Hassan Shirvani (USA), Shahram Taj (USA), Bahman Mirshab (USA)

A new approach to data envelopment analysis with an application to bank efficiency in Turkey

Abstract

The conventional data envelopment analysis (DEA) relies on linear averages of outputs and inputs to measure operational efficiency, a process that renders the direct application of linear programing techniques untenable. To overcome this difficulty, the average input is often normalized to equal one. This paper offers an alternative approach in which the use of nonlinear averages results in log-linear relationships, thus making it possible to directly use linear programing for optimization purposes. The paper also illustrates the new approach by analyzing the efficiency of a sample of Turkish banks, where it is shown that the extent of inefficiency among these banks is much smaller than implied by the use of the more conventional approach.

Keywords: data envelopment analysis, banking efficiency.
JEL Classification: C61.

Introduction

The operational efficiency of a service entity is often defined in terms of the "output" produced per unit of the "input" used. For example, a bank can be said to be operationally efficient if it produces either the same level of output with fewer inputs or if it uses the same level of inputs to produce a higher level of output. While economists interpret the concept of efficiency in an absolute sense, assessing the efficiency of individual units in light of an ideal production possibilities frontier, the well-known data envelopment analysis (DEA) approaches the issue from a relative sense, relying on actual firm performances to empirically derive this frontier. More specifically, the DEA constructs the production efficient frontier from the existing input-output combinations that arise from the available production technologies (Yue, 1992). In so doing, it rates all entities whose input-output combinations lie on the frontier as operationally efficient, while treating firms with input-output combinations under the frontier as inefficient. In short, the DEA singles out the best-practice entities as benchmarks against which the performances all other units are to be evaluated.

Given the foregoing, the DEA has proved itself a powerful tool of assessing operational efficiency in service organizations. For these organizations, it is often a challenging task to improve their operational efficiency without sacrificing service quality. Unlike manufacturing concerns, these organizations face a number of subjective factors that can seriously impact their service quality and customer satisfaction. Among the most important of these factors are customer needs and attitudes towards the services provided, the judgments and skills by which the services are offered, and the changing mix of the services themselves. The best service providers are characterized by both the high quality of their services as well as the efficient application of their resources. In an increasingly competitive business environment, it is thus of vital interest for many service providers to avail themselves of the existing analytical tools to assess their operational efficiency.

Since its introduction by Charnes et al. (1978), the DEA has been the subject of extensive theoretical refinements and empirical applications (see Cook and Seiford, 2009 for an excellent review of the relevant literature over the past thirty years, covering over 130 citations). Although most applications of DEA are centered in the service sector, there are also several applications in manufacturing and supply chain sectors. For example, Duzakin and Duzakin (2007) measure the performance efficiency of 500 industrial enterprises in Turkey, using a modified version of DEA called the super slack-based DEA. Likewise, Saranga (2009) uses DEA to estimate various operational efficiencies from publicly available financial data on a representative sample of 50 firms in the Indian auto components industry. Finally, within the service sector, the application of DEA has recently been extended to the financial services sector. In particular, Sherman and Ladino (1995) report how some banks using DEA have managed to score savings of up to 20 percent in their operational costs without any sacrifice of quality. In a similar vein, Kao and Liu (2009) apply the so-called stochastic DEA to measure bank efficiency in Taiwan. Having shown that bank efficiency in Taiwan has a stochastic distribution, they obtain this distribution through simulation analysis for 25 banks, using three inputs and three outputs in the process. Their results again indicate significant cost savings through the adoption of DEA. Sufian (2007) also uses DEA to study the trends in the efficiency of Singapore's commercial banking groups.

