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NUMERICAL AND ANALYTICAL DIAGRAM OF DISTRIBUTED SIMULATION OF DYNAMIC SYSTEMS

The study is an attempt to solve one of priority tasks in financial mathematics, namely, definition of arbitrage-free option prices. Herein, differential of European-style option price model is based on the Black-Scholes equation. Using a priori information about the smoothness of a solution, great attention is paid to construction of high-accuracy solutions. The proposed approach eliminates the recurrent structure calculations for desired vectors' decisions which leads to accumulation of rounding errors. Parallel form of the algorithm is the maximum, and therefore demands the shortest possible time for implementation on parallel computing systems.

Keywords: *option; option price; economic-mathematical model; high order accuracy; parallel computing; dynamic system.*

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ЧИСЛОВО-АНАЛІТИЧНІ СХЕМИ РОЗПОДІЛЕНОГО МОДЕЛЮВАННЯ ДИНАМІЧНИХ СИСТЕМ

У статті розв'язано одну з пріоритетних задач фінансової математики, а саме — визначення безарбітражних цін опціонів. При цьому на основі рівняння Блека-Шоулса побудовано диференціальну модель ціни опціону європейського типу. Використовуючи апріорну інформацію про гладкості рішення, велику увагу приділено побудові рішень високого порядку точності. Запропонований підхід виключає рекурентну структуру обчислень шуканих векторів рішень, яка призводить до накопичення помилок округлення. Побудована паралельна форма алгоритму є максимальною, і, отже, має мінімально можливий час реалізації на паралельних обчислювальних системах.

Ключові слова: *опціон; ціна опціону; економіко-математична модель; високий порядок точності; паралельні обчислення; динамічна система.*

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ЧИСЛЕННО-АНАЛИТИЧЕСКИЕ СХЕМЫ РАСПРЕДЕЛЕННОГО МОДЕЛИРОВАНИЯ ДИНАМИЧЕСКИХ СИСТЕМ

В статье решена одна из приоритетных задач финансовой математики, а именно — определение безарбитражных цен опционов. При этом на основе уравнения Блека-Шоулса построена дифференциальная модель цены опциона европейского типа. Используя априорную информацию о гладкости решения, большое внимание уделено построению решений высокого порядка точности. Предложенный подход исключает рекуррентную структуру вычислений искомых векторов решений, которая приводит к накапливанию ошибок округления. Построенная параллельная форма алгоритма является максимальной, следовательно, имеет минимально возможное время реализации на параллельных вычислительных системах.

Ключевые слова: *опцион; цена опциона; экономико-математическая модель; высокий порядок точности; параллельные вычисления; динамическая система.*

Introduction. Problems economists face are mostly complex ones. This could be explained by the fact that they depend on a number of factors, which do not only

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interact, but also are determined by time behavior (Ivanilov and Lotov, 2007). For this reason, this class of problems is investigated by means of economic and mathematical modeling (Troianovskiy, 2001; Kolomaev, 2005). A mathematical model allows taking into account a variety of parameters that effect economic systems as a whole.

However, during economic and mathematical modelling a situation often arises when a system under study has an extremely complex structure. In most cases this is due to multidimensionality of their description. Multidimensional models are used in marketing research and management. In addition, they can be also applied in relation to segmentation problems and to predict the market, to study economic depression, for analysis and prediction of social and economic phenomena etc. The above class of problems is crucial for the development of economics in general and in connection with this, the development of effective methods for their solution appears to be an actual scientific and practical problem.

In recent decades, in the research on economic systems dynamics one can notice a steady tendency of transition to continuous time (Tsarkov, 2007; Petrov, 2010). The class of economic problems using continuous time allows adapting methods and models, experience of linear and nonlinear dynamical systems' study, accumulated in technical sciences (especially in the theory of control action) (Besekerskiy and Popov, 2007).

