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Modeling value at risk of financial holding company: time varying vs. traditional models

Abstract

This paper models the Value at Risk of financial holding company. Two portfolios are formed, and the daily profit and loss are computed for each portfolio. We find that only the Historical Simulation and GARCH (1,1)-AR (1) models produce the number of the failures within the non-rejection region of BASLE, while all other models are failed for both portfolios at both confidence levels. At a 99% confidence level, both models perform equally well in both portfolios, with only one failure in an out sample test. The Historical Simulation model slightly outperforms the GARCH (1, 1)-AR (1) model at a 95% confidence level.

Introduction

Financial holding companies have been a global trend since the repeals of sections 20 and 32 of the Glass-Steagall Act in 1999. Key market players aim to leverage the size and the complexity of their business lines across conglomerates to achieve both economies of scale and economies of scope. These efforts are also aimed at marginalizing the operations of small to medium size holding companies and the non-financial holding companies.

Unlike industrial corporations, the primary function of financial institutions is to actively manage financial risks. As a result, they need to precisely measure sources of risk and to control and price them properly. The prudential regulation of financial institutions requires the maintenance of minimum levels of capital as reserves against financial risks (Keegan, 2008). U.S. and international banking authorities, e.g., the Basle Committee on Banking Supervision, the U.S. Securities and Exchange Commission, and regulators in the European Union have sanctioned various Value at Risk (VaR) models for determining market risk capital requirements for large banks through the 1996 Market Risk Amendment to the Basle Accord.

Initially, VaR was limited to measuring market risk, but now it is used to actively control and manage both credit risk and operational risk (Marek, 2008). VaR has become a key measure of a firm-wide risk management indictor. VaR supplies an accumulating view of a portfolio's risks, and accounts for leverage, correlations, and current positions (Jorion, 2001).

Because trading data is highly confidential, most studies compare VaR modeling approaches and implementation procedures using illustrative portfolios (Hendricks, 1996; Marshall and Siegel, 1997; Pritsker, 1997). Berkowitz and O'Brien (2002) launched a pioneer study, examining the

statistical accuracy of VaR forecasts. They analyzed the distribution of historical trading P&L (profit and loss) data and the daily VaR performance estimated by six large U.S. banks with the criterion of "large trader" under Basle, for which trading activities equal at least 10 percent of total assets or $1 billion. They showed that, unconditionally, the VaR estimates tend to be conservative relative to the 99th percentile of P&L. However, losses sometimes exceed the VaR, and such events tend to be clustered. This implies that the VaR has difficulty in forecasting changes in the volatility of P&L. The GARCH model performs better at predicting changes in volatility in P&L. Thus, the GARCH model permits comparable risk coverage with less regulatory capital.

All Taiwanese banks implement an 8% capital adequacy on the balance sheet. Nevertheless, this rule is not equally applicable to securities firms and insurance firms. Financial institutions and financial holding companies only need to report the profit or loss of their market activities until the holding positions of different kinds of structured products are squared. In recent years, the trading accounts of large commercial banks and financial holding groups have grown rapidly and become progressively more complex. To a large extent, this reflects the sharp growth in the over-the-counter derivatives markets, such as TWD, NDF, IRS, CCS, Futures and Options etc.

The purpose of this study is to evaluate the best statistic fit VaR models for the trading portfolios of financial holding companies. We compare the empirical performance of time-varying models and widely-used traditional models, such as RiskMetrics[TM], the Historical approach and the Monte Carlo simulation approach. Based upon the nature of risk adjusting time varying models, we expect that they outperform traditional models in modeling value at risk. Based on some assumptions, we form two simulated portfolios. These two

portfolios were simulated from the holding portfolios of two leading financial holding groups. Basically, these two portfolios contain three types of asset classes, including foreign exchange, equity and government bonds. Moreover, both portfolios have exactly the same instruments in each asset class and the same total portfolio amount.

We adopt five different market risk VaR models, including GARCH (1,1)-AR(1), GARCHM, RiskMetrics[TM], the Historical approach and the Monte Carlo simulation approach, to test in-sample and out-of-sample model performance. Both portfolios are tested under a one-day horizon at 95% and 99% confidence levels. Both portfolios are tested under a one-day horizon. Empirical results indicate a strong preference of model efficiency of time varying GARCH over the widely used traditional RiskMetric[TM] model across both portfolios at both 95% and 99% confidence levels. Only for the Historical Simulation and GARCH (1,1)-AR (1) there is the number of the failures within the non-rejection region of BASLE. This finding is consistent with Berkowitz and O'Brien (2002), who found that a simple ARMA+GARCH model outperforms traditional bank models.

At a 99% confidence level, both models performed equally well on both portfolios, with only one failure in 217 observations. At a 95% significance level, the Historical Simulation model slightly outperforms the GARCH (1,1) -AR (1) model because P&L is aggregated on the financial holdings instead of the commercial bank level, and there is no structure break during the sample period.

We go through section 1 with reviews of previous literatures about Basle Accord, traditional Risk Metric model of VaR, time varying GARCH models and both historical & Monte Carlo simulation approaches. In section 2, we discuss the P&L assumptions and process, while in section 3, the model methodologies will be discussed. Section 4 presents the empirical results and a conclusion is in the final section of the paper.

1. Modeling value at risk

Risk management is the process by which various risk exposures are identified, measured, and controlled. Guldimann introduced the concept of value at risk at J.P. Morgan in the late 1980s. VaR describes the quintile of the projected distribution of gains and losses over the target horizon. For example, at a 95% confidence level, VaR should be such that it exceeds 5% of the total number of observations in the distribution. Today, many universal banks and markets regulators have widely applied and endorsed statistical-based risk-management systems such as VaR to gauge their financial risk.