While a powerful tool, the standard DEA model, however, suffers from the way in which the outputs and inputs of the DMUs (Decision-Making Unit) are aggregated. Specifically, by using simple weighted averages of outputs and inputs, the model assumes that all outputs and inputs are perfect substitutes for each other. For example, in the context of bank efficiency, and with regards to inputs, a bank employee will be considered as a perfect substitute for a bank branch. This is clearly an unwarranted assumption. In addition, the use of linear averages renders the relevant efficiency ratios nonlinear fractions, a process that makes the direct application of linear programing techniques untenable. To overcome this difficulty, the model is then forced to standardize the denominator of the efficiency ratio (the weighted average input) to equal one. This paper offers an alternative approach, in which the use of nonlinear (geometric) weighted averages results in a log-linear relationship among the relevant variables, thus making it possible to directly use linear programing for optimization purposes. In addition to the ease of calculation, the nonlinear averaging method also avoids the restrictive perfect substitution assumption of the inputs and outputs mentioned earlier. In fact, the nonlinear averages of outputs and inputs are widely used (under the name of Divisia indexes) in the study of factor productivity measurements, and the present paper is simply an attempt to extend them to the DEA analysis. To illustrate the advantages of the new approach, the paper applies and compares the results of applying both the standard and the alternative DEA approaches to analyze the efficiency of a sample of Turkish banks. Our main finding is that the extent of inefficiency among these banks is much smaller than implied by the use of the standard approach.

The rest of this paper is organized as follows. Section 1 presents our modified DEA mode and discusses its relationship to the conventional DEA approach. Section 2 offers our empirical findings, and the last Section 3 concludes.

1. Model

The standard DEA model is based on a linear programing formulation by Ragsdale (2007). Specifically, the efficiency an arbitrary bank i (i = 1,…,k) is defined as follows:

H_i = (Weighted sum of bank i's outputs) / (Weighted sum of bank i's inputs)

$$H_i = \sum_{j=1}^{n} O_{ij}\, W_j \,/\, \sum_{j=1}^{m} I_{ij}\, V_j, \tag{1}$$

where O_{ij} represents the output j for bank i, I_{ij} represents the input j for bank i, W_j is a nonnegative weight assigned to output j, V_j is a nonnegative weight assigned to input j, n is the number of outputs, and m is the number of inputs. The problem in DEA is to determine values for weights W_j and V_j that will maximize the efficiency of bank i subject to the constraint that, at these same weights, the efficiencies of all banks, including bank I, will be greater than 100%. Thus, we have:

$$\text{Maximize: } H_i \tag{2}$$

Subject to:

$$H_j \leq 1 \text{ for } j = 1, 2, \ldots, k. \tag{3}$$

A separate optimization problem is solved for each bank to obtain the best possible weights to maximize the efficiency of that bank, subject to the similar constraints.

In addition, to be able to apply the linear programing techniques to the above optimization problem, as well as to prevent unbounded solutions, DEA requires the sum of the weighted inputs for each bank to equal one.

$$\sum_{j=1}^{m} I_{ij}\, V_j = 1. \tag{4}$$

As the foregoing indicates, the use of linear weighted averages of outputs and inputs renders the efficiency ratios nonlinear and, thus, necessitates the constraint that the denominators of these ratios are all equal to one. More importantly, the use of linear averages involves the unrealistic assumption, not explicitly stated, that all outputs and inputs are perfect substitutes. In the context of inputs for banks, for example, the assumption asserts that bank employees and branches are perfectly substitutable, so that instead of adding to the number of its branches, a bank may as well add new employees to its existing branches. In realty, of course, while there is some degree of substitutability among outputs and inputs, this substitutability is far from perfect.

To overcome the above difficulties, the standard DEA can be slightly modified by using nonlinear (geometric) weighted averages of outputs and inputs in measuring the efficiency ratios. Thus measured, the log of each efficiency ratio can be expressed as a linear function of the logs of all outputs and inputs for each DMU. This means that the linear programing techniques can now be directly used to solve our optimization problems. The use of this new approach has the added advantage that it makes no restrictive assumptions about the perfectly substitutability of outputs and inputs. However, to prevent unbounded solutions, we need to add the linear constraint that the sum of all (nonnegative) weights, both for outputs and inputs, is one for each DMU. In light of the above, we can present the reformulation of our standard optimization problem as follows:

$$H_i = \frac{\Pi_j O_{ji}^{Wj}}{\Pi_i I_{ji}^{Vj}} \text{ for } i = 1,2,\dots, k \quad (5)$$