In developed countries the most important component of financial market is the market of futures and option contracts, with its turnover tenfold greater than the volume of trading at the underlying assets market. For this reason, options are important for economic activity of the enterprise. Essentially, option is a contract that in return for a premium (option price) entitles its holder in the implementation of certain conditions to sell or buy a certain financial asset at a fixed price.

The theory of options can be applied not only at financial market but also in real economy. In particular, the real options method allows calculating the cost of an investment project. Also real options can be found in customs tariffs and tax policy.

At that we notice that simultaneously with the start of the first trading at Chicago Stock Exchange back in 1973 the works of F. Black and M. Scholes, devoted to options pricing appeared. On the basis of these studies the theory of derivatives' pricing using Gaussian processes for financial markets simulation was developed. According to this theory, fair prices options represent the defined functional of simulation.

Thus, we state a problem of determining the practical value of derivative securities depending on the asset value which is its financial basis. Let us consider the essence of determining the value of call option of European-style, which is a derivative of valuable security of share. We introduce the share price as function $x = x(t)$, which depends on time t. Assuming that at certain point of time a deal was consummated (i.e., a call option purchased), at the moment t_1 ($t_1 > t$) option holder is entitled to buy (and the owner is obliged to sell) a share at price x_1 agreed according to the option at the moment t, despite the stock price at the moment t_1. As seen from this reasoning, option price depends on the price of an asset and its time. In this regard, the function $y = y(x,t)$, which determines the option price, must be found as a function of two variables. Calculation of fair price of an option is described by the

Black-Scholes method (Black and Scholes, 1973). This equation is reduced to solutions of differential equations in partial derivatives with certain initial and boundary conditions or free boundary.

Thus, evaluation of options is currently one of the most important and urgent tasks in financial mathematics. In addition, we note that the process of fair option price modelling is much more complicated if price depends on a number of underlying assets. Under such circumstances numerical procedures have high computational complexity. For this reason, an effective research of the given class of problems can be based on the use of multiprocessor systems (Bashkov et al., 2011).

Recent research and publications analysis. Currently, macroeconomic processes are being studied as transients to dynamic systems (Besekerskiy and Popov, 2007). Therefore, the study of macroeconomic processes is carried out by means of mathematical methods and models, primarily, with the help of the dynamic systems theory (mainly automatic control theory), which is based on a range of differential equations and Laplace transforms. In the investigation of transients in unstructured macroeconomics they use a dynamic model of Keynes and Samuelson-Hicks (Kolomaev, 2005). Moreover, in the process for monetary and material accumulation modelling, dynamics securities are treated as large systems (Petrov, 2010).

One of the most popular mathematical methods for approximate solution of option price problems are finite-difference schemes. In the process of building up a finite-difference scheme digitization at spatial and temporal variables takes place. Thus, from the standpoint of numerical algorithms solving of differential equations is distributed on explicit and implicit schemes (Godunov and Riabenkiy, 1973). In the explicit scheme the value of an unknown function is determined successively, layer by layer. In this connection, this approach cannot be used for parallel computing. The implicit schemes allow conducting calculations with a big step without a significant loss in accuracy, but such an approach requires larger computation.

To solve the problem of finding the fair value for a wide range of options in certain cases the Monte Carlo method and some of its modifications (Boyle, 1977) are used. The Monte Carlo method for estimating the option value is based on probability distribution of the total process history of the underlying asset. The size of expected payments at risk-neutral conditions is calculated using a method of selective research. After that, the size of expected payments is discounted at risk-free interest rate. In this case the Monte Carlo method provides fairly accurate results. However, its main limitation is slow convergence.

The analysis shows that the methods of solving tasks of this class should be not only diverse but also combine quantitative estimates with analysis capabilities. At present, certain trends can be outlined in the development of numerical and analytical methods with a complex logical structure, but in comparison with piecewise-difference methods they have higher order of accuracy and the possibility of constructing algorithms with adaptation for orders of approximation (Shvachych, 2006; 2008). In calculations this approach is cumbersome, but it shows a kind of standard for comparison with other practices. At the same time despite the fact that computer experiment is carried out on a multiprocessor system, it could be argued that the fact that hindered the development of a numerical and analytical approach, is now losing its relevance.

