

TO A GENERALIZED EQUATION OF MACHINE TOOLS MOTION

A.V. Maltsev, P. N. Pavlishin
Odessa State Agrarian University

We obtain the most general form of the equations of motion of machine units with infinitely-adjustable and non-adjustable drive.

Key words: machine, drive, equation, link.

Introduction. CVT drive provides up technological and industrial machines for optimal conditions under varying effects on executive power and the input units of machines. Such systems can be reduced to a dynamic model consisting of a motor, CVT drive and machinery. In our controlled system various types of drives - electromechanical, hydraulic and mechanical, are used.

Purpose. Creating a mathematical model of the machine set in the most general terms of change in the inertial characteristics when moving of drive units describes generalized coordinates.

Materials and methods. The analysis of dynamic processes in systems CVT drive [1] defines them as nonintegrable kinematic (nonholonomic) connection. The study of such systems can be performed using the Lagrange equations of the second kind with uncertain factors (equation Ferrers)

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_i} \right) - \frac{\partial T}{\partial q_i} = Q_i + \sum_{j=1}^m \lambda_j \frac{\partial f_j}{\partial \dot{q}_i} \quad (1)$$

where: T - kinematic energy of the system; q_i , \dot{q}_i - generalized coordinate and its first time derivative; λ - undetermined factor; f - nonholonomic constraint equation, i - the number of generalized coordinates; j - the number of constraint equations ($j = 1, 2, \dots, m$); Q_i - generalized force.

Considering the machine set as a system whose motion is described by two coordinates, the nonholonomic connection, and reduced inertia characteristics of the output link is a known function of the generalized coordinates of the driven member, to its derivatives and the time t. We introduce the following notation: - q_1 , q_2 - generalized coordinates of the input and output drive units; $\dot{q}_2 - u\dot{q}_1 = 0$ nonholonomic constraint equation, where u is a function of the time-dependent, generalized coordinate q_2 and p its derivatives; H_1 , H_2 - inertial characteristics of the units cast.

We assume that the reduced inertia characteristics of the H_1 leading machine set is unchanged, ie $H_1 = const$, and the reduced inertia characteristics of the leading machine set H_2 may be dependent on the time t, the generalized coordinate q_2 and p its derivatives, ie,

$$H_2 = H_2^{(p)}(q_2, \dot{q}_2, \ddot{q}_2, \dots, q_2, t)$$

Using equation (1) subject to acceptance of the conditions set by the notation of the equation of motion of the two mass nonholonomic constraint c , we can write

$$\begin{aligned} & \left[H_1 U^2 + H_2 + 2\dot{q}_2 \frac{\partial H_2}{\partial q_2} \right] \ddot{q}_2 + \left[H_1 U \cdot \dot{U} + \frac{\partial H_2}{\partial t} + \sum_{\xi=2}^p \frac{\partial H_2}{\partial q_2} \cdot^{(\xi+1)} q_2 \right] \dot{q}_2 + \\ & + \frac{\partial^2 H_2}{\partial q_2 t} + \sum_{\xi=1}^p \frac{\partial^2 H_2}{\partial q_2 \partial q} \cdot^{(\xi+1)} q_2 = U \cdot Q_M - Q_C \end{aligned} \quad (2)$$

From the last equation, with appropriate restrictions, the equations of motion of mechanical systems with nonholonomic constraints can be prepared. So, assuming $H_2 = const$, where reduction link perform a rotational motion with angular displacements $\varphi - q_1 = \varphi_1$, $q_2 = \varphi_2$ input and output units, the generalized forces, torques is given in the input and output parts of the system that is - $Q_1 = T_M$, $Q_2 = T_C$, and inertial characteristics is reduced moment of inertia - $H_1 = I_1$; $H_2 = I_2$, we obtain the equation [2

$$(I_1 U^2 + I_1) \ddot{\varphi}_2 + I_1 U \dot{U} \dot{\varphi}_2 = U T_M + T_C$$

From the dependence of (2) we obtain the equation of motion for any system configuration. So if the drive system does not comprise a control relationship, that is, $U=const$, then (2), describing the motion of a single mass of the system is given by

$$H \ddot{q} + 2 \frac{\partial H}{\partial \dot{q}} \dot{q} \ddot{q} + \dot{q} \left[\sum_{\xi=2}^p \frac{\partial H}{\partial q} \cdot^{(\xi+1)} q + \frac{\partial H}{\partial t} \right] + 0,5 \dot{q}^2 \left[\sum_{\xi=2}^p \frac{\partial^2 H}{\partial \dot{q} \partial q} \cdot^{(\xi+1)} q + \frac{\partial^2 H}{\partial \dot{q} \partial t} + \frac{\partial H}{\partial t} \right] = Q'_M - Q_C, \quad (3)$$

where: $H \sim (H_2 + H_1 U_2)$; $Q'_M \sim U Q_M$; $q \sim q_2$.

