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## Computing Model for Vibratory Digging-Out of Sugar Beet Roots

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The new design mathematical model of the sugar beet roots vibration digging-out process with the plowshare vibration digging working part has been created. In this case the sugar beet root is simulated as a solid body, while the plowshare vibration digging working part accomplishes fluctuations in the longitudinal-vertical plane with the given amplitude and frequency in the process of work. The **aim** of the current research has been to determine the dependences between the design and kinematic parameters of the sugar beet roots vibration digging-out technological process from soil, which provide their non-damage. **Methods.** For the aim accomplishment, the methods of design mathematical models constructing based on the classical laws of mechanics are applied. The solution of the obtained differential equations is accomplished with the PC involvement. **Results.** The differential equations of the sugar beet root's motion in course of the vibration digging-out have been comprised. They allow to determine the admissible velocity of the vibration digging working part's forward motion depending on the angular parameters of the latter. In the result of the computational simulation i.e., the solution of the obtained analytical dependence by PC, the graphic dependences of the admissible velocity of plowshare vibration digging working part's forward motion providing the extraction of the sugar beet root from soil without the breaking-off of its tail section have been determined. **Conclusions.** Due to the performed analytical research, it has been established that  $\gamma = 13 \dots 16^\circ$ ,  $\beta = 20 \dots 30^\circ$  should be considered as the most reasonable values of  $\gamma$  and  $\beta$  angles of the vibration digging working part providing both its forward motion optimum speed and sugar beet root digging-out from the soil without damage. On the ground of the data obtained from the analytical research, the new vibration digging working parts for the sugar beet roots have been designed; also the patents of Ukraine for the inventions have been obtained for them.

**Keywords:** vibration digging working part, sugar beet root, simulation analysis, differential equations, computing solution.

### INTRODUCTION

The wide use of the vibration digging working parts in the contemporary beet-harvesting machines is caused and explained by their considerably smaller tractive resistance, and also ability to extract root-crops from the soil actually without the damages and losses. The fluctuations of the digging working part create conditions for intensive stabbing of stuck soil from the root-crops during their excavation,

which contributes to the high level of quality indicators. Therefore, the design of new vibration digging working parts, and also study of their functioning for determining the optimum design and kinematic parameters is the urgent task of sugar-beet raising mechanization.

The analytical study of the process of interaction between the working elements of the vibration digging working part and a root will allow to obtain the

kinematic, design and technological characteristics and also – to determine their optimum values.

Basic theoretical and experimental researches of the sugar beet root's vibration excavation are published in source [1], where the root-crop is simulated as the body like to an elastic rod of variable cross section with one fixed end. The lateral oscillations of a root examined in the current research are described by means of the differential equation in the partial derivatives of the fourth order. The direct technological process of the root-crop digging-out of the soil during the vibration application of efforts is not actually discussed in the paper. Though, it is mentioned that the conditions for the root digging-out of the soil under the action of the perturbing force applied in the transverse-vertical plane are found by means of the specially comprised dynamic force analysis equations. In the mentioned source it is considered that precisely this direction of fluctuations will most fully contribute to the high-quality excavation of the sugar beet root from the soil.

In the literature [2] the theory of usual plowshare digging operating unit is developed. The method of the root digging-out of the soil at forward motion of digger taking into account the conditions for the root non-damaging has been discovered. In the above mentioned paper the expression for determining the admissible velocity of digging working part's forward motion at given design parameters is obtained. In this case the root digging-out process is achieved under the action of the forces appearing on the working surfaces of plowshares as a result of the digging working parts' forward motion along the rows of root-crops.

The developed natural and forced root body oscillations theory [3] is necessary for the evaluation of the influence of the indicated fluctuations on the process of destroying the connections of root-crops with the soil.

Numerous studies currently existing in the scientific literature concern issues of the root vibration digging-out of the soil [2–10]. However, the data of developments are nevertheless insufficient for the complete analysis of the root direct digging-out of the soil.

The purpose of the current research was to establish the analytical dependences between the design and kinematic parameters of the sugar beet root vibration digging-out technological process out of the soil by a plowshare digger ensuring conditions for their non-damage.