Maximize:

$$\log H_i = \sum_j Wj \log O_{ji} - \sum_j Vj \lg I_{ji} \quad (6)$$

Subject to:

$$\log Hj \le 0 \text{ for } j = 1, 2, \dots, k, \quad (7)$$

$$\sum_{j=1}^{n} W_j + \sum_{j=1}^{m} V_j = 1. \quad (8)$$

Having outlined the basic structures of the standard and modified DEA models, we can now proceed to apply these alternative approaches to assess bank efficiency in Turkey. This is done in the next section.

2. Empirical results

To find efficiency or productivity ratings for k banks, k optimization problems are solved. The best-practice units are relatively efficient and are identified by a DEA productivity rating of 100%. The inefficient (less-productive) units are identified by a productivity rating of less than 100%.

As to the choice of the banking industry inputs and outputs, there exists a voluminous literature, excellently surveyed by Milma and Hjalmarsson (2002), which discusses the costs and benefits of using the various definitions of banking inputs and outputs. Since the main purpose of the present paper is to offer a new nonlinear approach to the aggregation of banking inputs and outputs, we wish to eschew the input/output choice debate, by using some standard definitions used in the literature. Specifically, we adopt the "production" approach, in which the actual inputs and outputs in the banking production process are used. Thus, the inputs for our DEA models are the number of bank branches, the total bank deposits, and the number of bank employees, while the outputs are the total bank loans and the total bank non-loan assets. In keeping with the usual practice, it is further assumed that for outputs, more is better, whereas for inputs, less is better. Any output or input variables that do not conform to these rules should be transformed before applying the DEA methodology (Ragsdale, 2007). The data, used in this study, presented in Table 1, are for 27 Turkish banks for the year 2008, are taken from the Turkish Banking Association (2009), and all financial data are in USD million.

Table 1. The 2008 data and the standard DEA efficiency ratings for 27 Turkish banks

		Outputs		Inputs		
Banks	Total loans	Non-loan assets	# of employees	# of branches	Total deposits	DEA efficiency
ZIRAAT	20263	48,348	21299	1269	55121	55.6%
HALK	16977	16,598	12467	622	26463	62.9%
VAKIFLAR	20310	14,254	9567	525	24782	92.5%
AKBANK	32234	27,126	15127	868	37834	91.6%
ALTERNATIFBANK	1560	903	1006	46	1743	74.0%
ANADOLUBANK	1499	937	1718	77	1610	43.7%
SEKERBANK	3126	2,130	4089	250	3944	32.1%
TEKSTIL	1071	885	1410	60	1007	39.6%
TURKISH BANK	118	408	292	26	262	56.1%
TURK EKONOMI	6065	4,095	6400	336	6897	43.0%
GARANTI	34663	25,650	16350	726	38086	100.0%
IS	33966	32,817	20924	1039	41390	75.1%
YAPI KREDI	25992	16,461	14795	861	28928	74.8%
ARAP TURK	214	311	170	3	64	100.0%
CITIBANK	1651	1,931	2315	56	2779	48.5%
DEUTSCHE	90	359	94	1	182	100.0%
EUROBANK	718	1,570	661	42	1129	72.8%
FINANSBANK	13099	5,714	9986	458	12502	63.9%
FORTIS	5588	3,073	5378	300	4383	48.8%
HSBC	6390	3,267	6853	335	6018	44.5%
ING BANK	7506	3,587	6357	366	6556	50.7%
MILLENNIUM	581	212	320	18	648	78.3%
TURKLAND	391	280	457	25	380	39.5%
ABN AMRO	130	716	205	8	430	87.4%
BANK MELLAT	119	96	50	3	47	100.0%
SOCIETE GENERALE	69	243	234	16	166	46.1%
WESTLB A.G.	24	521	42	1	408	100.0%

The Table 1 also presents the standard DEA efficiency ratings of the sample banks. As stated earlier, these ratings are the linear programing solutions to the 27 standard DEA models discussed above. As the table indicates, there are five banks (Garanti, Arap Turk, Deutsche, Bank Melat and Westlb A.G.) that have productivity ratings of 100%. In contrast, the Sekerbank, Turkland, and Tekstil banks have the lowest DEA efficiency scores of 32.1%, 39.5% and 39.6%, respectively, thus being the least efficient commercial banks in our sample. These findings further indicate that 22 of the banks in the sample could make substantial productivity improvements.