1.1. BASLE Accords. This Committee investigated the possible use of banks' proprietary in-house models for the calculation of market risk capital as an alternative to a standardized measurement framework. The results of this study were sufficiently reassuring for it to envisage the use of internal models to measure market risks.

The BASLE Accord represents a landmark financial agreement for the regulation of commercial banks. The main objective is to strengthen the soundness and stability of the international banking system by providing a minimum standard for capital requirements.

The 1988 Basle Accord defined a common measure of solvency (the Cooke ratio) that only covers credit risks. The Cooke ratio requires capital to be equal to at least 8% of the total risk-weighted assets of the bank. Capitals not limited by usual definition of equity book value consist of two components. Tier 1 capital (or "core" capital) includes paid-up stock issues and disclosed reserves, most notably from after-tax retained earnings. Of the 8% capital charge, at least tier 1 capital must cover 50 percent. Tier 2 capital (or "supplementary" capital) includes perpetual securities, undisclosed reserves, subordinate debt with maturity greater than 5 years, and shares redeemable at the option of the issuer. Risk capital weights were classified into four categories, depending on the nature of the asset.

In 1996, the Basle Committee amended the Basle Capital Accord to incorporate market risks. This amendment added a capital charge for market risk based on either of two approaches, the standardized method or the internal models method. It separated the bank's assets into two categories, which are trading book, banks' portfolio for intentionally held for short-term resale and typically marked-to-market, and banking book, mainly loans.

The amendment adds a capital charge for the market risk of trading books, as well as the currency and commodity risk of the banking book. To obtain total capital-adequacy requirements, banks should add their credit risk charge to their market risk charge. Besides banks were allowed to use a new class of capital, i.e. tier 3 capital, that consists of short-term subordinated debt. The amount of tier 3 capital (tier 2 capital or both) is limited to 250 percent of tier 1 capital allocated to support market risks.

1.2. VaR and models. VaR is a method of assessing risk that uses standard statistical techniques and measures the worst expected loss over a given horizon under normal market conditions at a given confidence level. Importantly it measures risk using the same units as the bank's bottom line – dollars. Hence, shareholders and managers can then decide whether they feel comfortable with this level of risk.

Due to the data confidentiality, most of VaR related empirical studies use the dummy data for a specific asset class. Berkowitz & O'Brien (2002) release the first study to present the first direct evidence on the performance of VaR models for six US large trading firms. They adopt an ARMA+GARCH to model VaR from the daily reported P&L data to compare the results with their VaR forecasts. The major empirical finding is that GARCH model of P&L is suited for lower VaRs and is better at predicting changes in volatility. The inference from the latter is the GARCH model permits comparable risk coverage less regulatory capital.

Christoffersen and Diebold (2000) try to provide a framework for risk managers to evaluate the best statistics fit VaR tools to their various holding portfolios. The paper used the daily records of S&P 500 index from November 1985 to October 1994 to test the performance of GARCH (1,1), Risk Metrics, implied and re-projected of four different risk coverage probabilities. The testing results indicated that different VaRs might be optimal for different levels of coverage. The statistics showed Risk Metrics is preferred to the re-projected volatility VaR at 1% confidence level and the implied volatility VaR is preferred to Risk Metrics at 10% confidence level. The GARCH and Risk Metrics models typically provide very similar short-term variance forecasts, but they have very different implications in the longer term.

Britain's Financial Services Authority has revealed that 42 percent of banks use the covariance matrix approach, 31 percent use historical simulation, and 23 percent use the Monte Carlo approach.

1.2.1. The GARCH approach. The estimation of time-varying covariance and implicitly of the entire covariance matrix between asset returns is crucial for asset pricing, portfolio selection and risk management. To that end, a wide variety of multivariate volatility models have been proposed. For example, Bollerslev, Engle and Wooldridge (1988) proposed the diagonal GARCH process. In risk management field, exponentially-weighted moving averages of past portfolio returns are commonly used as a simple model of asset variances and covariance.

For modeling financial returns and determining the down-side risk of financial positions, exact frequency of extreme events is crucial. In the literature, two different approaches can be distinguished: unconditional versus conditional modeling. In unconditional models, based on extreme value theory (EVT), the tail behavior of the return distribution is modeled over time assuming that the tail events are independent and uncorrelated. Usually a huge data set is necessary to derive reasonable tail estimates, but the tail characteristics seem to be stable over time.

Tail estimates are used to approximate Value-at-Risk for certain horizons and confidence levels.

In contrast to the unconditional approach, the class of GARCH models has been very successful in modeling the significant volatility clustering and non-i.i.d. properties of the data (Bollerslev et al., 1988). Many studies show the improvement in VaR estimations associated with GARCH models driven by fat tailed distributions.

1.2.2. The Risk Metrics^{TM} approach. Risk Metrics takes a pragmatic approach to model risk. Variances are modeled using an exponentially weighted moving average (EWMA) forecast. The exponential models place geometrically declining weights on past observations and assign greater importance to recent observations. This model can be viewed as a special case of the GARCH process. The exponential model is particularly easy to implement because it relies on one parameter only.

The Risk Metrics Group's proprietary LongRun^{TM} methodology provides an integrated methodology, for generating market rate scenarios over long horizons using two forecasting methodologies: one based on current market information, the other based on econometric models. The forecasts based on current market prices make intensive use of spot, futures, forwards and options price data and apply some derivatives theory to extract information from price data, while the forecasts based on economic fundamentals rely on historical time series of financial and economic data and the econometric modeling of time series.

1.2.3. Historical simulation method. The historical simulation method, a straightforward implementation of full valuation, consists of going back in time, such as over the last 220 days, and applying current weights to a time-series of historical asset returns. This approach is sometimes called *bootstrapping* because it involves using the actual distribution of recent historical data without replacement. When the goal is to model returns on a horizon longer than data frequency, Monte Carlo simulation or bootstrapping techniques can be seen as sensible choices.