## MATERIALS AND METHODS

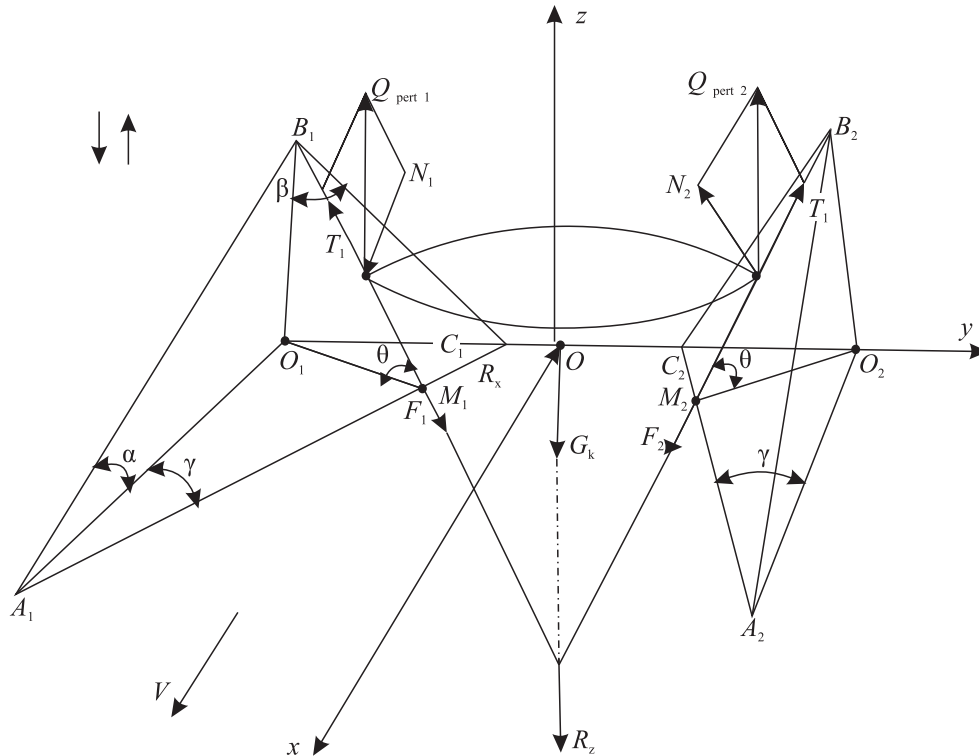
For end of the mentioned purpose the methods of design mathematical models construction and computer simulation are applied.

## RESULTS AND DISCUSSION

Let us examine the process of beet roots digging-out with a vibrating digger performed under the action of the vertical harmonic perturbing force transferred to plowshares from an actuator. The principle of the usual plowshare digging working part functioning lies in the fact that during the motion on the row of the beet roots its wedges destroy the layer of soil penetrating together with the roots the reduced digger canal formed by the internal surfaces of wedges. Since the wedges are fixed under appropriate  $\alpha$ ,  $\beta$  and  $\gamma$  angles, interaction of the layer of soil together with the roots with the sides of wedges in course of their forward motion occurs in such a way that the layer of soil is compressed from both sides, deformed, and of the corresponding efforts of digging-out from the soil are created for the beet roots during their further motion between the working surfaces of wedges (i.e., through the reduced canal).

It should be noted that in the digging-out roots process by the usual plowshare digging working part, the soil support forces play the important role, therefore the layer of soil is compressed in the reduced digger canal, which causes the appearance of necessary forces of vertical digging-out. Thus, the presence of the soil support forces is the necessary condition for the work of the usual plowshare digging working part.

In case of the beet roots digging-out by a vibration digger as a result of the fluctuating motion of plowshares the soil in the zone of the digger working canal is strongly loosened, and therefore, the mentioned root digging-out forces do not appear, since the necessary compressive strain of soil in the digger canal does not occur. As indicated in [3], the presence of soil in the vibration digger working canal is not a basic condition for creating the root digging-out efforts. This is an essential difference between the vibration digger and the known types of the digging working parts. In literature [3] it is also noted, that if in the canal of the usual disk or plowshare digging working part the root at presence of the soil support forces, is inclined in the direction of motion, so then in the canal of vibration digger the axis of roots at its digging-out always takes the position almost perpendicular to the row axis.



**Fig. 1.** Force interaction between the sugar-beet root and wedges of the vibration digging working part

This is caused by the fact that the support forces, which are the necessary condition of the usual plow-share digger operating, are no longer important for the vibration digger work. Therefore, the probability of the root breaking-off as a result of its axis inclination in the direction of the forward motion of vibration digger considerably decreases. During the vibration digging-out in the process of vertical displacement the roots are intensively cleaned of the stuck soil due to providing them with a significant acceleration.

