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Optimal investment with transparency costs and tax evasion

Abstract

This paper discusses relations among firms size, “transparency costs” and tax evasion. Tax evasion is here performed by allocating resources into an underground sector. The authors address these issues within the context of theory of investment, explicitly incorporating tax evasion, transparency costs and the relationship with credit markets. The theoretical model is calibrated for two countries: Italy, as a proxy for full developed countries with a significant underground sector, and Turkey, as a proxy for transition countries, still with a large underground share of the economy. In this context, the article discusses the consequences of selected economic policies acting over transparency costs, credit market constraints and anti-evasion policies. It obtains the following results: the growing of transparency costs disincentivate capital accumulation inducing a reduction in firms size. The relative weight of underground activities increases over produced output. Vice-versa, a reduction in transparency costs supports an increase in firms size and a reduction of the weight of underground activities. A tighter credit market reduces the average firm size but increases the share of regular economy. An easier access to the credit market, on the other hand, increases the average size of firms but incentivates the companies to hide more revenues from the Tax Offices.

Keywords: optimal investment, credit market, tax evasion and underground activities.

JEL Classification: E32, E13, H20, E26.

Introduction

This paper discusses relations among firm’s size, “transparency costs” and tax evasion. Tax evasion is here interpreted as an opportunity that firms have for allocating resources into a relatively more flexible market, the underground sector, and for reducing costs associated to be transparent¹. Transparency costs are here represented by all those costs that firms sustain when growing in size (i.e., balance sheet certification, administration costs, monitoring costs, etc.). Operating into the underground sector does not require any of these additional services that become crucial for investors/shareholders operating into the regular financial markets.

Our model focuses on the class of the so-called moonlighting firms. Following Cowell (1990) the underground production can be carried out either by fully irregular firms (the so-called ghost firms), or by firms operating in both sectors and choosing, on a period by period basis, how many resources allocate in each sector (the so-called moonlighting firms).

From an empirical point of view, according to a recent survey carried out by Censis (2005), the largest part of Italian firms operating in the underground sector, are moonlighters. The relative percentage is 81.7% of the sample examined. Hibbs and Piculescu (2006) support these conclusions at an international level while using enterprise level data from the World Bank Business Environment Surveys.

The main intuition we model in the paper goes as follows. We argue that tax evasion represents an opportunity for reducing “transparency costs” and an imperfect substitute for credit. Specifically, a firm may have an incentive to structure its aggregate production into a regular side and into an underground side; on the one hand this would reduce “transparency costs” and, at the same time, it would generate a tax saving, unless the firm is detected evading. On the other hand, it would worsen the credit condition of the firm because banks would have difficulties in finding sufficiently large collateral. In such context this is not necessarily a bad news because the internal finance flow produced by the (illicit) tax saving might offset the worsened credit conditions.

This discussion suggests that there exist a trade-off between operating into the underground economy and being fully transparent. A firm completely operating in the regular economy has access to favorable credit conditions but, on the other hand, it would pay full taxes and it would be subject to the transparency costs. On the contrary, a firm that chooses, more or less intensively, to evade taxes has a reduced tax base, and therefore pays less taxes, but it cannot have an easy access to credit as a large and transparent company. Depending on which one of these effects dominate, a firm might have incentives to reallocate more or less resources from the underground sector to the regular one or vice versa. Our model shows that there exist

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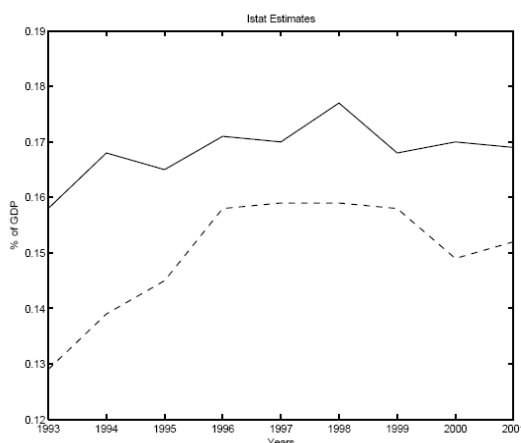
We are grateful to Amedeo Argentiero for the research assistance and to Leonardo Becchetti for many conversations. We wish also to thank the participants at XIII Tor Vergata Conference in Banking and Finance for criticisms and suggestions on earlier drafts of this paper. Francesco Busato acknowledges financial support from the Department of Economics, University of Rome “Tor Vergata”. Of course all errors are ours.

