International Review of Financial Analysis*, 20(5): 258–268.
- Lee, T, Lun, V., Yan, H.* (2013). Price volume relativity in the dry bulk shipping market. *International Journal of Shipping and Transport Logistics*, 5(4/5): 551–563.
- Mintz, I.* (1972). *Dating American Growth Cycle*. Edited by Victor Zarnowitz in the *Business Cycle Today*, National Bureau of Economic Research, New York, pp. 30–88.
- Scott, R.E.* (1993). Chaos Theory and the Justice Paradox. *William and Mary Law Review*, 35: 329–351.
- Stopford, M.* (2009). *Maritime Economics*. 3rd Ed. Oxon, UK: Routledge.
- Tinbergen, J.* (1939). *Statistical Testing of Business Cycle Theories: Volume II: Business Cycles in the United States of America, 1919–1932*. Geneva: League of Nations.

Стаття надійшла до редакції 17.04.2015.