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ALTERNATIVE APPROACH TO PUBLIC DEBT SUSTAINABILITY [*]

This paper suggests an alternative approach to measuring public debt sustainability. It offers a way how to measure the limit on both domestic and external public debt. Special attention is paid to external public debt, the importance of which has increased in the last decades and which is connected with lower sustainability. By appropriate measuring of public debt sustainability we can avoid potential economic problems, first of all countries dependent on foreign capital.

Keywords: *public debt; external debt; GDP; closed economy; open economy.*
JEL codes: C68; E61; F34; H59; H63.

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АЛЬТЕРНАТИВНИЙ ПІДХІД ДО СТІЙКОСТІ ДЕРЖАВНОГО БОРГУ

У статті запропоновано альтернативний підхід до вимірювання стійкості державного боргу, а також спосіб вимірювання ліміту заборгованості для внутрішнього та зовнішнього державного боргу. Особливу увагу приділено зовнішньому державному боргу, економічне значення якого збільшилось в останні десятиліття. Крім того, зовнішній борг пов'язан із зовнішніми ризиками, порівняно з внутрішнім. За допомогою вірного вимірювання стійкості державного боргу можна уникнути потенційних економічних проблем, особливо це стосується країн, залежних від іноземного капіталу.

Ключові слова: *державний борг; зовнішній борг; ВВП; закрита економіка; відкрита економіка.*
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АЛЬТЕРНАТИВНЫЙ ПОДХОД К УСТОЙЧИВОСТИ ГОСУДАРСТВЕННОГО ДОЛГА

В статье предложен альтернативный подход к измерению устойчивости государственного долга, способ измерения предела задолженности для как внутреннего, так и внешнего государственного долга. Особое внимание уделено внешнему государственному долгу, экономическое значение которого увеличилось в последние десятилетия. Кроме того, внешний долг связан с более высокими рисками, нежели внутренний. С помощью правильного измерения устойчивости государственного долга можно избежать потенциальных экономических проблем, особенно в странах, зависимых от иностранного капитала.

Ключевые слова: *государственный долг; внешний долг; ВВП; закрытая экономика; открытая экономика.*

Introduction. At present, the public debt-to-GDP ratio is monitored and paid more attention in most countries. Considerable amount of literature has been published on the topic of origins and macroeconomic impacts of fiscal deficits in which mostly the public debt-to-GDP ratio is applied. However, there is a wide range of factors which affect the value of this ratio. Actually, an increase in a public debt-to-GDP ratio does not mean automatically that financial and economic situation became worst.

The aim of this paper is to suggest a method of measuring the limit on the overall public debt and as well limit on individual forms of a debt separately. Therefore, domestic and external public debt are distinguished in calculations (Atique, 2012;

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Guzman, 2002; Pattillo, 2015). On the basis of the proposed method both relative and absolute versions of a public debt constraint can be calculated. The suggested method reflects the ability of government and national economy pay off its liabilities. It enables evaluating in appropriate way public debt sustainability. The settled public debt constraint can be used in other research for example in dynamic models in which it is essential to be able to define behaviour of individual economic agents according to changing conditions of economic environment. A fixed public debt constraint can be used in public finance modelling and forecasting.

Theoretical framework and assumptions. The theoretical framework is based mainly on public economics, the debt trap theory which includes several approaches to public debt sustainability and the intertemporal choice theory.

The final equation for the overall public debt constraint and for domestic and external public debt constraints are derived on the basis of several scenarios in which various assumptions are applied. In total, there are 8 scenarios and individual assumptions are removed one by one. These assumptions are:

a) accumulated debt is non-debt-financed;
b) accumulated debt is non-interest bearing;
c) economy is closed;
d) accumulated debt is held only in the form of long-term bonds;
e) taxes are the only source of budget revenues;
f) tax rate is greater than the interest rate from public debt.

Individual scenarios for closed economy which will be considered are:

1) non-debt-financed non-interest bearing accumulated debt in closed economy;
2) non-debt-financed interest bearing accumulated debt in closed economy;
3) debt-financed non-interest bearing accumulated debt in closed economy;
4) debt-financed interest bearing accumulated debt in closed economy.

Individual scenarios for an open economy which will be considered are:

5) non-debt-financed non-interest bearing accumulated debt in open economy;
6) non-debt-financed interest bearing accumulated debt in open economy;
7) debt-financed non-interest bearing accumulated debt in open economy;
8) debt-financed interest bearing accumulated debt in open economy.

Government bonds are related to an obligation of a government to pay regularly (every year) interests which are set by an interest rate. Besides interests, on maturity, a government is supposed to pay off the principal sum as well. Therefore, two limits are measured; the minimum and maximum limits. The maximum limit on public debt represents a situation when only interests are paid off by non-debt-financed sources and the minimum limit on public debt represents a situation when both the interests and the principal sum are paid off.

Budget deficit is nothing but a consequence of public expenditures being higher than public revenues. It means that public debt is an alternative solution of funding a fiscal deficit. It can be described as in (Jackson, 2003):

$$D = -(R_G - E_G), \tag{1}$$

where D refers to fiscal deficit; R_G represents the non-debt-financed government revenues; E_G represents the overall government expenditures. The total revenues are

divided into non-debt-financed and debt-financed revenues. The non-debt financed revenues consist mainly of taxes (Bonney, 1995). If we ignore the existence of autonomous part of taxes or other sources as subsidies or property income, the overall non-debt-financed revenues can be described as:

$$R_G = t \times Y. \quad (2)$$

It means that potential non-debt-financed revenues are limited by the total national income and the public debt constraint can be modified according to the considered tax rate. Therefore, Y and t must be monitored. If public expenditures exceed non-debt-financed revenues, fiscal deficit is created. Then deficit is debt-financed. The funding sources for fiscal deficit can be expressed by a modified equation of macroeconomic equilibrium (Romer, 2001; Soukup, 2007):

$$-(R_G - E_G) = (S - I) + (M - X), \quad (3)$$

where typical $(T - G)$ was replaced by $(R_G - E_G)$ and it represents net savings of a public sector; $(S - I)$ refers to net saving of the private sector and $(X - M)$ represents foreign savings (NX). The equation states that if fiscal deficit is created, the higher deficit is, the lower must be private domestic consumption or the greater must be the financial inflow from abroad. When external public debt is created, NX increases and when it is paid off, NX decreases. However, because interests are paid, the final effect is a decrease in NX. Therefore, NX must be monitored as well.