¹ There is no universal agreement on what defines the underground economy. Tanzi (1983) presents two definitions: one is related to the production missed in the official statistics and the other refers to tax evasion. Alternatively, Fleming et al. (2000) distinguish between the “Definitional Approach” and the “Behavioral Approach”. The former defines the underground economy as non-recorded production activities while the latter interprets it as a change in agents behavior in response to institutional policies. Finally, the System of National Accounts and the European System of National Accounts define the “non-observed” economy in three ways: (1) underground production, which is the area of production activities not directly observed due to both economic than statistical reasons; (2) informal production, representing the economy characterized by low levels of organization and no division between labor and capital inputs; (3) illegal production, describing those activities that are prohibited by law.

equilibrium conditions that equate marginal benefits deriving from the allocation of resources into the underground sector, with corresponding marginal costs.

This paper addresses this issue within the context of optimal theory of investment, explicitly incorporating tax evasion, transparency costs, and the relationships with the credit market. The theoretical model is calibrated for two countries: Italy, as a proxy for fully developed countries with a significant percentage of underground sector and Turkey, as a proxy for a transition country, still with a large underground sector. In this context we discuss the consequences of selected economic policies acting over transparency costs, over credit market constraints and over anti evasion policies. We highlight consequences over effective firm size (i.e., the actual capital stock), over the declared firm size (i.e., the declared capital stock) and over two different measures of welfare: one related to the capital stock and the other to what firms may have incentive to announce. The idea is to distinguish between what is the actual welfare of a firm and what the firm might strategically announce.

Here is a summary of our results. First, transparency costs disincentivate capital accumulation, inducing a reduction in firm size. The relative weight of underground activities increases over produced output. A reduction in transparency costs supports an increase in firm size and a reduction of the weight of underground activities. It has, moreover, an asymmetric effect compared to the increased transparency cost scenario: indeed, a reduction produces a percentage welfare increase equal to 11.34%, 3.09% and 16.84%, respectively, while the symmetric increase lowers the welfare by 8.25%, 3.78% and 11.34%.

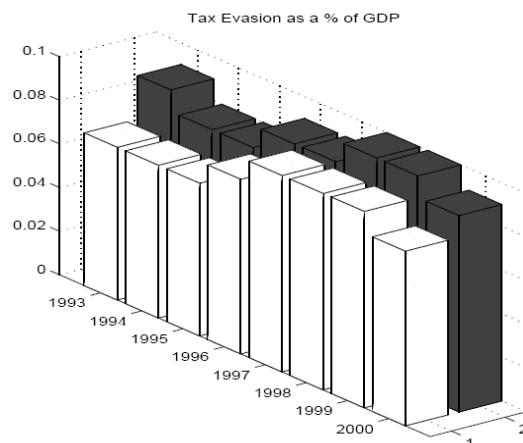


Second, a tighter access to credit market reduces the average firm's size but increases the share of the regular economy; an easier access to the credit market, on the other hand, increases the average size of firms, but incentivates the corporate to abscond more revenues to the IRS (Internal Revenue Service). Both policy experiments appear to be as welfare reducing, when evaluating them with the welfare measure defined over declared capital stock; that is because the value of declared capital stock falls in both cases. The "true" welfare measure instead, the measure defined over the actual capital stock, presents a different picture. A tighter credit market has a negative impact on welfare, while an easier access makes it growing.

The paper is organized as follows. Section 1 presents stylized facts, sections 2 and 3 present the model and the dynamical system. Section 4 contains the calibration exercise. Section 5 discusses selected results, and the last section concludes.

1. Stylized facts: underground economy, tax evasion and liquidity constraints

The following pages suggest that tax evasion is a large and widespread phenomenon, and that there exist a negative correlation between firms size and percentage of firms that are liquidity constrained. We here present evidence for the Italian economy, because it possesses a large underground sector. This allows to understand better the impact of underground activities on the overall economy. This analysis, however, is addressed not only to European countries like Belgium, Denmark, Greece, Portugal and Spain, but to the United States as well¹. Figure 1 below presents estimates for the size of underground economy, and for tax evasion. All series are reported as a percentage of aggregate GDP.



Notes: Left panel presents the underground economy as a percentage of GDP; the solid (dashed) line represents the highest (lowest) estimate. Right panel shows tax evasion as a percentage of GDP; the darker (white) series represents the (lowest) highest estimate. Source: Italy's National Statistical Institute (ISTAT) over the sample 1993-2000 (left panel). Authors' calculations, sample 1993-2000 (right panel).