If the reduction link perform a rotational motion, then putting in relation (3) $q \sim \varphi$; $H \sim I$; $Q'_M \sim T_M$; $Q_C \sim T_C$ write

$$T_M - T_C = I \ddot{\varphi} + 2 \frac{\partial I}{\partial \dot{\varphi}} \dot{\varphi} \ddot{\varphi} + \frac{\partial I}{\partial t} \dot{\varphi} + \sum_{\xi=2}^p \frac{\partial I}{\partial \varphi} \cdot^{(\xi+1)} \varphi + 0,5 \dot{\varphi}^2 \left[\sum_{\xi=0}^p \frac{\partial^2 I}{\partial \dot{\varphi} \partial \varphi} \cdot^{(\xi+1)} \varphi + \frac{\partial^2 I}{\partial \dot{\varphi} \partial t} + \frac{\partial I}{\partial t} \right] \quad (4)$$

Depending on the type of the function I (4), a set of equations describing the motion of the machine aggregate by movement of one unit reduction can be obtained.

So, when $I = I(\varphi, \dot{\varphi}, \ddot{\varphi}, t)$

We have :

$$T_M - T_C = I \ddot{\varphi} + 2 \frac{\partial I}{\partial \dot{\varphi}} \cdot \dot{\varphi} \ddot{\varphi} + \frac{\partial I}{\partial t} \dot{\varphi} + \frac{\partial I}{\partial \varphi} \cdot \ddot{\varphi} + 0,5 \dot{\varphi}^2 \left[\frac{\partial^2 I}{\partial \varphi \cdot \partial \dot{\varphi}} \cdot \dot{\varphi} + \frac{\partial^2 I}{\partial \dot{\varphi}^2} \ddot{\varphi} + \frac{\partial^2 I}{\partial \dot{\varphi} \cdot \partial \ddot{\varphi}} \ddot{\varphi} + \frac{\partial^2 I}{\partial \dot{\varphi} \cdot \partial t} \ddot{\varphi} + \frac{\partial I}{\partial \varphi} \right], \quad (5)$$

and in $I = I(\varphi, \dot{\varphi}, t)$ equation (4) takes the form obtained in [1]:

$$T_M - T_C = I \ddot{\varphi} + 2 \frac{\partial I}{\partial \dot{\varphi}} \dot{\varphi} \ddot{\varphi} + \frac{\partial I}{\partial t} \dot{\varphi} + 0.5 \dot{\varphi}^2 \left[\frac{\partial^2 I}{\partial \varphi \cdot \partial \dot{\varphi}} \dot{\varphi} + \frac{\partial^2 I}{\partial \dot{\varphi}^2} \ddot{\varphi} + \frac{\partial^2 I}{\partial \varphi \cdot \partial t} + \frac{\partial I_n}{\partial \varphi} \right], \quad (6)$$

Suppose that $I = I(\varphi, t)$ or $I = I(t)$ then we obtain successively:

$$T_M - T_C = I \ddot{\varphi} + \frac{\partial I}{\partial t} \dot{\varphi} + 0.5 \dot{\varphi}^2 \frac{\partial I_n}{\partial \varphi},$$

$$T_M - T_C = I \ddot{\varphi} + \frac{\partial I}{\partial t} \dot{\varphi}, \quad (7)$$