Thus, the root digging-out during the vibration excavation is achieved due to the direct seizure of roots by the vibration digging working part under the action of the perturbing force transferred by working part from the actuator. As followed from the mentioned above, it is necessary to examine the direct contact of roots with the working surfaces of plowshares during their seizure for investigating the digging-out process with the aid of the vibration digging working part. This contact can be achieved either directly between the working surfaces of digger and a root body, or through the sufficiently thin layer of soil.

Let us draw the equivalent diagram of interaction between a root and the working surfaces of the vibration digging working part. For that end, the vibra-

tion digging working part is represented in the form of two wedges –  $A_1B_1C_1$  and  $A_2B_2C_2$ , each of which has an inclination in the space under  $\alpha$ ,  $\beta$ ,  $\gamma$  angles, fixed to each other in the way that the working canal is formed, whose tail end is narrowed (Fig. 1). The indicated wedges accomplish fluctuating motions in the longitudinal-vertical plane (the actuator of plowshares to the fluctuating motion is not represented). The direction of the forward motion of the vibration digging working part is shown with a pointer. Let us connect with the vibration digging working part the Cartesian rectangular coordinate system  $Oxyz$ . Its  $O$ -center is located in the middle of the narrowed digger canal,  $Ox$ -axis coincides with the direction of the forward motion of digger,  $Oz$ -axis is directed upward, and  $Oy$ -axis is directed to the right side of the digger. Projections of  $B_1$  and  $B_2$  points to  $Oy$ -axis are marked with  $O_1$  and  $O_2$  correspondingly.

It is considered that the root approximated with the body of cone-shaped form located inside the digging working part, whose own axis is parallel to  $Oz$ -axis interacts with the surfaces of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$  in the two corresponding points. The straight lines drawn through the contact points of roots with the planes of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$  and  $B_1$  and

$B_2$  points create the corresponding  $M_1$  and  $M_2$  points at the intersection with the wedges sides  $A_1C_1$  and  $A_2C_2$ . Thus,  $\theta$  is a dihedral angle ( $B_1M_1O_1$ ) between the lower basis  $A_1O_1C_1$  and the working surface of the wedge  $A_1B_1C_1$  or, ( $B_2M_2O_2$  angle) between the lower basis  $A_2O_2C_2$  and the working surface of wedge  $A_2B_2C_2$ .

The forces appearing as a result of interaction of the root with the vibration digging working part is indicated. At the contact points of the root with the appropriate surfaces of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$  the perturbing forces  $Q_{pert1}$  and  $Q_{pert2}$  perform. Because of the action of these forces at the indicated contact points the normal responses  $N_1$  and  $N_2$  appear from the side of  $A_1B_1C_1$  and  $A_2B_2C_2$  wedges surfaces correspondingly, and  $T_1, T_2$  i.e., the tangential components of perturbing forces  $Q_{pert1}$  and  $Q_{pert2}$ . In addition, at the indicated contact points there are the friction forces of  $F_1$  and  $F_2$  appearing during the motion of roots along the working surfaces of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$  correspondingly. Since the vibration digging working part has a symmetry axis, assuming that the root in course of its digging-out of the soil is located precisely on this symmetry axis it is considered that the modules of the paired forces appearing on the appropriate planes of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$  between themselves. In the root gravity center the  $G_k$ -force acts i.e., the mass of a root, its binding force (cohesion) with the soil is designated with the common response of general of  $R_z$ .

The differential equation of the beet root motion in the process of its digging-out of the soil will take the vector form:

$$m\bar{a} = \bar{N}_1 + \bar{N}_2 + \bar{F}_1 + \bar{F}_2 + \bar{R} + \bar{G}_k, \quad (1)$$

where  $m$  – a beet root mass;  $\alpha$  – root digging-out acceleration.

The value of the forces included into equation (1) is determined below.  $T_1$  and  $T_2$  components of perturbing forces  $Q_{pert1}, Q_{pert2}$  never make a direct impact on a root, though they only cause the soil loosening around the root, and so, are not included into the differential equation of a root motion.