The aim of this paper is to set a long-term sustainable limit on public debt. The idea of sustainability is inspired by previous studies which first of all have been dedicated to the public debt-to-GDP ratio and to the risk of a debt trap.

Debt trap refers to a situation when despite a balanced fiscal deficit the public debt-to-GDP ratio increases. The equation measuring debt-to-GDP ratio from which the risk of a debt trap can be described as in (Dvorak, 2008; Roubini, 1989):

$$B_1 = B_0 \times \frac{1 + r_1}{1 + g_1} + \frac{PD_1}{GDP_1} - \frac{\Delta MB_1}{GDP_1}, \quad (4)$$

where $B_1 = \frac{GDEBT_1}{GDP_1}$ refers to the public debt-to-GDP ratio. It is affected by 4 factors – the initial value of public debt-to-GDP ratio at the beginning of an examined year (B_0), the fiscal deficit during the examined year excluding interest payments (so-called primary fiscal deficit) (RD_1), the money financed debt-to-GDP ratio $(\frac{\Delta MB_1}{GDP_1})$ and the ratio of real interest rate to GDP growth $(\frac{1 + r_1}{1 + g_1})$. If r is greater than g, the public debt-to-GDP will increase despite public budget being balanced (or even in surplus). This situation is called debt trap, and government is not able to influence none of the mentioned factors.

The equation (4) measures only values but it doesn't show sustainability. The question of public debt sustainability (again, using only public debt-to-GDP ratio) was raised by J.A. Bispham (1987). As we here assume 8 scenarios to estimate the public debt constraint, J.A. Bispham proposed 3 scenarios. On the basis of two of them he was able to compute a sustainable level of public debt-to-GDP ratio (interest rate was lower or equal to GDP growth). The last scenario was describing the si-

tuation when a public debt-to-GDP ratio is not sustainable (when interest rate is greater than GDP growth rate). However, this approach still ignores public debt maturity and the obligation to pay off the principal sum of public debt.

Another approach to sustainability states that public debt stock is sustainable if a government is not discouraged to pay its current and future liabilities. It means that a government will be able to cover its expenditures and interest payments with its disposable revenues (Corsetti, 1991). If the present value of future revenues exceeds the present value of future expenditures, then the accumulated public debt must not be paid off and it can be debt-financed (the so-called snowballing).

Another approach states that sufficient requirement for a sustainable public debt is the situation when the primary budget surplus covers interest payments. It means that this approach supposes the snowballing (Wyplosz, 2011).

Another important approach to debt creation is the intertemporal choice theory (Romer, 2001; Soukup, 2007). On the basis of this theory, the importance of interest rate and of the total national income is shown. The intertemporal choice is a theory dealing with decisions how to divide a disposable income and consumption at different points in time. The decision about current consumption above its disposable revenues depends on two determinants; the real interest rate (which influences the slope of the budget line and therefore as well the potential debt constraint) and the current and future disposable income (which influences the conditions for a situation when entire current and future incomes are spent and no debt is created).

Public debt constraint estimation: absolute version.

1. Non-debt-financed non-interest bearing accumulated debt in closed economy. Three requirements are considered in the first scenario – the accumulated debt is non-debt-financed, it is not connected with interest payments and the economy is closed. In this case taxes are the only source of revenues to cover a public deficit and to pay off the accumulated debt. If there are no government expenditures but interests from the accumulated debt, then interest payments will represent all public expenditures.

Because it is not necessary to pay any interests and the only way how to pay off the debt are taxes, the public debt constraint reflects the possible amount of taxes which the economy is able to produce during one year. If the tax rate is 1, the overall revenues are equal to total national income. The equation of the public debt constraint for the first scenario can be described as:

$$GDEBT^{1}_{\max} = Y. \tag{5}$$

2. Non-debt-financed interest bearing accumulated debt in closed economy. Under this scenario, the requirement of non-interest bearing accumulated debt is removed. Tax revenues are still the only way to finance fiscal deficit and to pay off the accumulated public debt. However, we have to pay the interests too. Therefore, the total public debt stock which can be created must be lower. Public expenditures are divided between accumulated debt and interest payments.

$$GDEBT^{2}_{\max} \times (1+i) = Y. \tag{6}$$

If we adjust the equation (6), then the limit on public debt according to the second scenario is:

$$GDEBT^{2}_{\max} = \frac{Y}{1+i}. \quad (7)$$

3. Debt-financed non-interest bearing accumulated debt in closed economy. In this and next scenarios besides tax revenues the government can pay off old liabilities by issuing new bonds. Because debt is non-interest bearing, there are no interest payments. Therefore, the replacement of the old debt by a new one must not result in an increase of the total public debt. Actually, the constraint on the total public debt is estimated by the capacity of the economy to create a new debt. In case of bank loans, domestic monetary financial institutions (MFI) must hold an adequate part of the total reserves on the basis of minimum reserve requirement. The amount is derived by the cash reserve ratio (*CRR*). If we suppose that the MFI must hold part of reserves (given by *CRR*) invested into public bonds as well, then the public debt constraint can be described as:

$$GDEBT^{3}_{\max} = \frac{Y}{CRR}. \quad (8)$$

4. Debt-financed interest bearing accumulated debt in closed economy. In the fourth scenario, the accumulated public debt can be repaid off by a new debt but in order to comply with the long-term sustainability requirement, it should not result in an increase in public debt stock. Therefore, only the principal sum can be paid off by a new debt and the interests which exceed the principal sum, they must be non-debt-financed. Because we still operate in a closed economy, if we use both debt and non-debt financing, we will receive the total domestic debt constraint.