There are several drawbacks in this approach. First, it assumes that there is a sufficient history of price changes. Nonetheless, some assets may have short histories or there may not be a record for an asset's history. Second, there is only one same path in uses. The sample might omit important events or contain events that will not reappear in the future. Third, it may be very slow to incorporate structural breaks.

1.2.4. Monte Carlo simulation method. In theory, the Monte Carlo approach can alleviate all these technical difficulties. It can incorporate nonlinear

positions, non-normal distributions, implied parameters, and even user-defined scenarios. The method proceeds in two steps. First, the risk manager specifies a stochastic process for financial variables as well as process parameters; parameters such as risks and correlations can be derived from historical or options data. Second, fictitious price paths are simulated for all variables of interest. The portfolio is marked-to-market using full valuation.

It is similar to historical simulation method, except that the hypothetical changes in prices for asset are created by random draws from pre-specified stochastic process. The biggest potential weakness, except the concern of the most expensive to implement, is model risk.

2. Data

There has been a high sensitivity and confidentiality in trading related data and these have been not part of the public accessible information. Based on the constraints, we form two simulated portfolios. These two portfolios were simulated from two leading financial holding groups under a set of rules described in Section 2.1. These two portfolios all contain three types of assets, which are foreign exchange, equity and government bonds. There are exactly the same instruments under each asset class and total portfolio amount. The only difference is the investment dollar amount for individual instrument based on the ratio and computed from the rules set below.

2.1. Formation of portfolios A and B – process and assumptions. Raw data have been sourced from the reported operational income listed on the quarterly financial reports of individual subsidiary under these two financial holding companies. In order to simulate the portfolios for both of them under a consistency, we make the following assumptions:

♦ For banks: We assume that only 20% out of their operational income come from the treasury department, while the main trading activities come from foreign exchange (FX) department (Devjak, 2007). For easy to do the back/stress testing, we only choose the vanilla FX transaction, i.e. no forward rate agreement or option products in the list. All currency pairs will be booked in at their initial trading pairs but will be converted to TWD base, for accounting purpose, on calculation period.

♦ For securities: Majority of the revenue comes from the brokerage. We assume 80% of their revenue from brokerage and 15% from equity trading and 5% from the government bonds.

♦ Property & life insurance: We assume that 85% of the revenue is from the insurance related brokerage. 15% of the revenue is from trading of their own portfolio and the split for instruments are 5% for government bond and 10% of stocks.

♦ Investment trust: We assume 50% of the revenue is from trading of the stocks.

♦ No derivatives for bonds and securities have been included in the portfolio for simplicity and also for the purpose of back testing accuracy.

From the above assumptions, we draw the trading activities size of both portfolio A and B spread over the three major asset classes of foreign exchange, equity and bonds. Market risk VaR comes from the activities of trading and the price changes of the underlying instruments. All brokerage incomes don't require a capital reserve for these types of activities.

Furthermore, we also exclude all the revenue from the retail mortgage business lines for two main reasons. First, majority of the retail loans in Taiwan are prefixed at a "floated" foundation, i.e. banks are allowed to adjust the rates whenever the primary rates are adjusted. Banks are always protected by a spread and there is less price volatility driven by VaR. Second, retail funding is more on credit VaR area based on Basle II amendments.

From all the steps and assumptions adopted above, we create two dummy portfolios with various ratios among different asset classes, while the total asset size is also different between them. In summary, portfolio B is about 2 times larger than portfolio A by total asset size. Portfolio B has higher allocation, over 59% of the total asset, in equity related investment, while portfolio A is comparatively more spread over the three types of tools.

Further, we assume portfolios A and B with the same total asset value. The only difference between them is the distribution of their investment in the three asset classes of foreign exchange, government bond and stock. The presumption in this process is that size of the portfolio will not impact the performance result of VaR forecasting tool. In Table 1, we get a summary for asset investment allocation of both portfolios A and B.

Then we select stocks, government bonds and currency pairs in comparison to establish the positions for both portfolios. The positions are bought and held for both portfolios from Nov. 28, 2001 though April 15, 2003. There are 617 observations in our sample period. The portfolios are marked to market everyday to obtain the 617

observations of daily P&L. In order to perform out-sample testing, we use 400 observations for model

fitting and the rest of 217 observations for out-sample testing.

Table 1. Size and allocation of portfolios A and B among investment asset classes

Upon some assumptions, we draw the trading activities size of both portfolios A and B spread over the three major asset classes of foreign exchange, equity and bonds from Nov. 28, 2001 through April 15, 2003. There are 617 observations in our sample period. The table presents the size and allocation of portfolios A and B among investment asset classes. Panels A and B describe the size and allocation of portfolios A and B among investment asset classes. The asset allocation percentage for portfolios A and B is compared in Panel C.

Panel A. Size and allocation of portfolio A among investment asset classes		
Portfolio A	Asset value (TWD)	Percentage
FX	$7,370,520,600	42%
Government bond	$3,291,360,300	18%
Stock	$7,207,263,950	40%
Total	$17,869,144,850	100%
Panel B. Size and allocation of portfolio B among investment asset classes		
Portfolio B	Asset value (TWD)	Percentage
FX	$6,656,714,200	12%
Government bond	$16,172,832,850	29%
Stock	$32,345,665,700	59%
Total	$55,175,212,750	100%
Panel C. Asset allocation percentage for portfolios A and B		
	Portfolio A	Portfolio B
FX	42%	12%
Government bond	18%	29%
Stock	40%	59%
Sum	100%	100%

3. Methodology

In this study, we test five models, which are time varying models of GARCH-AR(1) and GARCHM, traditional and widely used RiskMetric^{TM}, Historical approach and Monte Carlo simulation.