It should be noted that today solution of complex, large-volume tasks requires powerful computers and is characterized by a word "parallel", in other words, there exist parallel computers, computing systems, parallel computing methods etc. (Ivaschenko et al., 2013; 2014). Emergence of new communication means and more advanced element base in computer systems has stimulated the development of high-performance computing based on standard technologies and publicly available technologies and components (Bashkov et al., 2011).

In this paper, for computational experiments we have used the so-called "blade" server solutions for multiprocessor systems (Bashkov et al., 2011). On the basis of IB network technologies implemented "blade" server solution multiprocessor system has been realized at which several similar modules are installed in the same place. Practice shows that blade systems are more compact and easy to maintain, and their implementation is not much more expensive as compared to multi-processor computer systems. The key features of its design architecture have been presented in (Bashkov et al., 2011).

Thus, it can be argued that to date fundamental problems of potentially infinite peak increase performance for computers disappeared. But a really serious problem is how to use this enormous potential. This paper shows the possibility of constructing maximum parallel computing algorithms for solving the problem of determining the fair value of European-style options.

Unresolved issues. The existing methods for solving the problem of determining the fair value of European-style options are not always applicable for the reasons of accuracy, speed, memory requirements, algorithms' structure, applicability for multiprocessor computer systems. In this context, new ideas arise and find use in the field of computational financial mathematics. Ultimately, for the most perfect mathematical models it is necessary to design new methods of numerical experiments implementation.

The purpose of the article is to construct the maximum parallel algorithms for solving the problem of determining the fair value of European-style options described by dynamic models. In this case we consider the problems of mathematical modeling of this class on parallel computing systems of the cluster type. Majority of ordinary algorithms for solving these problems (sweep methods, decomposition of a matrix into a product of two diagonal matrices, doubling etc.) in the presence of several processors work usually no faster than at a single processor. The reason for this is substantial sequence of computations for these algorithms.

Key research findings. Creation of parallel computing systems requires a mathematical concept for parallel algorithm development, i.e., algorithms adapted for implementation on such systems. Basis for algorithm parallel development can be a sequential algorithm, and also as a task itself (Ivaschenko et al., 2014). At parallelizing of sequential algorithm the pragmatic approach seems to be the most rational, namely, sequential algorithm identifies common elements, which are then transformed into a parallel form.

In (Voloshchuk, 2015) it is shown that the differential model of option price (Black and Scholes, 1973) can be reduced to the form:

$$\frac{\partial Y}{\partial t}-\frac{1}{2}\sigma\frac{\partial^2 Y}{\partial x^2}=0. \tag{1}$$

Equation (1) is a homogeneous parabolic equation with two variables. In this paper, such an equation satisfies the initial condition

$$Y(x,o) = \varphi(x) \tag{2}$$

and the boundary conditions

$$Y(0,t)=\mu_0(t), \quad Y(x_L,t)=\mu_L(t), \tag{3}$$

where $\mu_0(t),\ \mu_L(t),\ \varphi(x)$ are the defined functions. It is known that on certain assumptions smoothness in the solution of problem (1)—(3) exists and is unique.

Let us introduce grid the variable x steps between nodes:

$$Dx1_p = \frac{x_L}{2m}, \quad p=\overline{1,2m-1}, \quad m\in Z, \tag{4}$$

where m — the integer parameter sampling. For uniformly distributed nodes

$$\begin{cases} Dx1 = x_p - x_{p-1} = const, \\ x_p = x_{p-1} + p\times Dx1, \quad p=\overline{1,2m-1}. \end{cases} \tag{5}$$