When “I” do not depend on t, and $I = I(\varphi, \ddot{\varphi})$ then from equation (4) we have:

$$T_M - T_C = I \ddot{\varphi} + \frac{\partial I}{\partial \ddot{\varphi}} \ddot{\varphi} \ddot{\varphi} + 0.5 \dot{\varphi}^2 \frac{\partial I_n}{\partial \varphi}, \quad (8)$$

if $I = I(\varphi, \dot{\varphi})$ we get:

$$T_M - T_C = I \ddot{\varphi} + 2 \frac{\partial I}{\partial \dot{\varphi}} \dot{\varphi} \ddot{\varphi} + 0.5 \dot{\varphi}^2 \left[\frac{\partial^2 I}{\partial \varphi \cdot \partial \dot{\varphi}} \dot{\varphi} + \frac{\partial^2 I}{\partial \dot{\varphi}^2} \ddot{\varphi} + \frac{\partial I}{\partial \varphi} \right], \quad (9)$$

if $I = I(\varphi)$ we get:

$$T_M - T_C = I \ddot{\varphi} + 0.5 \dot{\varphi}^2 \frac{\partial I}{\partial \varphi}, \quad (10)$$

if $I = I(\dot{\varphi}, \ddot{\varphi})$ we get:

$$T_M - T_C = I \ddot{\varphi} + 2 \frac{\partial I}{\partial \dot{\varphi}} \dot{\varphi} \ddot{\varphi} + \frac{\partial I}{\partial \ddot{\varphi}} \ddot{\varphi} \ddot{\varphi} + 0.5 \dot{\varphi}^2 \left[\frac{\partial^2 I}{\partial \dot{\varphi}^2} \ddot{\varphi} + \frac{\partial^2 I}{\partial \dot{\varphi} \cdot \partial \ddot{\varphi}} \ddot{\varphi} \right], \quad (11)$$

if $I = I(\dot{\varphi})$ we get:

$$T_M - T_C = I \ddot{\varphi} + 2 \frac{\partial I}{\partial \dot{\varphi}} \dot{\varphi} \ddot{\varphi} + 0.5 \dot{\varphi}^2 \frac{\partial^2 I}{\partial \dot{\varphi}^2} \ddot{\varphi}, \quad (12)$$

if $I = I(\ddot{\varphi})$ we get:

$$T_M - T_C = I \ddot{\varphi} + \frac{\partial I}{\partial \ddot{\varphi}} \ddot{\varphi} \ddot{\varphi}, \quad (13)$$

if $I = \text{const}$, we get:

$$T_M - T_C = I \ddot{\varphi}, \quad (14)$$

Thus from equation (4), we can get all the known equations for machine tools, whose motion can be described by the rotational motion level of reduction. Equation (7), under certain assumptions, can also be obtained from the equation

Meshchersky. If the reduction link performs forward movement, $q \sim S$ - linear displacement of drive link, $Q_m \sim F_m$ - the driving force, $Q_c \sim F_c$ - resistance force, $H \sim m$ - reduced mass and the equation (4) is:

$$F_M - F_C = m \ddot{S} + 2 \frac{\partial m}{\partial \dot{S}} \dot{S} \ddot{S} + \frac{\partial m}{\partial t} \dot{S} + \sum_{\xi=2}^{\rho} \frac{\partial m}{\partial \dot{S}^{\xi}} \dot{S}^{\xi} \ddot{S} + 0,5 \dot{S}^2 \left[\sum_{\xi=0}^{\rho} \frac{\partial^2 m}{\partial \dot{S}^{\xi} \partial S} \dot{S}^{\xi} + \frac{\partial^2 m}{\partial \dot{S} \partial t} + \frac{\partial m}{\partial S} \right], \quad (15)$$

An analogous analysis of the equation (15) can give different forms of the equations of motion of machine units for ongoing progress level of reduction. Dependence (6) and (14) cover the entire range of the equations of motion of machine units, which are listed in the technical literature [1, 2].

Conclusions. The resulting mathematical model for computer units with infinitely variable drive and drive, leading to a system with one degree of mobility, equation (2), (3), will be available, including all the known equations of motion machine units with infinitely-adjustable and non-adjustable drive .

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К ОБОБЩЕНИЮ УРАВНЕНИЙ ДВИЖЕНИЯ МАШИННЫХ АГРЕГАТОВ

Мальцев А.В., Павлишин П.Н.

Ключевые слова: агрегат, привод, уравнение, звено.

Резюме

Получены наиболее общие формы уравнений движения машинных агрегатов с бесступенчато-регулируемыми и не регулируемыми приводами.

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Summary

The most general forms of equalizations of motion of machine aggregates have been got with the step-managed and not managed drives.