3.1. GARCH (1,1) model. A simple GARCH process, GARCH(1,1) model is as follows:

$$R_t = \alpha + \varepsilon_t$$

$$\varepsilon_t \mid \phi_{t-1} \sim N(0, h_t) \tag{1}$$

$$h_t = \alpha_0 + \alpha_1 \times \varepsilon_{t-1}^2 + \beta_1 \times h_{t-1},$$

where R_t is the return in period t; h_t is the conditional variance in period t; ε_t is the error term in period t.

While a GARCH (1,1)-AR(1) model can be expressed as below:

$$R_t = \alpha + AR * R_{t-1} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, h_t)$$

$$h_t = A + B * h_{t-1} + C\varepsilon_{t-1}^2. \tag{2}$$

where R_t is the return in period t; h_t is the conditional variance in period t; ε_t is the error term in period t, AR is the estimate of autoregressive, t-1.

Engle, Lilien, and Robins (1987) have included the conditional variance to the conditional mean equation and formed the ARCH- in mean (ARCH-M) Model. An ARCH(1,1)-M model is as follows:

$$R_t = \alpha + \beta\sqrt{h_{t-1}} + \varepsilon_t$$

$$\varepsilon_t \mid \phi_{t-1} \sim N(0, h_t) \tag{3}$$

$$h_t = \alpha_0 + \alpha_1 \times \varepsilon_{t-1}^2 + \beta_1 \times h_{t-1},$$

where R_t is the return in period t, h_t is the conditional variance in period t; ε_t is the error term in period t.

3.2. Back testing and forward testing. Back testing is also central to the Basle Committee's ground-breaking decision to allow internal VaR models for capital requirements. When the model is perfectly calibrated, the number of observations falling outside VaR should be in line with the confidence level set. Model back-testing involves systematically comparing historical VaR measures with the subsequent returns.

For small values of the VaR parameter p (the risk coverage), it becomes increasingly difficult to confirm deviations. For instance, based on Basle (1996) rules, the back-testing non-rejection region

under p=0.01 and T=255 (number of observations) is N<7 (N denotes the number of failures that could be observed in sample size T without rejecting the null hypothesis that p is the correct probability at a 95% confidence level). There is no way to tell if N is abnormally small or the model systematically overestimates risk. Intuitively, detection of systematic biases becomes increasingly difficult for low values of p because these correspond to very rare events. This explains why some banks prefer to choose a higher value for p, e.g., 0.05 (which means at a 95% confidence level). Hence, in this study we also test both p=0.01 and 0.05.

Forward testing is to compare the actual return of one day with its forecasted VaR. If the actual return exceeds VaR, the sample of this day is an "outlier". The same process has been repeating 200 times to get the number of outliers and, furthermore, this number has been compared against benchmark to validate how well the model is.

In this study, we use back testing to validate the results of historical simulation, Monte Carlo simulation and RiskMetrics, while we adopt the forward testing for the two GARCH models by nature. There are totally 617 observations (sample

period from 2000/11/28 to 2003/04/15) for portfolio A and B, respectively, and the same number of results are used on the back testing of the three models. In the forward testing, we take 400 observations to get GARCH estimates. From the obtained estimates, we predict the mean and the variance for the remaining 217 observations (from 2002/06/16 to 2003/04/15) day by day.

4. Empirical results

4.1. Daily P&L time series pattern. The distributions of the two portfolios are illustrated in Figures 1 and 2. Neither of them demonstrates a standard normal distribution. However, most of the returns cluster around their mean values on the observation periods. Table 2 presents the statistics for daily P&L. There is the leptokurtosis in distributions; i.e., there are few chances for the extreme spikes in the changes of values on both portfolios. In theory, the leptokurtoses make distributions very different to a normal distribution. Therefore, we need to adopt time varying models to calculate and predict VaRs to check whether the risk exposure is too violated to avoid extreme losses. Moreover, the correlation coefficient of daily P&L is 0.71. High correlation may reflect similarity in portfolio composition.
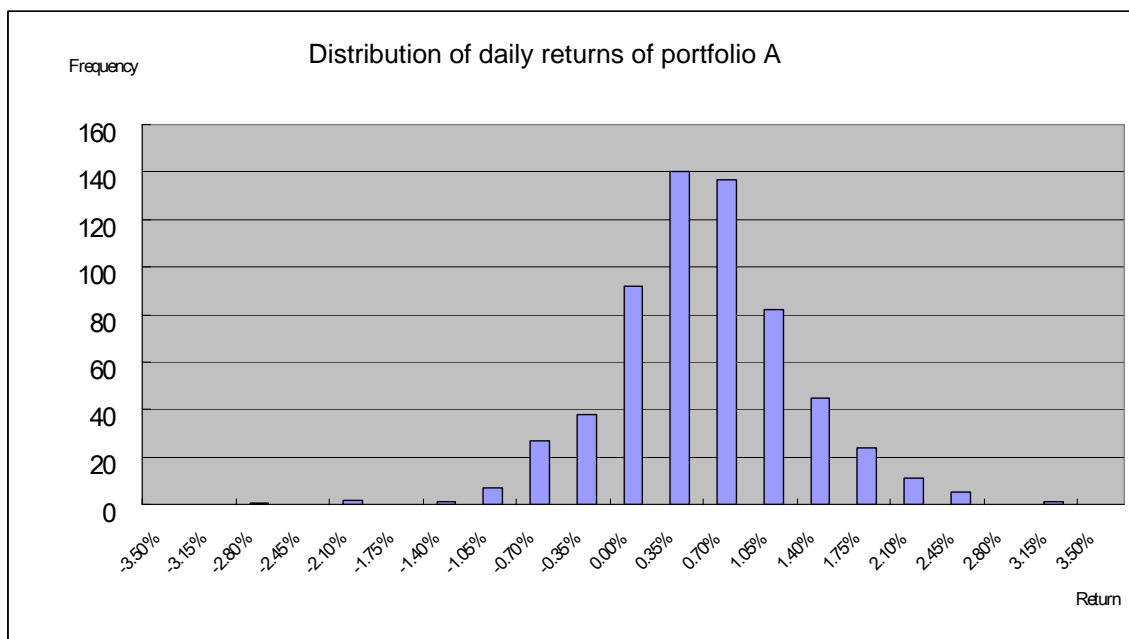
Note: The figure shows the distribution of daily return of portfolio A from 2000/11/28 to 2003/04/15. There are totally 617 observations for portfolio A.