The total public debt stock which the economy is able to operate with is given by the overall interest payments which can be paid off with tax revenues.

$$GDEBT^{4}_{\max} \times i = Y. \quad (9)$$

The maximum long-term sustainable limit of a public debt in a closed economy can be defined as:

$$GDEBT^{4}_{\max} = \frac{Y}{i}. \quad (10)$$

The maximum limit on public debt according to (10) cannot be exceeded because the economy in fact would not be able to finance the interest payments in any future period. If the public debt stock was likely to exceed this limit, the government must decrease other expenditures or pay off a part of accumulated debt. Actually, it can happen that nobody at capital markets is willing to lend money to the government. In this case the government would not be able to pay off its liabilities. Therefore, it is necessary to calculate not only the limit which compute with interest payments but as well the limit for a situation when the overall accumulated public debt is paid off. This limit represents *the minimum long-term sustainable limit on public debt in a closed economy* and it can be defined as:

$$GDEBT^{4}_{\min} = \frac{Y}{(1+i)}. \quad (11)$$

According to (10) and (11) we can estimate a range for public debt under closed economy. The public debt stock in a closed economy can be within the range

$\left\langle 0;\frac{Y}{(1+i)};\frac{Y}{i}\right\rangle$. Actually, public debt in a closed economy becomes dangerous when stock exceeds the value $Y/(1+i)$ and the debt gets unplayable when stock exceeds Y/i.

If tax revenues are included instead of Y, then the *modified version of the maximum limit on public debt in a closed economy* is:

$$GDEBT_{\max}^{modified}=\frac{t\times Y}{i_e}. \tag{12}$$

Analogously, *the modified version of the minimum limit on public debt in a closed economy* is:

$$GDEBT_{\min}^{modified}=\frac{t\times Y}{(1+i)}. \tag{13}$$

The public debt stock in a closed economy ranges within $\left\langle 0;\frac{t\times Y}{(1+i)};\frac{Y}{(1+i)};\frac{t\times Y}{i};\frac{Y}{i}\right\rangle$, where three conditions must be achieved. The first condition is that $t \succ i$, because if $t \le i$, the order can differ. The values of t and i are paid more attention later in the text. Another condition is that neither t, nor i equals to zero $(t\neq 0 \ and \ i\neq 0)$. The last condition is that neither t, nor i equals to one $(t\neq 1\wedge i\neq 1)$. If both values equal to 1, then all the values equal to Y/i.

5. Non-debt-financed accumulated debt under open economy. Both scenarios for open economy in which non-debt-financed accumulated debt is considered result in the same conclusions as in the case of a closed economy. It does not matter whether the interests must be paid or not. Tax revenues can be the only financial source, therefore the limit according to the fifth scenario is:

$$GDEBT_{\max}^{5}=GDEBT_{\max}^{1}; \tag{14}$$

$$GDEBT_{\max}^{5}=Y. \tag{15}$$

The public debt constraint according to the sixth scenario can be described as:

$$GDEBT_{\max}^{6}=GDEBT_{\max}^{2}; \tag{16}$$

$$GDEBT_{\max}^{6}=\frac{Y}{1+i}. \tag{17}$$

6. Debt-financed non-interest bearing accumulated debt in open economy. This scenario is again similar to that under closed economy. The difference is that a government can borrow money from both residents and non-residents and therefore we must distinguish domestic and external public debts. In a closed economy, the limit on public debt is derived from domestic financial sources and the cash reserve ratio. It means that the limit is determined by the ability of domestic capital market to create debt. Under open economy it is limited by the ability of international capital markets. In fact, the limit does not exist.

$$EGDEBT_{\max} = \frac{Y_{world}}{CRR}. \quad (18)$$

*7. **Debt-financed interest bearing accumulated debt in open economy.*** If we remove all the previous requirements – non-debt-financing, non-interest bearing debt and the existence of a closed economy, we can describe the real limit on external public debt. Let's remind that for the case of a closed economy, the maximum and minimum limits on public debt are:

$$GDEBT^4_{\max} = \frac{Y}{i}; \quad (10)$$

$$GDEBT^4_{\min} = \frac{Y}{(1+i)}. \quad (11)$$

In the same way the we can define limit on public debt in an open economy but we have to include the determinants which represent the specifications of external debt. All the conditions are equal but one. Each financial flow between residents and non-residents is connected with currency exchange. Therefore, the interest payments are not limited only by Y but also by NX. If public debt is supposed to be long-term sustainable, we must count only with annual NX and we cannot count with accumulated foreign exchange reserves which are in possession of a monetary authority. Then, the overall limit on a debt-financed interest bearing accumulated debt in open economy equals to the equation (15) but the external public debt constraint can be described as:

$$EGDEBT_{\max} \times i_e = NX, \quad (19)$$

where i_e is the interest rate from external public debt and where conditions $NX \leq Y$ and $0 \prec NX$ must be complied. The condition $NX \leq Y$ guarantees that interests will not exceed the value of all domestic income. Actually, interests must be paid with taxes and they cannot be higher than all domestic incomes.

