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# The demand for excess reserves in Turkey

## Abstact

The main focus of this paper is to model the demand for excess reserves and present the intra-reserve maintenance period pattern for Turkey. The demand for excess reserves is a crucial part of a bank's liquidity management decision. It is also one of the most significant factors influencing the liquidity of the Turkish banking system. This paper introduces the ARMA-based approach to model the pattern of excess reserves within the maintenance period. The results indicate that there exists a systematic pattern of excess reserve balances within a maintenance period. The effect of uncertainty about liquidity shocks, the volume of payment flows and inefficiencies in the interbank money market borrowing have effects on the pattern of reserve holdings. Further development of the interbank money markets (such as reduction in trading costs, increased transaction volume) together with increased competitiveness between Turkish banks may induce banks to manage their daily reserves in a more cost efficient way and create a smoother demand for reserves.

## Introduction

The most widely quoted source of reserve demand is the existence of compulsory reserve requirement systems. Reserve excess or deficiency occurs simply because of the uncertainty related to flows in and out of banks' reserve accounts.

There have been discussions about the excess reserves of the commercial banking system related to both monetary policy and optimal reserve management. Most of the literature on commercial banking is based on a certainty model which causes banks to hold no excess reserves. With the introduction of uncertainty, banks tend to hold excess reserves because of ambiguity of cash flows.

Excess reserves are defined as the current account holdings of banks with the Central Bank beyond the required amount and serve as a buffer against a possible costly overnight overdraft. Since these balances earn no return and do not satisfy any regulatory requirement, they have an opportunity cost which is approximately equal to money market rates. Banks hold balances at Central Bank of Turkey (CBT) for two main reasons: 1) to meet reserve requirements; and 2) to provide buffer against unexpected liquidity shocks which serve as a cushion against overdrafts. Banks generally demand higher levels of excess reserves when flows in and out of their accounts are in greater volumes, in other words, demand for excess reserve is highly sensitive to levels of payment flows.

Although, excess reserves do not play a significant role in monetary economics, the open market desk needs daily estimates of excess reserves similarly to other autonomous liquidity factors like treasury accounts or volume of currency in circulation, in order to decide the reserve supply. The daily forecasts of excess reserves take into account market conditions, including expectations of high payment flows, the day of the week effect, and information gathered from banks. Since the central bank's objective of steering interest rates is achieved by managing the liquidity of the banking system, it is required to take into account the expected excess reserves while focusing on the reserve demand of the system. In this framework, understanding the dynamics of excess reserves becomes crucial in implementing monetary policy. A better understanding of bank reserve management may lead to easing of technical problems in daily liquidity management of central banks.

There exists an extensive theoretical literature focusing on bank's reserve management and the demand for excess reserves. Poole (1968) developed one of the early models of commercial bank reserve management in a stochastic model and showed that much of the demand for balances was related to uncertainty. Other researchers, including Clouse and Dow (1999, 2002), focused on the banks' optimal reserve management and provided numerical solutions to the dynamic programing problem. Clouse et. al (2002) used dynamic programing to solve the decision problem of a representative bank in a 14-day maintenance period with carry-over rules and showed the effect on uncertainty, the level of reserve requirements, and overdraft penalties on the pattern of reserve holding over the maintenance period. Longworth (1989) provided optimal reserve demand in a multi-day maintenance period in Canadian system. Furfine (1998) used an optimization model to examine the effect of variability of reserves over the maintenance period.

It should be noted that the model in this paper does not include prices; it is constructed in terms of quan-

tities of reserve balances. The researches by Spindt and Hoffmeister (1998), Griffiths and Winters (1995), Hamilton (1997, 1998), Carpenter and Demiralp (2004, 2006), Judson and Klee (2008) focus on the price response to reserve balances by analyzing patterns in the federal funds rate over the days of the week and maintenance period.

The remainder of the paper proceeds as follows. Section 1 provides information about the reserve market structure in Turkey. The data and econometric model are presented in Section 2. The daily pattern of excess reserve demand is also discussed in Section 2. The last Section presents some brief concluding remarks.

## 1. Reserve market structure in Turkey

This Section provides brief information about the reserve market in Turkey including the computation and maintenance periods, remuneration on reserve balances, penalty structure for deficiencies and provisions for carry-over of reserve surpluses and deficiencies from one maintenance period to the next.