Fig. 1. The distribution of daily returns of portfolio A
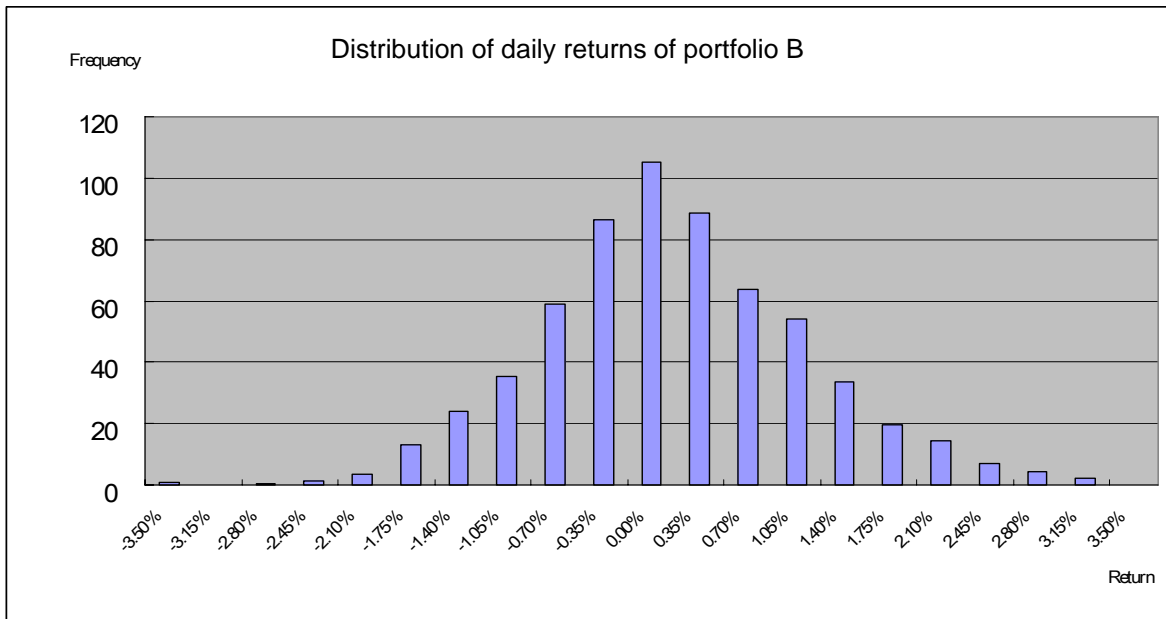
Note: The figure shows the distribution of daily return of portfolio B from 2000/11/28 to 2003/04/15. There are totally 617 observations for portfolio B.

Fig. 2. The distribution of daily returns of portfolio B

Table 2. The statistics for daily P&L of portfolios A and B

The table presents the statistics for daily P&L from 2000/11/28 to 2003/04/15. There are totally 617 observations for portfolios A and B, respectively.

	Obs	Mean	St. dev	99th percentile	Kurtosis	Skewness
Portfolio A	617	0.0093%	0.7046%	-1.6786%	33.4632922	-0.08928627
Portfolio B	617	0.0022%	0.9590%	-2.1910%	18.57438641	1.307275392

4.2. Performance testing results under various VaR approach. We test VaR prediction performance for five different approaches, i.e., GARCH (1,1)-AR (1), GARCHM, RiskMetrics[TM], Historical simulation and Monte Carlo simulation, and retraced their performance results in either back-testing or forward testing tactics. In Table 3, we attach all the results of the back testing for three approaches, which are RiskMetrics[TM], Historical simulation and Monte Carlo for both the portfolio A and B at the 95% and 99% confidence levels. Among the three models under the back-testing, historical simulation model demonstrates a better performance result under both scenarios at 95% and 99% confidence levels and across both portfolios.

Table 3. The number of outliers of portfolios A and B in the sample period

The table presents the number of outliers of portfolios A and B in back testing of historical simulation, Monte Carlo simulation and Risk Metrics[TM] at the 95% and 99% confidence levels. The sample period is from 2000/11/28 to 2003/04/15.

	Portfolio A			Portfolio B		
	Historical simulation	Monte Carlo	RiskMetrics	Historical simulation	Monte Carlo	RiskMetrics
95% confidence	25	33	29	29	38	31
99% confidence	2	12	7	2	11	7
Observations	617	617	617	617	617	617

We conduct the forward testing result of the two GARCH models for a period of 217 observations and at the same time we also compare the results with back testing ones in the same period. We put the comparison in Table 4. In two forward testing models, GARCH (1,1)-AR (1) indicates a better result than GARCHM. When we compare the results for all the five models in the same period, we find that historical simulation is better performed, then followed by GARCH (1,1)-AR (1), GARCHM. The results of Monte Carlo and RiskMetrics are very mixed between two portfolios and two confidence levels. We attach the results for all five tools, respectively, in Table 5. Additionally, we also graph their performance results by portfolios at 95% confidence level and present them in Figures 3 through 6.