Other condition $0 \prec NX$ guarantees there is a net inflow of foreign exchange reserves and therefore, in order to pay off the debt, the accumulated foreign exchange reserves of a monetary authority must not be used. Then, *the maximum long-term sustainable limit of external public debt* is defined as:

$$EGDEBT_{MAX} = \frac{NX}{i_e}. \quad (20)$$

If we assume that besides interests the principal sum of external public debt must be paid off too, then, *the minimum long-term sustainable limit of external public debt* is defined as:

$$EGDEBT_{MIN} \times (1+i_e) = NX; \quad (21)$$

$$EGDEBT_{MIN} = \frac{NX}{(1+i_e)}. \quad (22)$$

Now we can add up the limit on domestic and on external public debt. Because a part of Y (equal to NX) is used to interest payments to non-residents, only the rest

of Y can be used to interest payments to residents. Therefore, first, it is necessary to modify the individual equations of domestic and external public debt constraints.

8. Overall public debt constraint. If we do not distinguish between domestic and external public debt, the overall public debt constraint is defined by (10). We can rename $GDEBT^4$ as $GDEBT$, then the overall public debt is defined as:

$$GDEBT_{MAX} = \frac{Y}{i}. \tag{23}$$

If we want to distinguish between domestic and external public debts, then the equation will be:

$$GDEBT_{MAX} = DGDEBT_{MAX} + EGDEBT_{MAX}, \tag{24}$$

where $DGDEBT_{MAX} = \frac{Y^{DGDEBT}}{i_d}$ and $EGDEBT_{MAX} = \frac{Y^{EGDEBT}}{i_e}$. Y must be divided between the sources reserved to domestic public debt (Y^{GDEBT}) and those reserved to external public debt (Y^{EGDEBT}), therefore:

$$GDEBT_{MAX} = \frac{Y - NX}{i_d} + \frac{NX}{i_e}. \tag{25}$$

When we do not express the interest payments to non-residents as the absolute value of NX but as the NX-to-Y ratio, then the limit on external public debt is:

$$EGDEBT_{MAX} = w \times \frac{Y}{i_e}, \tag{26}$$

where $w = NX / Y$ and the values of w range from 0 to 1 $(w \in \langle 0;1 \rangle)$. Then we can modify the equation of the limit on domestic external debt in a form:

$$DGDEBT_{MAX} = v \times \frac{Y}{i_d}, \tag{27}$$

where $v = (Y - NX) / Y$ and the values of v range from 0 to 1 $(v \in \langle 0;1 \rangle)$. If $v, w \in \langle 0;1 \rangle$, then $v + w = 1$. Let's repeat as well the previous conditions that $NX \leq Y$ and $0 \prec NX$. Then *the maximum limit on long-term sustainable total public debt* in form which enables to distinguish domestic and external public debt is:

$$GDEBT_{MAX} = DGDEBT_{MAX} + EGDEBT_{MAX}; \tag{28}$$

$$GDEBT_{MAX} = v \times \frac{Y}{i_d} + w \times \frac{Y}{i_e}. \tag{29}$$

The minimum limit on long-term sustainable total public debt is:

$$GDEBT_{MIN} = DGDEBT_{MIN} + EGDEBT_{MIN}; \tag{30}$$

$$GDEBT_{MIN} = v \times \frac{Y}{(1+i_d)} + w \times \frac{Y}{(1+i_e)}. \tag{31}$$

These values we will transfer to Figure 1 in order to compare the individual limits. We will also include the modified versions of limits in which not the total income but only tax revenues are included. *The modified maximum limit on long-term sustainable total public debt* is:

$$GDEBT(T)_{MAX} = v \times t \times \frac{Y}{i_d} + w \times t \times \frac{Y}{i_e}. \quad (32)$$

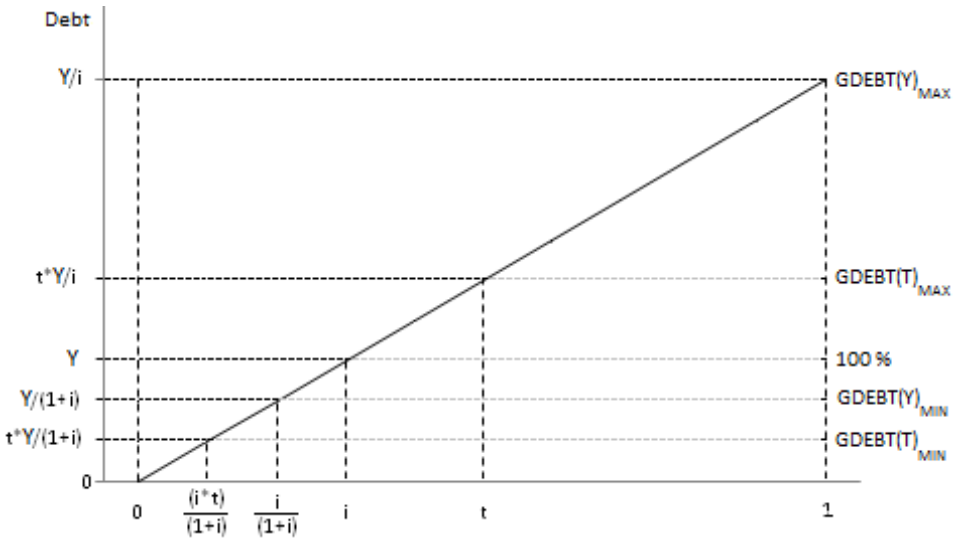

Figure 1. **Limits on the public debt stock according to various maturity and revenues scenarios,** *author's calculations and own construction*

The modified minimum limit on a long-term sustainable total public debt is:

$$GDEBT(T)_{MIN} = v \times t \times \frac{Y}{(1+i_d)} + w \times t \times \frac{Y}{(1+i_e)}. \quad (33)$$

On the y axis, the absolute values for specific kinds of limits are recorded. We assume that $t \succ i$. Then, the values range within $\left\langle 0; \frac{t \times Y}{(1+i)}; \frac{Y}{(1+i)}; \frac{t \times Y}{i}; \frac{Y}{i} \right\rangle$. The highest value for the limit on public debt is Y / i and it represents the maximum limit on the long-term sustainable overall public debt ($GDEBT(Y)_{MAX}$).