The expressions for normal  $N_i$  and tangential  $T_i$  components of the perturbing forces  $Q_{pert1}$  and  $Q_{pert2}$  is following:

$$N_1 = N_2 = Q_{pert1} \cos\theta, \quad (2)$$

$$T_1 = T_2 = Q_{pert1} \sin\theta, \quad (3)$$

The value of frictional forces  $F_1$  and  $F_2$  are estimated from the following equations:

$$F_1 = F_2 = fN_1 = fQ_{pert1} \cos\theta \quad (4)$$

It is obvious that taking into account the working conditions for the above mentioned vibration digging working parts [3] during the analytical study of the root digging-out process it is necessary to examine two sequential phases:

- 1) A root is solidly connected to the soil (it is fixed in it, exactly), and therefore, it cannot be moved along  $Oz$ -axis;
- 2) The connection of roots with the soil has already been weakened (almost destroyed) and it begins its upward motion i.e., in the direction of  $Oz$ -axis.

The first phase is further considered.

On purpose of the thorough analysis of the root digging-out process the differential equation (1) in the projections on the axis of the Cartesian coordination system of  $Oxyz$  is composed.

It should be noted immediately that since the components of normal responses  $N_i$  of the working surfaces  $A_1B_1C_1$  and  $A_2B_2C_2$  along  $Oy$ -axis are equal in value and directed oppositely, the root digging-out occurs actually only into  $xOz$ -plane, and therefore, the vector equation (1) is reduced to the system of two equations in the following form:

$$\left. \begin{aligned} m\ddot{x} &= N_{1x} + N_{2x} + F_{1x} + F_{2x} - R_x \\ m\ddot{z} &= N_{1z} + N_{2z} + F_{1z} + F_{2z} - R_z \end{aligned} \right\} \quad (5)$$

where  $R_x, R_z$  – the projections of the constraint forces of the root to the soil to the corresponding axes.

The projections of the forces included into the equation system (5) are determined as follows. As seen from Fig. 1, the projections of the normal components  $N_1$  and  $N_2$  to  $Ox$ -axis are equal:

$$N_{1x} = N_{2x} = N_1 \sin\theta \sin\gamma = Q_{pert1} \cos\theta \sin\theta \sin\gamma \quad (6)$$

Obviously, until the root is fixed in the soil, and when during the contact of the root with the wedges the plowshares  $A_1B_1C_1$  and  $A_2B_2C_2$  move upward in the process of fluctuations (in the positive direction of  $Oz$ -axis), though the root motion has not begun yet, the wedges can slip along the root body. Therefore, the frictional forces direction can be considered opposite to the direction of the  $T_i$ -force, ( $i = 1,2$ ) (parallel to the lines of  $B_1M_1$  and  $B_2M_2$  as for the working surfaces of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$  respectively). At that, the



slippages lengthwise to  $Ox$ -axis almost never occurs due to the absence of the soil support forces, also in connection with the digger forward motion (since its working surfaces are fixed at the appropriate angles to the direction of motion, what creates the condition for the soil layer shift) it is inclined to the forward motion at some small angle.

Thus, as follows from the above and on the basis of structure diagram of Fig. 1 it is possible to conclude that the projections of the  $F_1$  and  $F_2$  friction forces to  $Ox$ -axis will be equal to:

$$F_{1x} = F_{2x} = F_1 \cos \theta \sin \gamma$$

or, as considering (4), it will be:

$$F_{1x} = F_{2x} = fQ_{\text{pert1}} \cos^2 \theta \sin \gamma \quad (7)$$

The projections of  $N_1$  and  $N_2$  normal compounds to  $Oz$ -axis, as follows from the chart of Fig. 1, will be equal to:

$$N_{1z} = N_{2z} = N_1 \cos \theta$$

or, as considering (2):

$$N_{1z} = N_{2z} = Q_{\text{pert1}} \cos^2 \theta \quad (8)$$

The projections of  $F_1$  and  $F_2$  friction forces to  $Oz$ -axis, as also follows from the chart of Fig. 1, will be equal to:

$$F_{1z} = F_{2z} = F_1 \sin \theta$$

or, as considering (4), it will be:

$$F_{1z} = F_{2z} = fQ_{\text{pert1}} \cos \theta \sin \theta \quad (9)$$

While substituting equations (6), (7) in the first equation of the system (5), it will be:

$$m\ddot{x} = 2Q_{\text{pert1}} \cos \theta \sin \theta \sin \gamma + 2fQ_{\text{pert1}} \cos^2 \theta \sin \gamma - R_x \quad (10)$$

Whereas,

$$Q_{\text{pert1}} = \frac{1}{2} Q_{\text{pert1}} = \frac{1}{2} H \sin \omega t, \quad (11)$$

where  $H$  – perturbing force amplitude;  $\omega$  – perturbing force frequency, then the equation (10) will be, as follows:

$$m\ddot{x} = H \cos \theta \sin \theta \sin \gamma \sin \omega t + fH \cos^2 \theta \sin \gamma \sin \omega t - R_x \quad (12)$$

The process of the vibration digging-out of a root out of the soil should be analyzed examining separately the equations of the system (5). Thus, the obtained equation (12) describes the forced oscillations of the root in the soil along  $Ox$ -axis, while its lower end is rigidly fixed.