**1.1. Reserve computation and maintenance periods.** The rules governing the required reserve system in Turkey are described in Communiqué No. 2005/1. Turkish banks are required to hold a pre-determined amount of reserves (5 percent of reservable liabilities as of October 16, 2009) averaged over 14-day maintenance period beginning on a Friday and ending on a Thursday. Reserve levels on Fridays effectively count three times in computing the average maintained level of balances over the maintenance period.

The amount of a bank's required reserves is determined from its reservable liabilities two weeks earlier. Since the reserve maintenance period begins after the reserve computation period, banks face absolutely no uncertainty during a given maintenance period about the level of their required reserves. The amendment to Communiqué No. 2005/1 reserve requirement system made on December 15, 2009 permit banks to carry forward into the next accounting period a reserve excess or deficiency of not more than 10 percent of their required reserves. The carry-over facility became effective from the maintenance period started on January 8, 2010.

Based upon payment flows, each bank will forecast its expected reserve position on a daily basis. Banks adjust their reserve holdings by attending open market operations, borrowing or lending transactions in money markets, lending to CBT in interbank money market. Banks' forecasts about their reserve positions may differ from the realized levels due to shocks to liquidity which forces banks to have precautionary demand for reserves. For any realized

deficiency a bank will incur deficiency penalty costs and for excess reserves banks will face the opportunity cost.

**1.2. Remuneration and penalties.** The remuneration rate in average reserve balance of a maintenance period is set equal to 80 percent of CBT's overnight borrowing rate (5.20 percent since November 20, 2009). The system does not allow banks to have negative end-of-day balances (overnight overdrafts). Banks in Turkey are not permitted to hold negative reserve balances and can avoid overdrafts by using CBT's standing facilities which have higher costs compared to borrowing from money markets. If a bank fails to meet reserve requirements on time or with insufficient amounts, it is required to hold interest-free deposits in blocked accounts with the CBT in the amount double the deficient portion of required level. Instead of holding a deposit, the bank may choose to make an interest payment on the average deficient amount at a rate equal to 150 percent of the CBT's announced maximum overnight lending rate for late overnight liquidity window facility.

## 2. Data and model

The data covers 68 maintenance periods, with the first maintenance period beginning on December 28, 2007 and the last ending on August 5, 2010. The data span covers the period when the CBT conducts regular refinancing operations (1 week maturity repo auctions) and provide sufficient funds to the banking system because of the liquidity shortage.

In order to capture the day specific demand for excess reserves, dummy variables are used for each day of the maintenance period. Dummy variables are also introduced to capture the effect of some liquidity shocks, like quarter ends, when banks' balance sheet window-dressing activities become significant; public holidays, religious holidays (Feast of Ramadan and Feast of Sacrifice), when the increase in the volume of currency in circulation causes a negative liquidity shock; tax payments; treasury auction settlement days and salary payments. These are days of high payment flows through banks' reserve accounts and, consequently, represent days of increased uncertainty. Increased uncertainty should be associated with a greater demand for excess reserves. In order to capture the effects of policy rate decisions on the pattern of excess reserve demand, dummy variables for tightening and easing are included. The dataset consists of 3 rate hike and 15 rate cut decisions. Each maintenance period is independent from one another, assuming that no carry-over provisions exist. However, in case, where the first day of a maintenance period is a public holiday, the maintenance period

becomes dependent with the previous period. In order to control for this effect, dummy variables are included for the first day of the maintenance period and for Friday. The dummy for Friday is included because of the banks' tendency to hold high levels of reserve balances on this specific day for accounting conventions. In addition, dummy variable is included for the maintenance period when the failure of Lehman Brothers occurred to find if the financial turmoil had some effect on banks' demand for excess reserves. Finally, two dummies are included for the outlier data observed in the dataset.

**2.1. Daily patterns of excess reserve demand.**
Figure 1 plots the level of period average excess reserves held in each maintenance period. As can be seen from the graph, banks tend to hold 1 billion TL excess reserves on average, but there are two clear outliers when the demand reaches very high levels (~ 7 bio TL). The first outlier in the data caused by the increased uncertainty after the failure of Lehman Brothers in October 2008. The second upsurge in demand has resulted from a 5-day long public holiday when the banks could not adjust their reserve positions and stayed locked-in to a high level of reserves. Banks started to hold negative reserve positions after the introduction of the carry-over facility. The variation in excess reserves is expected to rise with the banks' better use of the carry-over provision when the interest rate expectations become significant.
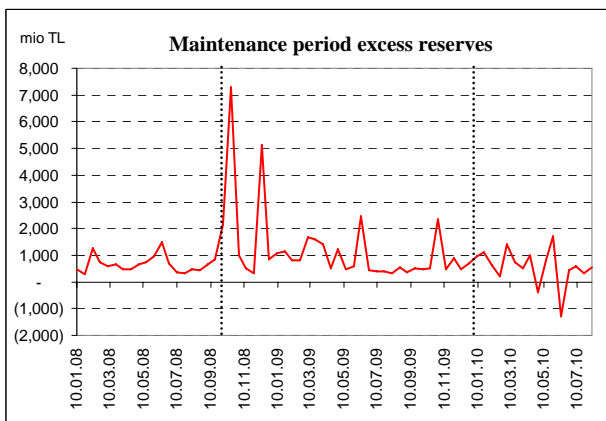