The values on the x axis represent the share of specific limits on the maximum limit on the overall public debt. According to this mechanism we receive values which range from 0 to 1 and which are easily comparable. The order of values depends on the values of i and t. If $t \succ i$, then the values can be written in the range $\left\langle 0; \frac{t \times i}{1+i}; \frac{i}{1+i}; i; t; 1 \right\rangle$. The values on the x axis are paid more attention later on. Because $w \in \langle 0;1 \rangle$, it is easily comparable with the values at the x axis and we can quickly estimate the external public debt constraint and assess if the existing current external public debt is sustainable.

The values of individual limits differ according to the interest and the tax rates. The higher is i, the lower is the limit on public debt (when tax rate is unchanged). This conclusion corresponds with the intertemporal choice theory. This relationship is shown in Figure 2.

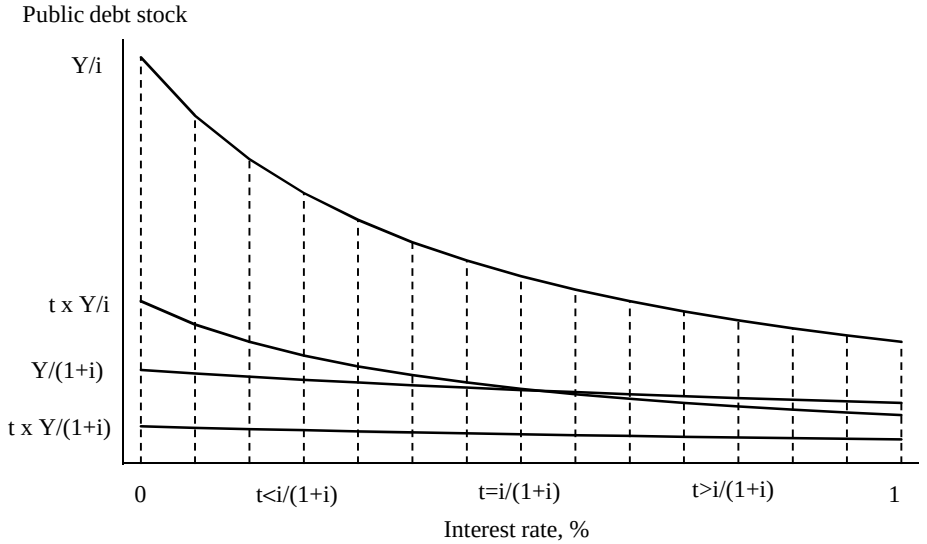

Figure 2. **Limits on public debt stock according to various maturity and revenues scenarios when the interest rate changes (in case of a constant tax rate),** *author's calculations and own construction*

The relationship between the limit on public debt and the tax rate is shown in Figure 3. Changes in t do not lead to any change in public debt limits. According to Figure 3, the higher is t, the more closed are the original and the modified versions of a limit on public debt. If t equals to 1, then, $\frac{Y}{i}=\frac{t\times Y}{i}$ and $\frac{Y}{(1+i)}=\frac{t\times Y}{(1+i)}$.

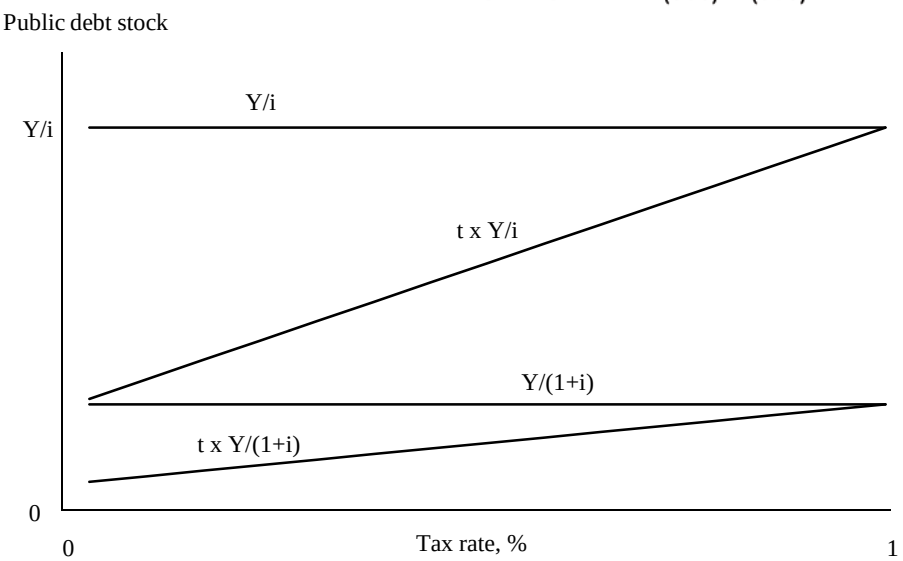

Figure 3. **Limits on public debt stock according to various maturity and revenues scenarios when the tax rate changes (in case of a constant interest rate),** *author's calculations and own construction*

Public debt constraint estimation: relative version. In this part, the assumption that the tax rate is always greater than the interest rate is removed. Actually, three possible situations can happen:

1) $t \succ i$;

2) $t = i$;

3) $t \prec i$.

For all the situations it is valid that the values on the y axis are within the range $\left\langle 0;\frac{1}{i}\right\rangle$ (in the absolute version it was $\left\langle 0;\frac{Y}{i}\right\rangle$) and the values on the x axis are within the range $\langle 0;1\rangle$. Each value on the x axis refers to a value on the y axis but the order depends on the value of *t* and *i*.