On the basis of a priori information the desired function is submitted in the form of the Taylor series

$$Y_{p+\varepsilon_x,1}(t,x) = \sum_{n=0}^{\infty} \varepsilon_x^n \times Y_{p,n+1}(t), \tag{6}$$

where

$$\begin{cases} \varepsilon_x = \dfrac{x-x_p}{x_{p+1}-x_p} \in [+1,-1], \\ Y_{p,n+1} = \dfrac{Dx1^n}{n!} \times \left.\dfrac{\partial Y}{\partial x^n}\right|_{x=x_p} . \end{cases}$$

After agreement (6) to the equation (1), equating coefficients of identical powers, we obtain a system of ordinary differential equations ODEs

$$Y'_{p,n+1}(t) = \frac{(n+1)(n+2)}{Dx1^2} \times Y_{p,n+3}(t) \tag{7}$$

having a Cauchy form

$$Y_{p,n+1}(0) = \varphi_{p,n+1}, \tag{8}$$

where $\varphi_{p,n+1}$ are the known values of the Taylor component of the initial function (2).

We restrict ourselves to the right side of the Taylor series (6) by a finite number of terms, we obtain

$$Y_{p+\varepsilon_x,1}(x,t) = \sum_{n=0}^{N} \varepsilon_x^n \times Y_{p,n+1}(t), \tag{9}$$

where N is integer. To approximate the equation (1) at point (x_p, t) we introduce trailing communications

$$\begin{Bmatrix} Y_{p,N+1} \\ Y_{p,N} \end{Bmatrix}. \tag{10}$$

Putting $\varepsilon_x = \pm 1$ in (9), we get on the three-point template system of two algebraic equations

$$\begin{cases} Y_{p,N+1} + Y_{p,N} = \left[Y_{p+1,1} - \sum_{n=0}^{N-2} Y_{p,n+1} \right], \\ Y_{p,N-1} - Y_{p,N-1} = (-1)^N \times \left[Y_{p-1,1} - \sum_{n=0}^{N-2} (-1)^n \times Y_{p,n+1} \right]. \end{cases} \qquad (11)$$

And find

$$\begin{Bmatrix} Y_{p,N+1} \\ Y_{p,N} \end{Bmatrix} = \frac{1}{2} \times \left\{ \left[Y_{p+1,1} \pm (-1)^N \times Y_{p-1,1} \right] - \sum \varphi_n^{\pm} \times Y_{p,n+1} \right\}, \qquad (12)$$

where

$$\varphi_n^{\pm} = 1 + (-1)^{n+N}, \quad N = 2,\ 3,\ 4,\ \ldots \qquad (13)$$

are the normalizing factors.

At $N = 2$ value the $n = \overline{0,0}$ and then we receive

$$\begin{cases} Y_{p,2} = \frac{1}{2} \times \left[Y_{p+1,1} - Y_{p-1,1} \right], \\ Y_{p,3} = \frac{1}{2} \times \left\{ \left[Y_{p+1,1} + Y_{p-1,1} \right] - 2 \times Y_{p,1} \right\}. \end{cases} \qquad (14)$$

After substituting (14) into (7) we obtain

$$Y'_{p,1}(t) = \frac{1}{Dx1^2} \times \left\{ \left[Y_{p+1,1}(t) + Y_{p-1,1}(t) \right] - 2Y_{p,1}(t) \right\}, \quad p = \overline{1,2m-1}, \qquad (15)$$

where $\{Y_{0,1}(t), Y_{2m,1}(t)\}$ are the boundary functions of the first kind.

For $N = 3$ and the value of the relations (7) and (14) we obtain ODEs of the higher order

$$\begin{cases} Y'_{p,1}(\tau) = \frac{1}{2 \times Dx1^2} \times \left[Y_{p+1,2}(\tau) - Y_{p-1,2}(\tau) \right], \\ Y'_{p,2}(\tau) = \frac{1}{Dx1^2} \times \left[Y_{p+1,2}(\tau) + Y_{p-1,2}(\tau) - 2Y_{p,2}(\tau) \right], \end{cases} \qquad (16)$$

where

$$\begin{cases} Y_{0,2}(\tau) = Dx1 \times gW(\tau), \\ Y_{2m,2}(\tau) = Dx1 \times gL(\tau), \end{cases} \qquad (17)$$

are known boundary functions of the second kind.