Obviously, this equation occurs upon condition of the root immediate contact with the surfaces of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$ , i.e., when they are moving upward. This will be possible upon the following condition:

$$0 \leq \sin \omega t \leq 1$$

If the opposite in-equation occurs:

$$-1 \leq \sin \omega t \leq 0,$$

then it means that the surfaces of wedges  $A_1B_1C_1$  and  $A_2B_2C_2$  move downward.

Since it is accepted that the root is cone-shaped, and then upon condition of fulfilling the last in-equation the root immediate contact with the surfaces of the digger wedges is lost. So, the root manages to return to the vertical position in time of the absence of contact due to its own elasticity. It is possible to estimate approximately considering that in time of the absence of contact with the surfaces of plowshares the root accomplishes dying oscillations along  $Ox$ -axis upon availability of resistance and in the perturbing force absence. With the advent of new contact due to a change in the fluctuating motion direction of the plowshares' surfaces and also, the digging working part's forward motion (a canal in the tail end of the digger is narrowed) everything is repeated. As follows from the mentioned above, at the vibration digging-out of the soil the root preserves almost vertical position, that is in line with the conceptual issues of the source [5].

Further on, the second equation of system (5) should be considered. Due to substitution of expressions (8), (9) in it, the equation will be, as follows:

$$m \ddot{z} = 2Q_{\text{pert1}} \cos^2 \theta - 2fQ_{\text{pert1}} \cos \theta \sin \theta - R_z - G_k$$

or, as considering (11), it will be:

$$m \ddot{z} = H \cos^2 \theta \sin \omega t - fH \cos \theta \sin \theta \sin \omega t - R_z - G_k \quad (13)$$

Obviously, during the first phase (the root is quite strongly stuck in the soil), the left half of the differential equation (13) is equal to zero. That is why, this very equation is changed into the static equation i.e., the root balance equation in the soil along  $Oz$ -axis, as follows:

$$(H \cos^2 \theta - fH \cos \theta \sin \theta) \cdot \sin \omega t - R_z - G_k = 0 \quad (14)$$

Upon further condition of the root immediate contact with the wedges' surfaces  $A_1B_1C_1$  and  $A_2B_2C_2$  (i.e., upon condition of  $0 \leq \sin \omega t \leq 1$ ) the induced longitudinal vibrations of the roots examined in the source [3] are performed. In the absence of the root contact with the indicated plowshares' surfaces the root accomplishes dying oscillations along  $Oz$ -axis at resistance.

Thus, upon condition (14) the root always accomplishes either longitudinal vibrations or forced, or damped at resistance. Besides, the impact loads appear in the moment of root's the next contact with the plowshares. All this causes the intensive destruction of the root's connections with the soil, and therefore, misbalance (14) as a result of the  $R_z$  force value decrease.

Thus, at the very moment the following inequality is fulfilled instead of the equality (14):

$$(H \cos^2 \theta - f H \cos \theta \sin \theta) \cdot \sin \omega t - G_k > R_z \quad (15)$$

Upon condition (15) the root move upward begins described with the differential equation (13).

Thus, in the second phase the root begins to move in the soil as described by the differential equation (13).

The differential equation (13) is following:

$$\ddot{z} = \frac{H}{m} (\cos^2 \theta - f \cos \theta \sin \theta) \cdot \sin \omega t - \frac{R_z}{m} - \frac{G_k}{m} \quad (16)$$

After double integrating of this very differential equation the root's velocity and displacement along  $Oz$ -axis as functions of time  $t$  will be obtained.

The first integral will be equal to:

$$\dot{z} = -\frac{H}{m\omega} (\cos^2 \theta - f \cos \theta \sin \theta) \cdot \cos \omega t - \frac{1}{m} (R_z + G_k) t + C_1 \quad (17)$$

The second one looks as follows:

$$z = -\frac{H}{m\omega^2} (\cos^2 \theta - f \cos \theta \sin \theta) \cdot \sin \omega t - \frac{1}{2m} (R_z + G_k) t^2 + C_1 t + C_2 \quad (18)$$

where  $C_1$  and  $C_2$  – arbitrary constants.