Fig. 1. The level of excess reserves

Figure 2 demonstrates the daily pattern of reserve balances by providing the ratio of reserves held to total required reserves. Banks tend to hold high excess reserves early in the maintenance period especially during the first week and adjust the level by holding low levels during the second week. Banks also prefer to hold high levels of excess reserves on the first Friday mainly because of uncertainty of negative liquidity shocks during the following week and weekend accounting conventions (the triple accounting of reserves on Fridays). Since the reserve requirements are satisfied over a two-week

period, there exists an option to wait until the latter part of the period to satisfy these requirements. However, the banks in Turkey do not use this option which is reflected both in the data and model.
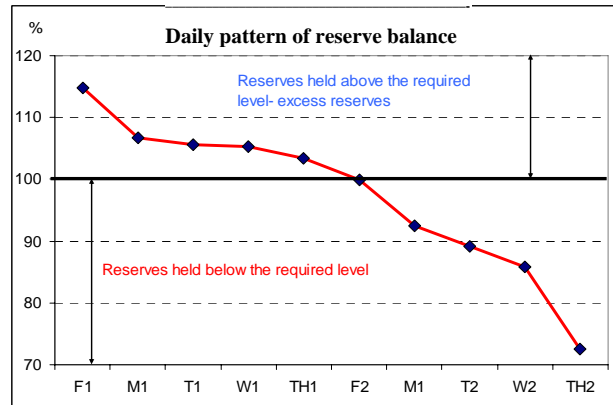


Fig. 2. Daily pattern of balances

Figures 3 and 4 display the banks' tendencies of holding excess reserves on the first day and the settlement day of the maintenance period. Banks hold approximately 115 % of their required reserves on the first day and 70 % on the settlement day.
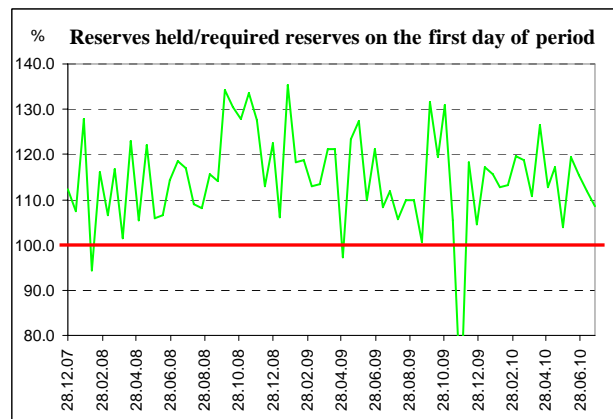


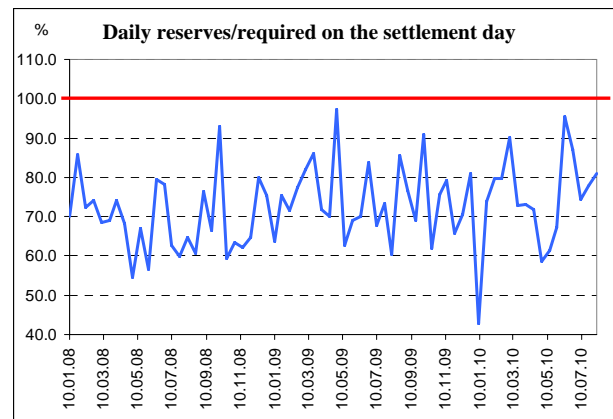Fig. 3. Level of reserves on the first day of period



Fig. 4. Level of reserves on settlement day

Autoregressive integrated moving average (ARIMA) model is used in this paper and the model is based on the methodology proposed by Box and Jenkins (1976). Bell and Hillmer (1983) suggest using the

model stated below for series with calendar variations, which is a linear regression model with errors following an ARIMA process:

$$y_t = D_{t,i} + \eta_t,$$

$$\eta_t = \frac{\theta(B)}{\phi(B)\delta(B)} \varepsilon_t,$$

$$\varepsilon_t \sim i.i.d(0, \sigma^2),$$

where $y_t$ is the daily excess reserves, $D_{t,i}$ is the linear regression part, $B$ is the backshift operator and $\theta, \phi, \delta$ are polynomials in $B$. The polynomials $\theta$ and $\phi$ are moving-average and autoregressive operators, respectively. The polynomial $\delta$ is a difference operator that can also include a seasonal difference operator.