As well as in the absolute version, the values of limits on public debt are recorded on the y axis. However, the values refer to the public debt-to-GDP ratio. The values are computed by dividing the original values of individual limits on public debt by GDP (*Y*). Then, the relative version of the maximum overall public debt constraint ($GDEBT(Y)_{MAX}$) is computed as (Y / *i*) / Y and it equals to 1 / *i*. When the *tax rate is higher than the interest rate*, then the values on the y axis are within the range $\left\langle 0;\frac{t}{(1+i)};\frac{1}{(1+i)};1;\frac{t}{i};\frac{1}{i}\right\rangle$. The individual limits are computed as:

1) $GDEBT(Y)_{MAX} = \frac{1}{i} \times 100(\%)$;

2) $GDEBT(T)_{MAX} = \frac{t}{i} \times 100(\%)$;

3) $GDEBT(Y)_{MIN} = \frac{1}{1+i} \times 100(\%)$;

4) $GDEBT(T)_{MIN} = \frac{t}{1+i} \times 100(\%)$.

On the x axis, the values range from 0 to 1 and refer to the share of individual limits on the maximum public debt constraint. It means that individual limits such as $\frac{t \times Y}{(1+i)}, \frac{Y}{(1+i)}, \frac{t \times Y}{i}$ and $\frac{Y}{i}$ are divided by $\frac{Y}{i}$. When the *tax rate is higher than the interest rate*, then the values on the x axis are ordered within $\left\langle 0;\frac{t \times i}{1+i};\frac{i}{1+i};i;t;1\right\rangle$. These values are easily mutually comparable without complicated calculations and adjustment. The i on the x axis represents the situation when public debt equals to 100% of GDP.

When *tax rate equals to interest rate*, then the limit on public debt in economy cannot exceed 100% GDP if the only considered disposable incomes are taxes. The values on the x axis are ordered within the range $\left\langle 0;\frac{t \times i}{1+i};\frac{i}{1+i};i;1\right\rangle$, where $i = t$ and to

these values correspond to the values on the y axis. These are ordered within the range $\left\langle 0;\frac{t}{(1+i)};\frac{1}{(1+i)};\frac{t}{i};\frac{1}{i}\right\rangle$, where $1 / t = 1$. Figure 5 describes this scenario.

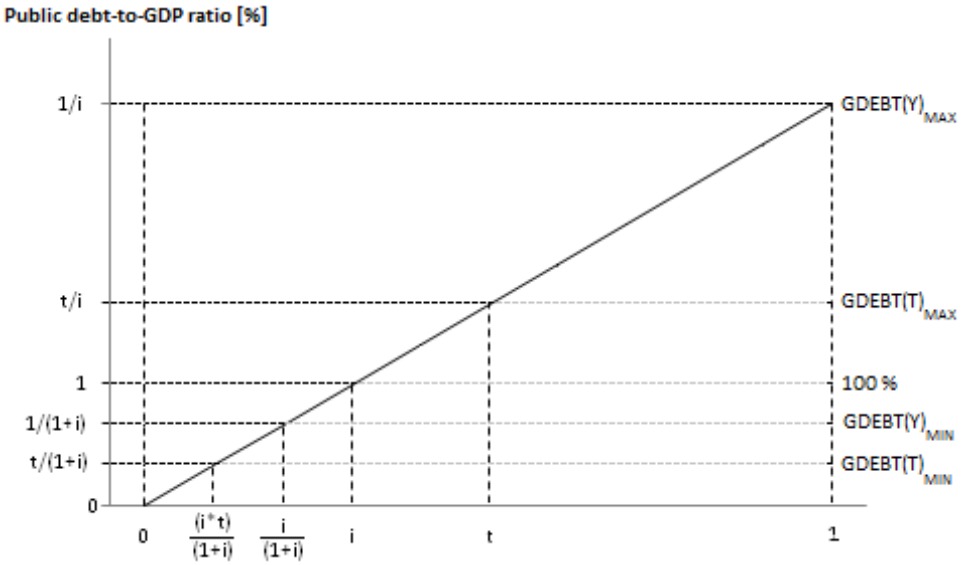

Figure 4. **Limits on public debt-to-GDP ratio according to various maturity and revenues scenarios in the situation when tax rate is higher than interest rate,** *author's calculations and own construction*

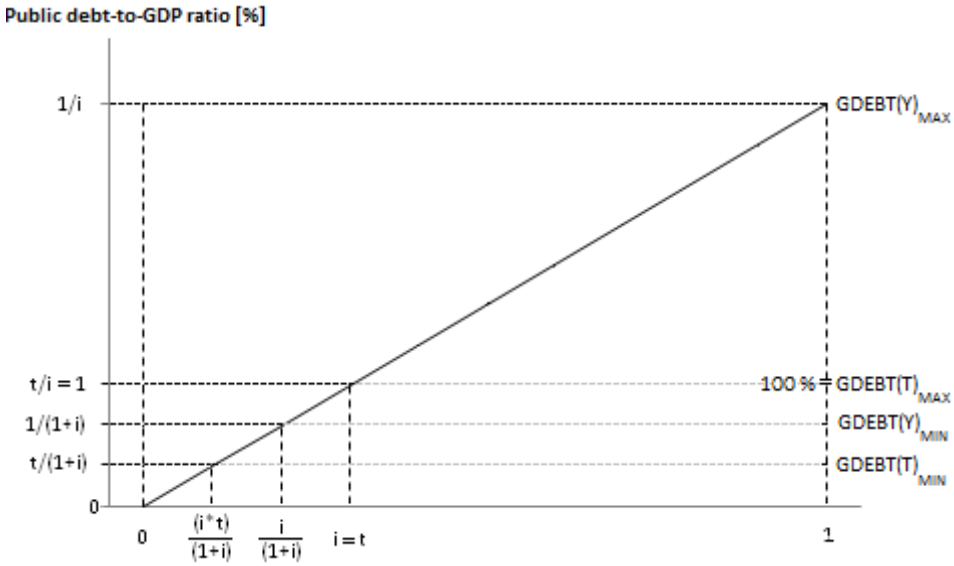

Figure 5. **Limits on public debt-to-GDP ratio according to various maturity and revenues scenarios in the situation when tax rate equals to interest rate,** *author's calculations and own construction*