For determination of the arbitrary constants the initial conditions are necessary to be established. If  $t = 0$ :

$$\begin{aligned} z &= -h_1, \\ \dot{z} &= 0, \end{aligned}$$

where  $h_1$  – root depth in the soil.

Considering the initial conditions, the arbitrary constants' values will be obtained:

$$\begin{aligned} C_1 &= \frac{H}{m\omega} (\cos^2 \theta - f \cos \theta \sin \theta), \\ C_2 &= -h_1 \end{aligned} \quad (19)$$

After substitution of (19) into the expressions (17) and (18) the following will be obtained:

$$\begin{aligned} \dot{z} &= -\frac{H}{m\omega} (\cos^2 \theta - f \cos \theta \sin \theta) \cdot \cos \omega t - \\ &- \frac{1}{m} (R_z + G_k) t + \frac{H}{m\omega} (\cos^2 \theta - f \cos \theta \sin \theta) \end{aligned} \quad (20)$$

Thus, the expressions for determining of the root's

velocity and displacement in the process of its digging-out of the soil along  $Oz$ -axis at the established initial conditions have been obtained.

$$\begin{aligned} z &= -\frac{H}{m\omega^2} (\cos^2 \theta - f \cos \theta \sin \theta) \cdot \sin \omega t - \\ &- \frac{1}{2m} (R_z + G_k) t^2 + \frac{H}{m\omega} (\cos^2 \theta - f \cos \theta \sin \theta) t - h_1 \end{aligned} \quad (21)$$

It is possible to determine the time of the root's digging-out of the soil  $t_1$  from equation (21). For that end, it is necessary to substitute value of  $z = 0$  into the left side of equation (21) and to estimate the obtained equation considering  $t_1$ .

Since this equation is transcendent, it is impossible to obtain the analytical expression for determination of  $t_1$ , though it is possible to estimate it with PC applying the known numerical methods. The calculated value of  $t_1$  is allowed to be used for determining the productivity of the aggregate for the roots excavation with the vibration digging working parts.

It is noted above, for the period of time, while the root is fixed in the soil, the equation (12) describes the forced oscillations of the root in the soil along  $Ox$ -axis. However, when the root loses ties with the soil and begins to move upward, the equation (12) describes its motion lengthwise  $Ox$ -axis until the complete digging-out.

Consequently, by analogue, after the double integrating of the differential equation (12) it is possible to determine root velocity and displacement along  $Ox$ -axis, i.e., in the direction of the vibration digging working part's forward motion. For that end, it is necessary to write down the differential equation (12) in the following form:

$$\ddot{x} = \frac{H}{m\omega} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta) \sin \omega t - R_x \quad (22)$$

The double integrating of this very equation gives the following results. The first integrating of the differential equation (22) results in the following:

$$\begin{aligned} \dot{x} &= -\frac{H}{m\omega^2} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta) \cos \omega t - \\ &- \frac{R_x}{m} t + L_1, \end{aligned} \quad (23)$$

The second one will give:

$$\begin{aligned} x &= -\frac{H}{m\omega^3} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta) \sin \omega t - \\ &- \frac{R_x}{2m} t^2 + L_1 t + L_2 \end{aligned} \quad (24)$$

where  $L_1$  and  $L_2$  – arbitrary constants.

For the arbitrary constants  $L_1$  and  $L_2$  to be determined, it is necessary to establish the initial conditions. Thus, at  $t = 0$ :

$$\dot{x} = 0, x = x_0,$$

where  $x_0$  – the distance of the root’s vertical axis from the initial point of the coordinate system ( $O$ -point) in the moment of time  $t = 0$ .

Considering the initial conditions, the following values of the arbitrary constants will be obtained:

$$L_1 = -\frac{H}{m\omega} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta), \quad (25)$$

and

$$L_2 = x_0 \quad (26)$$

The substituting of the obtained values of the arbitrary constants  $L_1$  and  $L_2$  in the expressions (23) and (24) will result in:

$$\begin{aligned} \dot{x} = & \frac{H}{m\omega} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta) \cos \omega t - \\ & - \frac{R_x}{m} t + \frac{H}{m\omega} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta), \end{aligned} \quad (27)$$

and

$$\begin{aligned} x = & -\frac{H}{m\omega^2} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta) \sin \omega t - \\ & - \frac{R_x}{2m} t^2 + \frac{H}{m\omega} \sin \gamma (\cos \theta \sin \theta + f \cos^2 \theta) t + x_0. \end{aligned} \quad (28)$$

Thus, the values of the projections of the root velocity and displacement to  $Ox$ -axis as the function of time  $t$ , which meets the given initial conditions, have been obtained.