The regression component is composed of dummy variables like day of the maintenance period, religious and public holidays, quarter ends, salary and tax payments, CBT's policy rate decisions. The dummy variables in $D_{t,i}$ specify the day-of-the-week effects of excess reserve demand. Apart from the seasonal effects, additional dummy variables are included for outliers and Lehman's failure.

The stationarity of daily series of excess reserves is tested by ADF test, the results[1] indicate that the series is stationary. The lags of the AR and MA processes are chosen with respect to ACF and PACF. After the final estimation ARCH LM test results reveal autoregressive conditional heteroscedasticity problem and, in order to eliminate heteroscedasticity, GARCH process is included in the final model.

Deterministic variables, ARMA and GARCH process variables are estimated simultaneously after the model specification by maximum likelihood ARCH (Marquardt) procedure. By including GARCH (1,2) process in the final model, autoregressive conditional heteroscedasticity problem is eliminated[2]. The specification of the model was finalized on the basis of significance of parameters and diagnostic tests on the structure of the residuals.

The model coefficient estimates of the excess reserve regression are reported in Appendix 2. There are several significant features of the intra-maintenance period demand for excess reserves:

1. The demand for excess reserves is significantly higher on the first day of the maintenance period and lower on the last day than all other days of the period. Excess reserves tend to start high early in the period, gradually decrease in the second half and reaching its lowest level on settlement Thursday. Reserve balances are generally the lowest (negative excess reserves) on the last few days of the period.

2. Banks tend to hold reserve balances below the required level on the two days prior to policy rate cut in order to facilitate arbitrage opportunities. Banks increase their reserve holdings one day after the MPC decision when the policy rate is lower to fulfill their requirements. However the rate hikes turn to have no significant effect on the demand for excess balances. This finding may be because the anticipation effect is not symmetric with banks predicted policy rate cuts better than rate hikes.

   Banks increase their reserve holdings one day before the Feast of Sacrifice and reduce their holdings on the day when the holiday begins to meet the upsurge in currency demand. Banks do not seem to have such precautionary demand for excess reserves before Feast of Ramadan, they only hold deficient reserve amounts on the day when the holiday begins and increase their reserve holdings for the following two days after the holiday.

3. The demand for excess reserves increases significantly on quarter-ends because of banks' balance sheet adjustments.

4. Banks hold low levels of excess reserves on the days before public holidays but tend to hold high levels when the day before a public holiday is the first day of the maintenance period. This tendency occurs because the reserves count four times over such periods. Banks tend to hold excess reserves on tax and salary payment days with the increased payment flows.

5. The increased distress in global money markets after Lehman's collapse caused Turkish banks to demand high levels of excess reserves.

6. The coefficient to capture the effect of the first day of the period being a holiday turned out to be statistically insignificant. It means that banks do not adjust their reserve position in a forward looking manner by taking into account the following period (assuming no carry-over provision). Banks hold higher excess reserve balances on Fridays in such maintenance periods starting with a holiday to fulfill their requirements.

It was stated by a number of studies that the demand for excess reserves is lower on Fridays than the rest of the week. Griffiths et. al (1995) claim that reserves held on Fridays are locked for three days, which causes banks to be unwilling to keep excess reserves on Fridays. Carpenter et al. (2006) also provide evidence about this issue in their

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[1] ADF test results are provided in Appendix A.
[2] ARCH-LM test results are provided in Appendix A.

model and state that on Friday the bank must pay three days of interest in borrowing reserve balances, but an overdraft on Friday is penalized for only one day. Therefore, overdrafts are cheaper on Fridays than on other days. Hamilton (1996) describes the same tendency by assuming a model in which reserves held at the end of the day provide liquidity on the following day. According to his model, reserves borrowed on Fridays cost three times more than borrowing on any other day of the week while the bank can use the borrowed funds on Mondays (only one day benefit). Clouse et. al (2002) also argues that the low demand for excess reserves on Fridays are because of the reserve market structure in the United States as in the work of Carpenter et al. (2006).