In both situations, when t was higher and when t was equal to i, for any t and any i the order on the x and the y axes was identical. When t is lower than i, three different situations can happen:

1) situation 1: $t \prec i$ and at the same time $t \succ \frac{i}{1+i}$;

2) situation 2: $t \prec i$ and at the same time $t = \frac{i}{1+i}$;

3) situation 3: $t \prec i$ and at the same time $t \prec \frac{i}{1+i}$.

In the case that $t \prec i$ and at the same time $t \succ \frac{i}{1+i}$; then the values on the x axis are ordered in the range $\left\langle 0; \frac{t \times i}{1+i}; \frac{i}{1+i}; t; i; 1 \right\rangle$. According to this scenario, the maximum limit on the overall public debt operating with tax revenues and minimum limit on the overall public debt operating with the total national income converge to 100% GDP. This scenario represents the left-side half of Figure 2.

The situation when $t \prec i$ and at the same time $t = \frac{i}{1+i}$; refers to the point where $\frac{t \times Y}{i}$ equals to $\frac{Y}{1+i}$ and therefore, the lines are crossing in Figure 2. The third scenario refers to the situation when $t \prec i$ and at the same time $t \succ \frac{i}{1+i}$; According to this condition, the overall public debt operating with tax revenues exceeds the minimum limit on the overall public debt operating with total national income. It refers to the right-side half of Figure 2.

Specific values for chosen countries in 2012 are presented in Table 1 and the development of the limits of PIGS countries are presented graphically Figure 6.

External public debt constraint estimation: relative version. In addition to Y or t x Y, an external public debt is limited by NX. The conditions which must be completed are $NX \leq Y$ and $0 \prec NX$. Then, a part of public revenues disposable to pay off external liabilities, is given by w. The value of w is within the range $\prec 0.1 \succ$ and it is possible to compare it with other values on the x axis. Therefore, it enables us set directly the maximum limit of a long-term sustainable external public debt (on the y axis). It represents the overall external public debt constraint which should not be exceeded to remain sustainable. According to w, the external public debt constraint can account for the values given by equations[2]:

1) when $w = 1$, then $EGDEBT_{MAX} = \frac{1}{i}$;

[2] The values are in the form of share. It is necessary to multiply by 100 to obtain form in %.