Table 4. The number of outliers of portfolios A and B in the out-sample period

The table presents the number of outliers of portfolios A and B in Historical, Monte Carlo simulation, RiskMetrics[TM], GARCH (1,1)-AR (1) and GARCHM (1,1) at the 95% and 99% confidence levels. The sample period is from 2002/06/16 to 2003/04/15.

	Portfolio A					Portfolio B				
	Historical simulation	Monte Carlo	RiskMetrics	GARCH-AR	GARCHM	Historical simulation	Monte Carlo	RiskMetrics	GARCH-AR	GARCHM
95% confidence	5	12	13	6	8	7	15	17	12	13
99% confidence	1	4	2	1	2	1	3	2	1	1
Observations	216	216	216	216	216	216	216	216	216	216

4.3. Model efficiency vs. capital charges. In Table 4, the results of Historical Simulation are 5(1) and 7(1) failures for portfolios A and B at 95% (99%) confidence level, while the results for GARCH (1,1)-AR (1) are 6(1) and 7(1) for portfolios A and B at 95% (99%) confidence level, respectively. We find that portfolio B has a dominating allocation for listed stocks, whereas portfolio A has a more spread allocation among all asset class. From perspectives of straight statistics failure rate, i.e., the number of the outliners, historical simulation presents the best result among the five approaches across both portfolios and at all confidence levels. GARCH (1,1)-AR (1) has a slightly more failure rate than historical simulation for both portfolios at 95% confidence level, while the results at 99% confidence level are the same for these two approaches. Nevertheless, the number of failures for both approaches is still within Basle non-rejection ranges. The time-series VaRs could deliver lower required capital levels without producing larger violations for GARCH model VaR's greater responsiveness to changes in P&L volatility.

If we take a more scrutinized checking on Figures 3 to 6, then we could find that the VaR prediction by historical simulation is always larger than others, which means that commercial banks or financial holding companies using historical simulation approach for capital requirements have a higher probability to reserve more capital charges than other models. This implies the financial institutions might impose a higher capital charge.

Table 5. The statistics of VaRs of portfolios A and B

The table presents the statistics of VaRs of portfolios A and B obtained from Historical simulation, Monte Carlo simulation, RiskMetrics, GARCH (1,1)-AR (1) and GARCHM (1,1) at the 95% and 99% confidence levels. The sample period in Panels A, B and C is from 2000/11/28 to 2003/04/15. The sample period in Panels D and E is from 2002/06/16 to 2003/04/15.

Panel A. The statistics of VaRs obtained from Historical simulation									
		99% confidence level				95% confidence level			
	Obs	Mean VaR	Violations	Mean Violation	Max Violation	Mean VaR	Violations	Mean Violation	Max Violation
Portfolio A	617	-2.2250%	2	-0.9598%	-1.3377%	-1.1659%	25	-0.3854%	-2.2570%
Portfolio B	617	-2.7396%	2	-0.5298%	-0.9273%	-1.6231%	29	-0.3205%	-1.8924%

Panel B. The statistics of VaRs obtained from Monte Carlo simulation									
		99% confidence level				95% confidence level			
	Obs	Mean VaR	Violations	Mean Violation	Max Violation	Mean VaR	Violations	Mean Violation	Max Violation
Portfolio A	617	-1.5814%	12	-0.3228%	-1.1708%	-1.1143%	33	-0.3900%	-1.5319%
Portfolio B	617	-2.2114%	11	-0.4110%	-1.7476%	-1.5455%	38	-0.4353%	-2.3833%

Panel C. The statistics of VaRs obtained from RiskMetrics									
		99% confidence level				95% confidence level			
	Obs	Mean VaR	Violations	Mean Violation	Max Violation	Mean VaR	Violations	Mean Violation	Max Violation
Portfolio A	617	-1.7765%	7	-0.3694%	-1.1648%	-1.2542%	29	-0.3459%	-1.5379%
Portfolio B	617	-2.4901%	7	-0.4296%	-1.7757%	-1.7580%	31	-0.4007%	-2.3123%

| Panel D. The statistics of VaRs obtained from GARCH (1,1)-AR (1)									
		99% confidence level				95% confidence level			
	Obs	Mean VaR	Violations	Mean Violation	Max Violation	Mean VaR	Violations	Mean Violation	Max Violation
Portfolio A	617	-1.4385%	1	-0.9524%	-0.9524%	-1.0177%	6	-0.3303%	-1.3835%
Portfolio B	617	-1.9984%	1	-1.6002%	-1.6002%	-1.4185%	12	-0.3372%	-2.1824%