It should be noted that the developed approach involves ordinary finite-difference methods as a particular case. The scheme (15) coincides with the classical Dirichlet problem, and (16) – with the Neumann problem. For the problem (16) it is indicative that transmission of information on the boundaries of the area in the natural scheme is implemented through internal point accurately without lowering the order of approximation.

With the reduction increase N-order approximation orders of connections (12) increase. Note that the integration of ODEs (15)–(17) having the form of Cauchy with explicit methods is the most developed procedure. Variety of standard programs allows us consider this procedure as an elementary one. From the viewpoint of economy on number of operations the last cannot be improved.

The developed procedure of numerical-analytical digitization can be quite simply generalized to other types of differential equations in mathematical physics. In particular, in stationary tasks it is easier to localize the features and use the high-order schemes at smooth areas.

The value of the order of approximation in conjunction with carrying out computation based on shredded grids allows be oriented in the evaluation of calculations' accuracy.

Let us show how to formulate the algorithm of approximate calculations based on operating functions as with formulas.

When constructing the computational algorithm (13)—(17) we have used a priori information available for task and above all — the information on membership to a particular class of smoothness, describing the task functions. Since smoothness specifically determines its widths, the value of widths gives indication of the optimally possible accuracy of computational algorithm.

Let us consider Cauchy data as dependent variables

$$\{Y_{p,1}(t),\ Y_{p,2}(t)\},\ p=\overline{1,2m-1}. \tag{18}$$

Rewriting ODEs (4) as following

$$Y_{p,n+3}(t)=\frac{Dx1^{2n}}{(n+1)(n+2)}Y'_{p,n+1}(t). \tag{19}$$

From ratios (18), (19) it follows

$$\begin{cases} Y_{p,3}(t)=\dfrac{Dx1^{2}}{2!}Y^{(1)}_{p,1}(t), \\ Y_{p,4}(t)=\dfrac{Dx1^{2}}{3!}Y^{(1)}_{p,2}(t), \\ Y_{p,5}(t)=\dfrac{Dx1^{4}}{4!}Y^{(1)}_{p,1}(t), \\ Y_{p,6}(t)=\dfrac{Dx1^{6}}{6!}Y^{(1)}_{p,2}(t), \\ \dots\dots\dots\dots\dots\dots\dots \end{cases} \tag{20}$$

Thus, the general solution of (6) can be represented as follows

$$Y_{p+\varepsilon_x,1}(t,x)=\sum_{n=0}^{\infty}\varepsilon_x^{2n}\times\frac{Dx1^{2n}}{(2n)!}\times Y^{(n)}{}_{p,1}(t)+\sum_{n=0}^{\infty}\varepsilon_x^{2n+1}\times\frac{Dx1^{2n+1}}{(2n+1)!}\times Y^{(n)}{}_{p,2}(t). \tag{21}$$

The first summand of (21) satisfies the condition of adiabatic wall and the second one corresponds to wall conditions with constant temperature.

In algebraic computation more convenient will be a mathematical model with the divided respective to Cauchy data (18) form

$$\sum_{n=0}^{\infty} Y^{(n)}{}_{p,1}(t) \times \frac{Dx1^{2n}}{(2n)!} = \frac{1}{2}\left[Y_{p+1,1}(t) + Y_{p-1,1}(t)\right]; \tag{22}$$

$$\sum_{n=0}^{\infty} Y^{(n)}{}_{p,2}(t) \times \frac{Dx1^{2n+1}}{(2n+1)!} = \frac{1}{2}\left[Y_{p+1,2}(t) + Y_{p-1,2}(t)\right]. \tag{23}$$

In general, for physical realizable unknown variables there exists fast convergence of infinite series (22), (23). We face this, e.g., in the case when derivatives of functions are limited by derivatives of exponential functions, which fortiori confirms differentiation termwise used in the analysis. However, in practical cases series should converge fast enough so that to make it possible to confine by only a few initial members of the series.