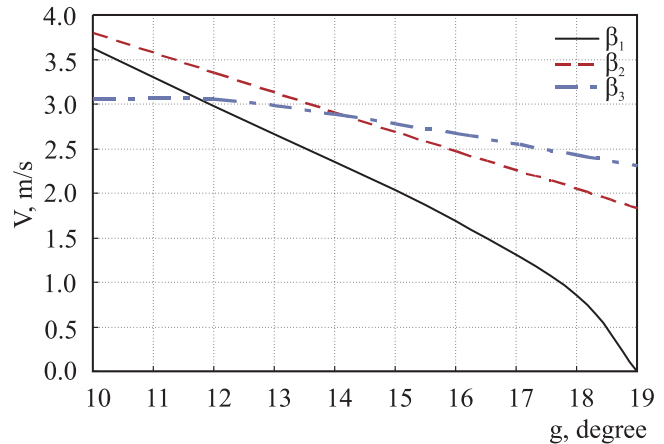
Owing to the specially determined software, the numerical computations of the admissible velocity  $V$  of the plowshare vibration digging working part’s moving upon condition of the non-damage of the sugar beet root and various values of angles  $\gamma$  and several fixed values of angles  $\beta$  have been accomplished. The initial data for the calculations are given in the Table.

The diagrams of the velocity change  $V$  for the forward motion of the plowshare digging working part depending on the different values of the angle  $\gamma$  have been drawn on a base of (Fig. 2).

As seen from the represented diagrams, the calculation results the dependences of the indicated parameters are close to the linear nature. By increasing in the angle of attack  $\gamma$  of the plowshare vibration digging working part, the value of the forward velocity  $V$  of its motion

**Estimating Parameters for the Plowshare Vibration Digging Working Part**

Parameter					
$a$	$[P_x]$	$\gamma$	$f$	$f_1$	$g$
0.12 m	200 N	11000 N/m <sup>3</sup>	0.60	0.50	9.81 m/s <sup>2</sup>



**Fig. 2.** Dependence of the admissible velocity  $V$  of the plowshare vibration digging working part’s forward motion upon the angle  $\gamma$ :  $\beta_1 = 15^\circ$ ,  $\beta_2 = 20^\circ$ ,  $\beta_3 = 30^\circ$

ensuring the digging-out of the sugar beet root of the soil without damaging is lowered. As concerning the influence of the camber angle  $\beta$  of the plowshare vibration digging working part onto the forward velocity  $V$  of its motion, as can be seen from the diagrams, the use of big values of such angles ensures the higher level of the forward velocity of motion. Considering that the statistical value of the taper angle  $\gamma_k$  of the sugar beet root composes 20... 28 degrees, then the use of a value of the camber angle  $\beta$  of the plowshare vibration digging working part close to 30 degrees, also ensures the higher level of the motion forward velocity.

Thus, the obtained estimation results demonstrate that the most reasonable values of the angles  $\gamma$  and  $\beta$ , wherein the high speed  $V$  of the plowshare vibration digging working part’s forward motion, and also digging-out of the sugar beet root from the soil without damage are ensured, should be  $\beta_1 = 15^\circ$ ,  $\beta_2 = 20^\circ$ ,  $\beta_3 = 30^\circ$ .

Thus, the PC estimation results of the obtained analytical dependences confirm their correctness and afford grounds for their practical application in course of both design and calculations of the new, more improved digging working parts for the beet-harvesting machines.

Taking into account the obtained results the new constructions of the vibration digging working parts [11–15] have been developed. Their use improves the quality of this technological process.

**CONCLUSIONS**

1. The new design mathematical model of the vibration digging-out of the beet root of the soil with the

plowshare vibration digging working part has been accomplished.

2. The root's velocity and displacement in the forward motion direction of the vibration digging working part have been determined taking into account its design-kinematic parameters.