Table 5 (cont.). The statistics of VaRs of portfolios A and B

| Panel E. The statistics of VaRs obtained from GARCHM (1,1)									
		99% confidence level				95% confidence level			
	Obs	Mean VaR	Violations	Mean Violation	Max Violation	Mean VaR	Violations	Mean Violation	Max Violation
Portfolio A	617	-1.3850%	2	-0.5303%	-1.0354%	-0.9853%	8	-0.2836%	-1.4391%
Portfolio B	617	-1.9399%	1	-1.6661%	-1.6661%	-1.3700%	13	-0.3420%	-2.2344%
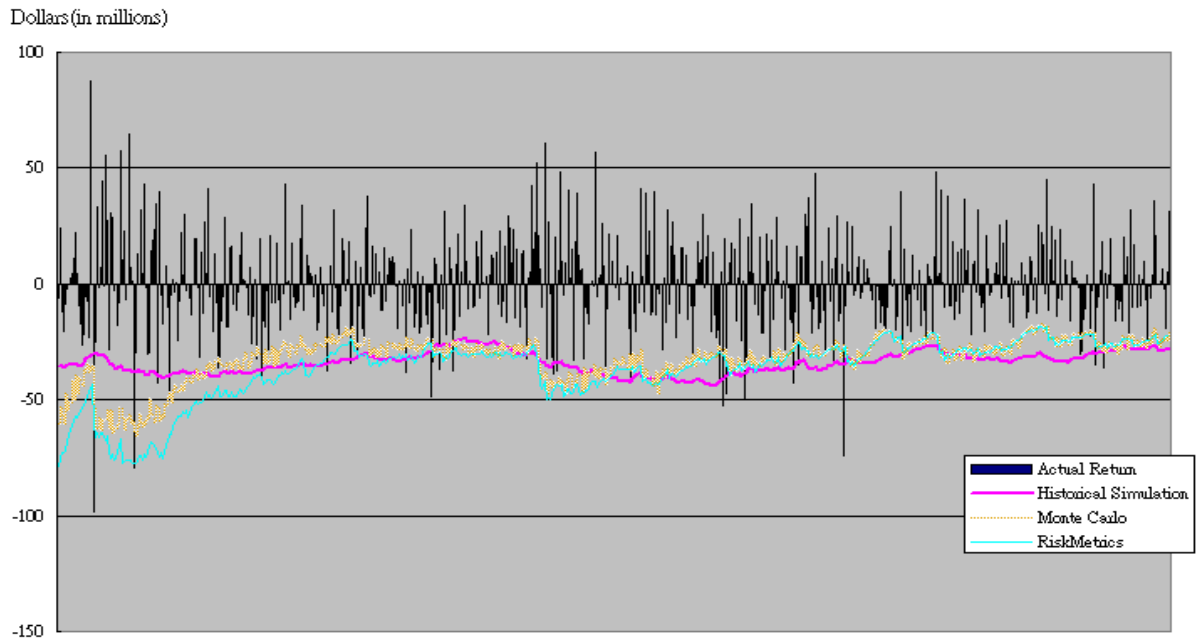
Note: The results are obtained from historical simulation, Monte Carlo simulation and RiskMetrics with 95% confidence level. The sample period is from 2000/11/28 to 2003/04/15.

Fig. 3. The actual returns and the VaR of portfolio A
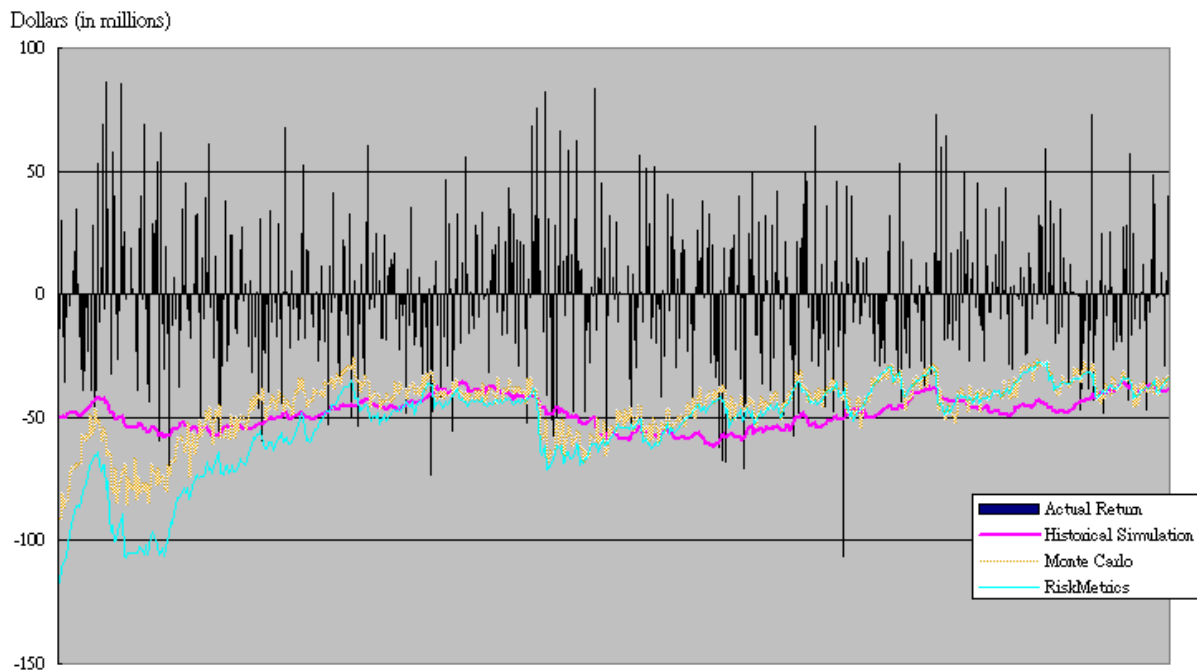
Note: The results are obtained from historical simulation, Monte Carlo simulation and RiskMetrics with 95% confidence level. The sample period is from 2000/11/28 to 2003/04/15.