Several studies on excess reserve demand state that cost minimising banks avoid getting locked into too large a cumulative reserve position early in the period because being locked into a high level of reserves is costly and it increases the possibility of the bank to end up with an excess reserve although it holds zero balances during the remaining of the maintenance period. Therefore, banks hedge against this possibility by running short on reserves during much of the maintenance period and adjust the reserve position especially on the last day. Banks, which manage their reserve accounts closely, tend to wait until late in the maintenance period to obtain more information about their remaining reserve demand and then hold sufficient balances to meet the requirements.

In other words, for a bank ending the period with minimal excess reserves is optimal. On the other hand, uncertainties about liquidity shocks and high cost of late borrowing (after 4:00 p.m.) induce Turkish banks to hold excess reserves early in the period and end the period with excesses.

## Conclusion

This paper introduces an econometric model to forecast the demand for excess reserves and present the intra-reserve maintenance period pattern for Turkey. By examining the behavior of excess reserves between January 2008 and August 2010, the paper provides a typical daily pattern of excess reserve demand of the Turkish banks. Consistent with the literature, the demand for excess reserves depends mainly on uncertainty of flows in and out of reserve accounts. The demand has been found to be high before high payment flow days such as religious holidays, salary and tax payments.

It is also evident that the uncertainty about the liquidity shocks during the maintenance period causes banks in Turkey to hold high levels of excess reserves both on the first day of the period (Friday) and during the first week. In other words, Turkish banks generally prefer to be locked in early in the maintenance period and accept the costs of holding non-interest earning excess reserves since the costs associated with failing reserve requirements or avoiding overnight overdraft are higher. This incident may be associated with the turbulent past of the Turkish economy which caused the banks that experienced high volatility episodes to act with a precautionary motive.

Further development of the interbank money markets (such as reduction in trading costs, increased transaction volume with the entry of some banks that tend to lend their liquidity surpluses to CBT) together with increased competitiveness between Turkish banks may induce banks to manage their daily reserves in a more cost efficient way. Stability both in financial sector and economy as well as increased transparency of the CBT specifically by the announcement of daily liquidity forecasts should give rise to a smoother demand for reserves by decreasing the uncertainty faced by reserve managers.

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## Appendix A

Table 1. ADF test results for excess reserves

| Null hypotesis: ER has a unit root<br>Exogenous: constant | | | |
|---|---|---|---|
| Lag length: 14 (Automatic – based on SIC , maxlag = 19) | | | |
| | | t-statistic | Prob* |
| Augmented Dickley-Fuller test statistic | | -11.9704 | 0.0000 |
| Test critical values | 1% level | -3.4403 | |
| | 5% level | -2.8658 | |
| | 10% level | -2.5691 | |

Note: * MacKinnon (1996) one-sided p-values.

Table 2. ARCH-LM test for the ARIMA - GARCH (1,2) models

| F-statistic | 0.52124 | Probability | 0.7202 |
|---|---|---|---|
| OBS*R-squared | 2.094498 | Probability | 0.7184 |
| Test equation: | | | |
| Dependent variable: WGT_RESID^2 | | | |
| Method: least squares | | | |
| Date: 09/06/10 | | Time: 10:47 | |
| Sample ( adjusted): 17 656 | | | |
| Included observations: 640 after adjustments | | | |
| Variable | Coefficient | Std. error | t-statistic | Prob. |
| C | 0.95693 | 0.11611 | 8.24174 | 0.00000 |
| WGT_RESID^2(-1) | -0.01806 | 0.03966 | -0.45527 | 0.64910 |
| WGT_RESID^2(-2) | 0.03028 | 0.03965 | 0.76365 | 0.44540 |
| WGT_RESID^2(-3) | -0.02583 | 0.03966 | -0.65134 | 0.51510 |
| WGT_RESID^2(-4) | 0.03385 | 0.03968 | 0.85317 | 0.39390 |