Computational experiments. Modelling was performed by means of multiprocessor computer systems (Bashkov et al., 2011). Direct computational experiment was carried out for European call option on a equity share on the following conditions: K = 40; T = 0.5; r = 0.1. For the mathematical model (1) the following conditions have been adopted:

$$\begin{cases} Y_{0,4}(t) = 1, Y_{2m,1}(t) = 1, x_{2m} = 2, \\ \varepsilon_x \in [-1, +1]. \end{cases} \tag{24}$$

Taking into account that $\varepsilon_x\big|_{x=0}$ and the symmetry of the problem on coordinate, we have $Y_{m,2}$ = 0, $Dx1$ = 1. To define $Y_{m,1}$ we use the boundary condition at $\varepsilon_x = \pm 1$. From the infinite sum in (21) we will retain n = 1, n = 2 term of series. Under such conditions, mathematical models will have the form

$$\begin{cases} N = 1,\ Y_{m,1}(t) + \frac{1}{2} Y'_{m,1}(t) = 1, \\ N = 2,\ Y_{m,1}(t) + \frac{1}{2} Y'_{m,1}(t) + \frac{1}{4} Y''_{m,1}(t) = 1. \end{cases} \tag{25}$$

Solution for models (25) is represented by the relationship:

$$\begin{gathered} N = 1,\ \ Y_{m,1}(t) = 1 - e^{-2t}; \\ N = 2,\ \ Y_{m,1}(t) = 1 - 1{,}37 e^{-2{,}54t} + 1{,}37 e^{-9{,}46t}; \\ N = \infty,\ \ Y_{m,1}(t) = 1 + \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{(2n-1)} e^{-(2n-1)^2 \frac{\pi^2}{4} t}. \end{gathered} \tag{26}$$

The results of computational experiments are presented in Table 1.

Analysis of the results shows that, on the one hand, at transition to limit at $N \to \infty$, the approximate solution coincides with the exact one. Thus, with the increase of the parameter N the error decreases rapidly. On the other hand, at setting as a goal to synthesize parallel algorithms of this method from relationship (26) it is obvious that it fits into the concept of infinite parallelism (Shvachych, 2008). In fact, at assigning one processor per node of the computational domain, it becomes possible to carry out calculations at all nodes in parallel.

Table 1. **Results of modelling the price for European option,** *authors'*

#	S_0	σ	*Exact solution*	*N = 1*	*N = 2*
1	30	0.1	0.0002	0.00018	0.0002
2	30	0.2	0.0925	0.0916	0.0924
3	38	0.1	1.0487	1.0478	1.0489
4	38	0.2	2.1192	2.1189	2.1193
5	40	0.1	2.3412	2.3421	2.3414
6	40	0.2	3.3123	3.3132	3.3126
7	42	0.1	4.0562	4.0558	4.0565
8	42	0.2	4.7597	4.7589	4.7599
9	44	0.1	5.9725	5.9731	5.9727
10	44	0.2	6.4081	6.4099	6.4083

Conclusions. In this paper we explored the differential model option price of European type, which is a financial asset share. Option price dynamics is described by the parabolic equation of Blake-Scholes. Particular attention was paid to numerical and analytical approach to determine the fair price of options. Higher acceleration of computations as compared with the finite-difference approach is explained through the use of analytical solutions that allow simultaneous calculations and in parallel on all temporary layers without using combined memory. This approach excludes the recurrent structure for unknown vectors' calculating which, as a rule, leads to accumulation of rounding errors. Thus, the constructed parallel form of the algorithm is maximal, hence, has the lowest possible time implementation of the algorithm on parallel computing systems.

The proposed approach can be used to study the dynamics of the call price and put options for prediction of the option value as well as to control the process of changes in the shares value.

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