By way of comparison, Table 2 shows the results of applying our new approach to the same sample of Turkish banks. It is seen from the table that there is considerable overlap between the two models in terms of the benchmark banks. Specifically, the same 4 efficient banks under the standard approach are also efficient under our approach (the only exception is the Garanti bank, which was efficient under the standard approach, but is not efficient under the new approach, with the score of 53%). The most inefficient bank is again the Sekerbank with DEA efficiency score of 37.8% followed by Turk Economi and HSBC banks. However, the level of inefficiency is relatively lower in the new approach, respectively 32.1% and 37.8% for Sekerbank. More generally, based on the results in Table 2, there seems to be a much smaller level of inefficiency for the banks in our sample, an indication that these banks are generally much more efficient than implied by the standard DEA results. Given the intensity of the competition among the Turkish banks, it seems unlikely that the inefficient banks would have survived for long, if they had been truly as inefficient as implied by the results of the standard approach. This leads us to believe that our own results are possibly more plausible.

A major advantage of the DEA is identification of best-practice operating units that could be used as benchmarks for the inefficient operating units. The sensitivity analysis in linear programing provides a best-practice-banks reference set for the less-productive banks. The logic of a DEA model, in other words, is to determine whether a hypothetical composite bank can achieve the same or more output while requiring less input. If more output with less input can be achieved, the facility being evaluated is judged to be relatively inefficient. Table 3 offers information on both the inefficient banks in our sample, as well as the best-practice reference set based on our new approach.

Table 2. Log (outputs) and log (inputs) and the DEA efficiency for 27 Turkish commercial banks in 2008

		Outputs		Inputs		
Banks	Loans	Non-loan assets	# of employees	# of branches	Total deposits	DEA efficiency
ZIRAAT	9.92	10.79	9.97	7.15	10.92	45.0%
HALK	9.74	9.72	9.43	6.43	10.18	47.0%
VAKIFLAR	9.92	9.56	9.17	6.26	10.12	55.1%
AKBANK	10.38	10.21	9.62	6.77	10.54	54.7%
ALTERNATIFBANK	7.35	6.81	6.91	3.83	7.46	59.7%
ANADOLUBANK	7.31	6.84	7.45	4.34	7.38	43.9%
SEKERBANK	8.05	7.66	8.32	5.52	8.28	37.8%
TEKSTIL	6.98	6.79	7.25	4.09	6.91	46.9%
TURKISH BANK	4.77	6.01	5.68	3.26	5.57	62.6%
TURK EKONOMI	8.71	8.32	8.76	5.82	8.84	39.8%
GARANTI	10.45	10.15	9.70	6.59	10.55	53.0%
IS	10.43	10.40	9.95	6.95	10.63	48.3%
YAPI KREDI	10.17	9.71	9.60	6.76	10.27	47.6%
ARAP TURK	5.37	5.74	5.14	1.10	4.16	100.0%
CITIBANK	7.41	7.57	7.75	4.03	7.93	44.1%
DEUTSCHE	4.50	5.88	4.54	0.00	5.20	100.0%
EUROBANK	6.58	7.36	6.49	3.74	7.03	68.1%
FINANSBANK	9.48	8.65	9.21	6.13	9.43	43.5%
FORTIS	8.63	8.03	8.59	5.70	8.39	43.5%
HSBC	8.76	8.09	8.83	5.81	8.70	39.1%
ING BANK	8.92	8.19	8.76	5.90	8.79	43.5%
MILLENNIUM	6.36	5.36	5.77	2.89	6.47	71.7%
TURKLAND	5.97	5.63	6.12	3.22	5.94	52.3%
ABN AMRO	4.87	6.57	5.32	2.08	6.06	78.2%
BANK MELLAT	4.78	4.56	3.91	1.10	3.85	100.0%
SOCIETE GENERALE	4.23	5.49	5.46	2.77	5.11	60.6%
WESTLB A.G.	3.18	6.26	3.74	0.00	6.01	100.0%