## Appendix B

Table 3. ML-ARCH model

| Dependent variable: excess reserves | | | | |
|---|---|---|---|---|
| Method: ML-ARCH | | | | |
| Sample ( adjusted): 13 656 | | | | |
| Included observations: 644 after adjustments | | | | |
| Convergence achieved after 74 iterations | | | | |
| MA Backcast: 7, 12 | | | | |
| Variable | Coefficient | Std. error | z-statistic | Prob. |
| $D_{First Friday}$ | 8214.61 | 319.2 | 25.7 | 0.00000 |
| $D_{Second Friday}$ | -1012.00 | 309.7 | -3.3 | 0.00110 |
| $D_{Second Tuesday}$ | -1472.37 | 388.8 | -3.8 | 0.00020 |
| $D_{Second Wednesday}$ | -2790.02 | 476.7 | -5.9 | 0.00000 |
| $D_{Settlement}$ | -4940.95 | 356.9 | -13.8 | 0.00000 |
| $D_{Rate cut}(-1)$ | -2132.20 | 775.0 | -2.8 | 0.00590 |
| $D_{Rate cut}$ | 3727.95 | 676.3 | -5.5 | 0.00000 |
| $D_{Rate cut}(1)$ | 6987.62 | 575.9 | 12.1 | 0.00000 |
| $D_{Sacrifice}(-1)$ | 5711.90 | 1078.7 | 5.3 | 0.00000 |
| $D_{Sacrifice}$ | -13320.05 | 884.0 | -15.1 | 0.00000 |

Table 3 (cont.). ML-ARCH model

| Variable | Coefficient | Std. error | z-statistic | Prob. |
|---|---|---|---|---|
| $D_{Ramadan}$ | -6077.74 | 3527.8 | -1.7 | 0.08490 |
| $D_{Ramadan}$ *(1)* | 4916.27 | 1585.9 | 3.1 | 0.00190 |
| $D_{Ramadan}$ *(2)* | 4709.49 | 1243.8 | 3.8 | 0.00020 |
| $D_{Quarter\ end}$ | 5828.75 | 609.4 | 9.6 | 0.00000 |
| $D_{Tax\ payment}$ | 922.65 | 351.1 | 2.6 | 0.00860 |
| $D_{Public\ holiday}$ | -2550.91 | 659.6 | -3.9 | 0.00010 |
| $D_{Salary\ payment}$ | 667.53 | 237.8 | 2.8 | 0.00500 |
| $D_{Lehman\ failure}$ | 542.73 | 315.3 | 1.7 | 0.08520 |
| $D_{Friday\ when\ First\ day\ holiday}$ | 13450.74 | 1384.2 | 9.7 | 0.00000 |
| $D_{Ourlier\ 1}$ | -19765.00 | 5061.0 | -3.9 | 0.00010 |
| $D_{Ourlier\ 2}$ | 19203.22 | 6109.8 | 3.1 | 0.00170 |
| Interaction dummy | | | | |
| $D_{Public\ holiday}$ * $D_{First\ Day}$ | 9002.21 | 3697.7 | 2.4 | 0.01490 |
| ARMA terms | | | | |
| AR(1) | 0.17 | 0.0 | 3.6 | 0.00040 |
| AR(10) | 0.15 | 0.0 | 4.6 | 0.00000 |
| AR(4) | 0.25 | 0.1 | 3.9 | 0.00010 |
| MA(6) | -0.14 | 0.0 | -3.6 | 0.00030 |
| MA(5) | -0.19 | 0.0 | -5.4 | 0.00000 |
| MA(4) | -0.44 | 0.1 | -7.4 | 0.00000 |
| MA(3) | -0.22 | 0.0 | -4.4 | 0.00000 |
| Variance equation | | | | |
| C | 5626259.0 | 1331929.0 | 4.2 | 0.00000 |
| RESID(-1)^2 | 0.15 | 0.0 | 4.9 | 0.00000 |
| GARCH(-1) | -0.33 | 0.1 | -3.4 | 0.00070 |
| GARCH(-2) | 0.45 | 0.1 | 6.6 | 0.00000 |

| | | | | | | |
|---|---|---|---|---|---|---|
| R-squared | | 0.700446 | Mean dependent var | | | 120.8977 |
| Adjusted R-squared | | 0.686808 | S.D. dependent var | | | 4991.112 |
| S.E. of regression | | 2793.203 | Akaike info criterion | | | 18.69295 |
| Sum squared resid | | 4.80E+09 | Schwarz criterion | | | 18.92189 |
| Log likelihood | | -5986.131 | Hannan-Quinn criter. | | | 18.78179 |
| Durbin-Watson stat | | 1.998168 | | | | |
| Inverted AR Roots | | 0.9 | .67 + .44i | | .67 - .44i | .24 - .81i |
| | .24 + .81i | | -.20 - .80i | | -.20 + .80i | -.64 + .45i |
| | -.64 - .45i | | | -0.85 | | |
| Inverted MA Roots | | 1 | .11 - .74i | | .11 + .74i | -.24 - .54i |
| | -.24 + .54i | | | -0.73 | | |