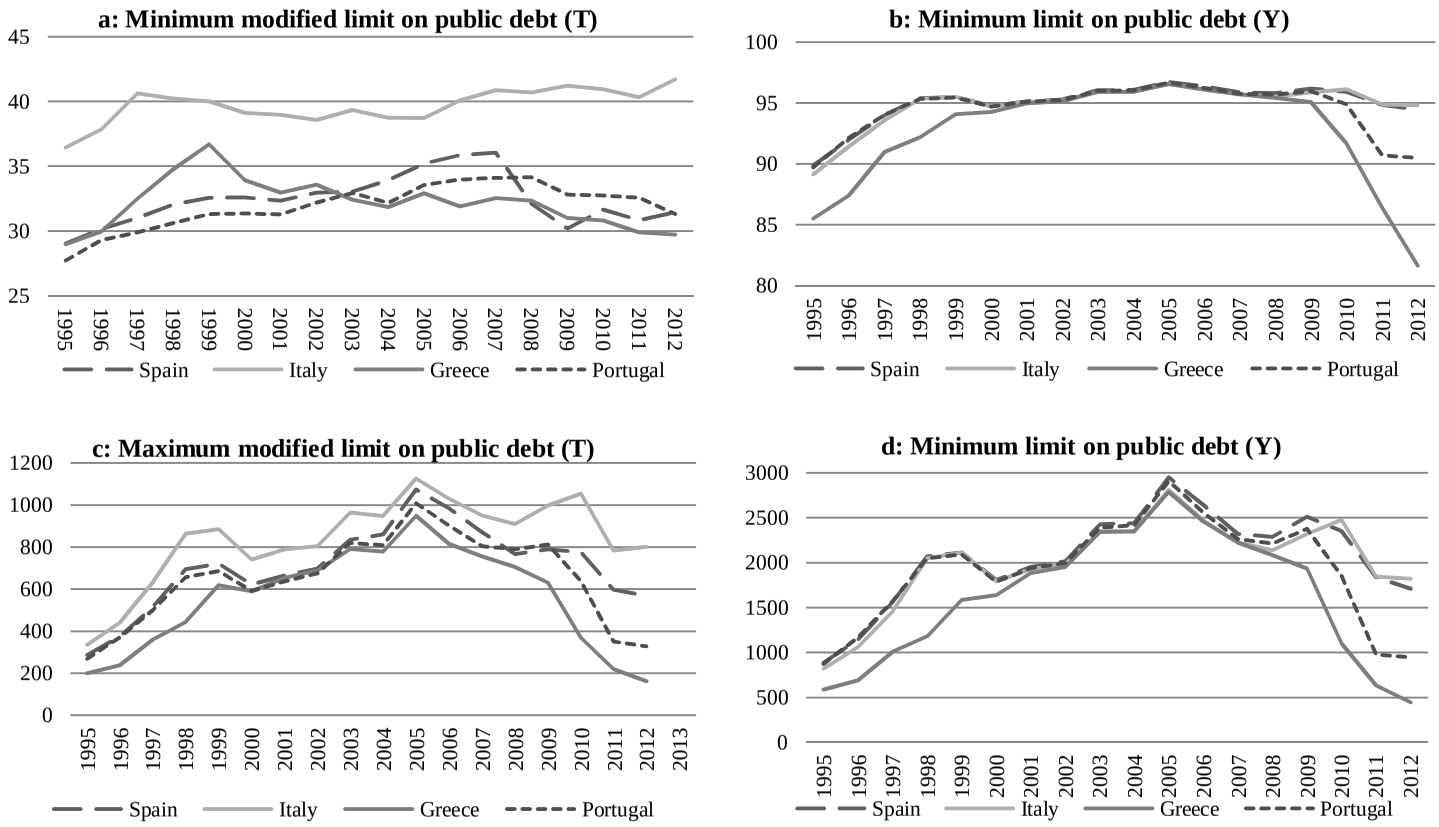

Figure 6. **Relative version of the limits on public debt of the PIGS countries between 1995 and 2012,** *% of GDP, author's calculations*

2) when $t \prec w \prec 1$, then $\frac{t \times 1}{i} \prec EGDEBT_{MAX} \prec \frac{1}{i}$;

3) when $w = t$, then $EGDEBT_{MAX} = \frac{t \times 1}{i}$;

4) when $i \prec w \prec t$, then $100 \prec EGDEBT_{MAX} \prec \frac{t \div 1}{i}$;

5) when $w = i$, then $EGDEBT_{MAX} = 1$;

6) when $\frac{i}{1+i} \prec w \prec i$, then $\frac{1}{1+i} \prec EGDEBT_{MAX} \prec 1$;

7) when $w = \frac{i}{1+i}$, then $EGDEBT_{MAX} = \frac{1}{1+i}$;

8) when $\frac{i \times t}{1+t} \prec w \prec \frac{i}{1+i}$, then $\frac{t}{1+i} \prec EGDEBT_{MAX} \prec \frac{1}{1+i}$;

9) when $w = \frac{i \times t}{1+t}$, then $EGDEBT_{MAX} = \frac{t}{1+i}$;

10) when $0 \prec w \prec \frac{i \times t}{1+t}$, then $0 \prec EGDEBT_{MAX} \prec \frac{t}{1+i}$;

11) when $w \leq 0$, then $EGDEBT_{MAX} = 0$.

Table 1. **Relative version of the limits on the overall public debt in 2012,** *% of GDP, author's calculations*

Countries	$GDEBT_{MAX}$	$GDEBT(T)_{MAX}$	100	$GDEBT_{MIN}$	$GDEBT(T)_{MIN}$
Belgium	3333	1577	100	97	46
Bulgaria	2222	611	100	96	26
Czech Republic	3597	1248	100	97	34
Denmark	7143	3493	100	99	48
Germany	6667	2680	100	99	40
Ireland	1621	486	100	94	28
Greece	444	162	100	82	30
Spain	1709	569	100	94	31
France	3937	1839	100	98	46
Italy	1821	801	100	95	42
Latvia	2188	611	100	96	27
Luxembourg	5495	2209	100	98	39
Hungary	1267	494	100	93	36
Portugal	948	328	100	90	31
Romania	1497	424	100	94	27
Slovakia	2198	620	100	96	27

If an economy is net importer and therefore the NX-to-GDP ratio is zero or negative, then the external public debt is not long-term sustainable at any level. Actually, in that case any interest payment to non-residents automatically leads to a decrease in accumulated foreign exchange reserves which is unsustainable in a long-

term period without negative macroeconomic implications and therefore, the only recommended form of public debt in that case is domestic public debt.

Table 2. **Relative version of the limits on external public debt in 2012***, % of GDP, author's calculations*

Countries	w	$EGDEBT_{MAX}$	$EGDEBT(T)_{MAX}$	$EGDEBT_{MIN}$	$EGDEBT(T)_{MIN}$
Belgium	0,011	37	17	1	1
Bulgaria	-0,031	x	x	x	x
Czech Republic	0,056	201	70	5	2
Denmark	0,051	363	177	5	2
Germany	0,059	395	159	6	2
Ireland	0,242	393	118	23	7
Greece	-0,048	x	x	x	x
Spain	0,007	13	4	1	0
France	-0,022	x	x	x	x
Italy	0,011	19	9	1	0
Latvia	-0,039	x	x	x	x
Luxembourg	0,291	1600	643	29	11
Hungary	0,073	93	36	7	3
Portugal	-0,006	x	x	x	x
Romania	-0,047	x	x	x	x
Slovakia	0,052	115	32	5	1

Conclusion. The main aim of this paper was to suggest a method of measuring the limit on long-term sustainable public debt which would enable changing the included determinants representing public revenues and public expenditures and which distinguishes external and domestic public debt. The suggested approach enables estimating the long-term sustainable external public debt constraint according to the values of the interest rate and the tax rate. For example, if the NX-to-GDP ratio is lower than the interest rate, then the external public debt should not exceed 100% of GDP. Actually, if an economy is net importer and there is no expected permanent annual increase in foreign exchange reserves, the external public debt is not long-term sustainable at any level.

The ongoing economic and financial crisis from 2008 showed how dangerous a dependency on foreign capital, such as the external debt, is. The solution can be the preference of domestic public debt the limit of which can account for hundreds of percents of GDP. Domestic public debt seems more stable form of a public debt. However, if the external public debt constraint is exceeded, it leads to negative macroeconomic implications. If the domestic public debt constraint is exceeded, public debt may become unplayable.

Despite public debt sustainability and the public debt composition is usually considered as a political issue in hands of the current government, the findings of this paper point out to the little leeway to deal with which the external public debt gives to a government. Actually, external public debt is related to lower sustainability.

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