Fig. 4. The actual returns and the VaR of portfolio B
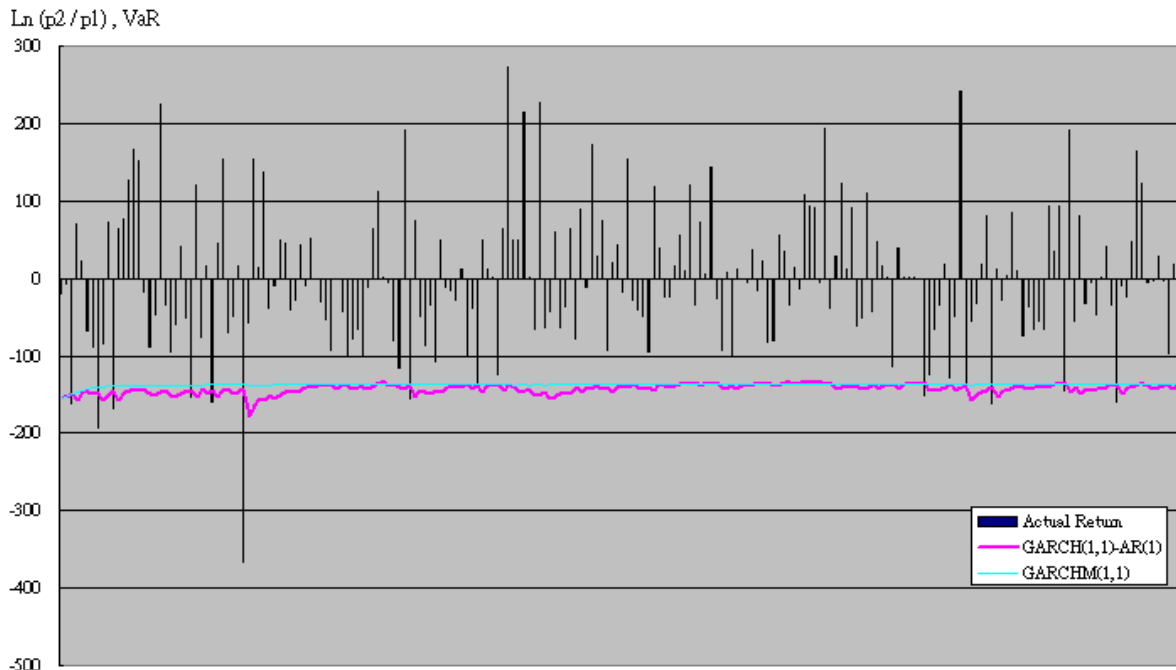
Note: The results are obtained from GARCH(1,1)-AR(1) and GARCHM(1,1) with 95% confidence level. The sample period is from 2002/06/16 to 2003/04/15.

Fig. 5. The actual returns and the VaR of portfolio A
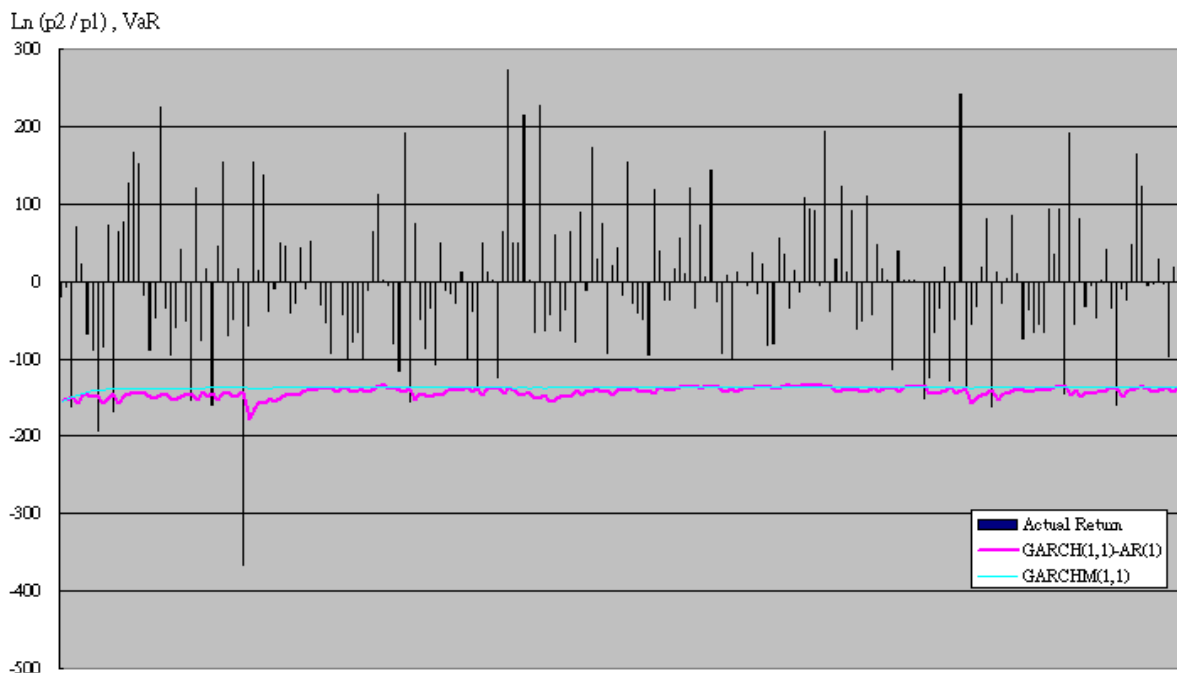
Note: The results are obtained from GARCH(1,1)-AR(1) and GARCHM(1,1) with 95% confidence level. The sample period is from 2002/06/16 to 2003/04/15.

Fig. 6. The actual returns and the VaR of portfolio B

Conclusion

This paper discusses Basle requirements, formalization of market risk, and value at risk of financial holding companies. Due to constraints of the confidentiality and unavailability for real trading data, we simulate two portfolios of two leading financial holding companies. We compute the daily P&L for both portfolios with a total of 617 observations. Then, we adopt five VaR methods at 95% and 99% confidence levels to compute the VaR forecasts and compare the performance based on back-testing or forward testing.

We find that the number of the failures in Historical simulation and GARCH (1,1)-AR (1) are within the non-rejection region of BASLE, while all others are failed for both portfolios at 95% and 99% confidence levels. At 99% confidence level, both models on both portfolios perform equally

well with only one failure for 217 observations. While at 95% significance level, we observe that

Historical simulation slightly outperforms GARCH (1,1)-AR (1).

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