Table 3. Target level for inefficient banks and the best-practice reference set

		Outputs		Inputs			
Banks	Loans	Non-loan assets	# of employees	# of branches	Total deposits	Bank #	Ref. banks
ZIRAAT	10.67	11.54	9.21	2.15	10.04	1	25,27
HALK	10.45	10.43	8.72	2.30	8.90	2	25,27
VAKIFLAR	10.47	10.12	8.61	2.38	8.56	3	25,27
AKBANK	10.94	10.77	9.07	2.44	9.15	4	25,27
ALTERNATIFBANK	7.84	7.49	6.42	1.80	6.32	5	25
ANADOLUBANK	8.12	7.75	6.64	1.87	6.54	6	25
SEKERBANK	8.99	8.61	7.37	2.06	7.27	7	25,27
TEKSTIL	7.72	7.53	6.51	1.74	6.17	8	14,25
TURKISH BANK	5.25	6.46	5.23	0.88	5.12	9	14,27
TURK EKONOMI	9.60	9.21	7.87	2.20	7.78	10	25,27
GARANTI	11.05	10.74	9.11	2.49	9.11	11	25,27
IS	11.12	11.08	9.26	2.45	9.45	12	25,27
YAPI KREDI	10.86	10.41	8.90	2.49	8.79	13	25,27
ARAP TURK	5.37	5.74	5.14	1.10	4.16	14	-
CITIBANK	8.21	8.37	6.94	1.77	7.13	15	14,25,27
DEUTSCHE	4.50	5.88	4.54	0.00	5.20	16	-
EUROBANK	6.94	7.72	6.13	1.35	6.67	17	14,25,27
FINANSBANK	10.28	9.82	8.41	2.36	8.28	18	25
FORTIS	9.43	9.06	7.79	2.16	7.58	19	14,25
HSBC	9.67	9.24	7.92	2.22	7.79	20	14,25
ING BANK	9.72	9.29	7.96	2.23	7.83	21	25
MILLENNIUM	6.67	6.37	5.46	1.53	5.37	22	25
TURKLAND	6.61	6.38	5.49	1.50	5.30	23	14,25
ABN AMRO	5.10	6.81	5.09	0.74	5.83	24	14,25,27
BANK MELLAT	4.78	4.56	3.91	1.10	3.85	25	-
SOCIETE GENERALE	5.05	5.98	4.97	0.90	4.63	26	14,27
WESTLB A.G.	3.18	6.26	3.74	0.00	6.01	27	-

In particular, the last column in Table 3 provides the reference set for less-productive banks. For example, the reference set for the Turkish bank (Bank #9) consists of the Arap Turk and Westlb A.G. banks (numbered 14 and 27). The target levels for the outputs and inputs of the Turkish bank, in order to render this bank efficient, are also provided in Table 3. It is interesting that the same best-practice banks, Arap Turk and Westlb A.G., also serve as benchmarks for a number of other less-productive banks.

Conclusion

This paper has offered an alternative approach to the standard data envelopment analysis (DEA), in which instead of relying on linear averages of outputs and inputs to measure operational efficiency, nonlinear averages are used. The use of nonlinear averages would result in log-linear relationships among the relevant variables, thus making it possible to directly use linear programing for optimization purposes, a task not possible under the standard DEA. Finally, the paper has illustrated the new approach by analyzing the efficiency of a sample of Turkish banks, where it has been shown that the extent of inefficiency among these banks is much smaller than implied by the use of the more standard approach. As part of any future research effort, it would be of interest to determine whether our findings concerning the Turkish banks can be extended to banks in other countries.

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