#### Математична модель вібраційного викопування коренеплодів буряка з ґрунту

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Побудовано нову розрахункову математичну модель процесу вібраційного викопування коренеплодів цукрових буряків лемішним вібраційним викопувальним робочим органом. При цьому коренеплід цукрових буряків моделюється як тверде тіло, а лемішний вібраційний викопувальний орган у процесі роботи здійснює коливання у повздовжньо-вертикальній площині із заданою амплітудою і частотою. **Мета** даного дослідження полягала у встановленні залежностей між конструктивними і кінематичними параметрами технологічного процесу вібраційного видалення коренеплодів цукрових буряків з ґрунту, що забезпечує умови їхнього неушкодження. **Методи.** Використано методи побудови розрахункових математичних моделей, засновані на класичних законах механіки. Вирішення отриманих диференціальних рівнянь здійснено за допомогою ПЕОМ. **Результати.** Складено диференціальні рівняння руху коренеплоду цукрових буряків при вібраційному викопуванні, що дають можливість визначити допустиму швидкість поступального руху вібраційного викопувального робочого органа залежно від геометричних параметрів копача. У результаті комп'ютерного моделювання, тобто вирішення отриманої аналітично залежності на ПЕОМ, визначено графічні залежності допустимої швидкості поступального руху лемішного вібраційного викопувального робочого органа, яка забезпечує вилучення коренеплоду цукрового буряка з ґрунту без обламування його хвостової частини, від кутових параметрів вібраційного робочого органу. **Висновки.** У результаті аналітичного дослідження встановлено, що найбільш раціональними значеннями кутів  $\gamma$  і  $\beta$  лемішного вібраційного викопувального робочого органу, при яких забезпечується оптимальна швидкість поступального руху і вилучення коренеплодів цукрового буряка з ґрунту без пошкоджень слід вважати

$\gamma = 13 \dots 16^\circ$ ,  $\beta = 20 \dots 30^\circ$ . На основі отриманих даних аналітичних досліджень спроектовано нові вібраційні викопувальні робочі органи для коренеплодів цукрових буряків, на які отримано патенти України на винаходи.

**Ключові слова:** лемішний вібраційний викопувальний робочий орган, коренеплоди цукрового буряку, математичне моделювання, диференціальне рівняння, рішення на ПЕОМ.

#### Математическая модель вибрационного выкапывания корнеплодов свеклы из почвы

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Построена новая расчетная математическая модель процесса вибрационного выкапывания корнеплодов сахарной свеклы лемешным вибрационным выкапывающим рабочим органом. При этом корнеплод сахарной свеклы моделируется как твердое тело, а лемешный вибрационный выкапывающий орган в процессе работы совершает колебания в продольно-вертикальной плоскости с заданной амплитудой и частотой. **Цель** данного исследования состояла в установлении зависимостей между конструктивными и кинематическими параметрами технологического процесса вибрационного извлечения корнеплодов сахарной свеклы из почвы, обеспечивающие условия их неповреждения. **Методы.** Для осуществления намеченной цели использованы методы построения расчетных математических моделей, основанные на классических законах механики. Решение полученных дифференциальных уравнений осуществлено с помощью ПЭВМ. **Результаты.** Составлены дифференциальные уравнения движения корнеплода сахарной свеклы при вибрационном выкапывании, дающие возможность определить допустимую скорость поступательного движения вибрационного выкапывающего рабочего органа в зависимости от угловых параметров копателя. В результате компьютерного моделирования т. е. решения полученной аналитически зависимости на ПЭВМ, определены графические зависимости допустимой скорости поступательного движения лемешного вибрационного выкапывающего рабочего органа, которая обеспечивает извлечение корнеплода сахарной свеклы из почвы без обламывания его хвостовой части, от угловых параметров вибрационного выкапывающего рабочего органа. **Выводы.** В результате проведенного аналитического



го исследования установлено, что наиболее рациональными значениями углов  $\gamma$  и  $\beta$  лемешного вибрационного выкапывающего рабочего органа, при которых обеспечивается оптимальная скорость его поступательного движения и извлечение корнеплодов сахарной свеклы из почвы без повреждения, следует считать  $\gamma = 13...16^\circ$ ,  $\beta = 20...30^\circ$ . На основе полученных данных аналитических исследований спроектированы новые вибрационные выкапывающие рабочие органы для корнеплодов сахарной свеклы, на которые получены патенты Украины на изобретения.

**Ключевые слова:** лемешный вибрационный выкапывающий рабочий орган, корнеплоды сахарной свеклы, математическое моделирование, дифференциальные уравнения, решение на ПЭВМ.

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