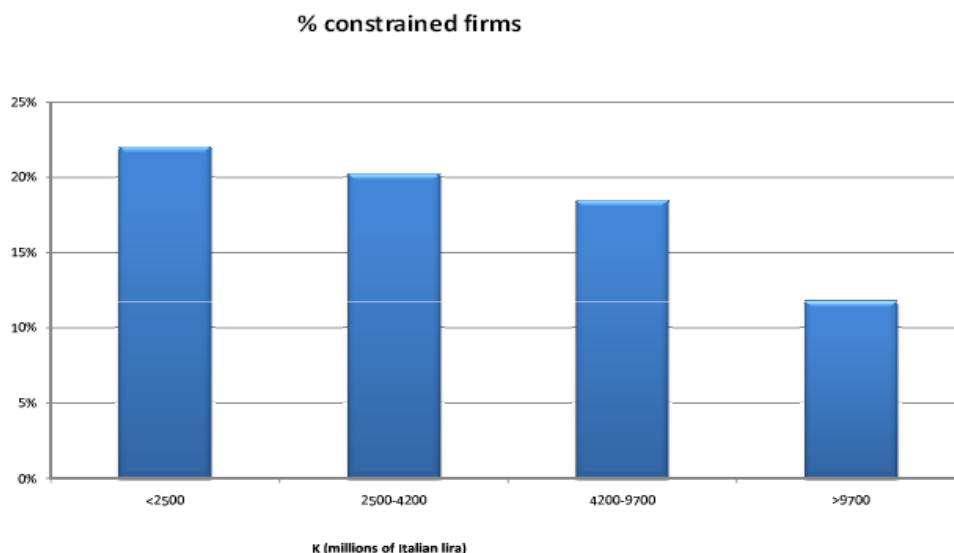
Fig. 1. Underground economy and tax evasion

¹ The average size of underground activities ranges between 5% of the United States GNP (in the Seventies) and 9% of the United States GDP (in the Eighties and early Nineties). See Tanzi (1980), Schneider and Enste (2000), Paglin (2001). Even if these figures are below the OECD countries average (17%), they still represent a significant amount of resources absconded from tax collection.

The size of the informal economy ranges between 15% and 20% of the GDP¹. Given the difficulty to obtain official time series statistics for tax evasion, we attempt a conservative estimate to give an idea of which are the figures we are talking about. Conservatively assuming that the effective tax rate for the whole economy is the minimum between the effective income (τ_t^Y) and the corporate tax rates (τ_t^I), we compute two approximate measures for tax evasion as $TaxEv_{min} = (\min(\tau_t^Y, \tau_t^I) \times u_t^{min}) \times GDP_t$ and $TaxEv_{max} = (\min(\tau_t^I, \tau_t^Y) \times u_t^{max}) \times GDP_t$, where u_t^{min} and u_t^{max} denote the lowest and the highest official estimates for the share of the underground economy as a percentage of GDP². The right panel of Figure 1 shows that tax evasion accounts for at least 5% of GDP. This is quite a big figure and, an analogous exercise for other European countries or for the United States, would generate qualitatively comparable numbers. In summary, if governments were effectively able to recollect unpaid taxes, this would

generate, on a yearly basis, a significant increase of government revenues³.

The idea that the benefit from transparency depends on firm size is consistent with previous results and evidence from Italian data. Gertler and Gilchrist (1994) show how small firms are more likely to face financial constraints such as credit rationing or very high cost of debt. Becchetti (1994), Bagella, Becchetti, Caggese (2001) demonstrate that Italian firms with financial constraints have half the ratio of net assets over liabilities and are half the size of the complementary set, while Fazzari, Hubbard and Petersen (1988) report almost the same conclusions for the United States. Becchetti, Castelli and Hasan (2010) find out that small firms are not able to raise funds from financial institutions as bigger firms are. Descriptive statistics on Capitalia Survey, which collects balance sheet, income statement and qualitative data on a sample of almost 5000 Italian firms for the 1998-2000 period, reported in Figure 2 below, show a negative relationship between firms size (on the x-axis) and easiness in credit access⁴.



Source: Authors' calculations on the Capitalia Survey (previously known as the Mediocredito Centrale Survey) over the sample 1998-2000.

Fig. 2. Relationships between firm size and liquidity constraints

2. The model's structure

Suppose there exists an homogenous good which can be produced by using a linear production function, we distinguish, however, between a regularly-produced output share, and an underground-produced output share; regular production is taxed while underground production is not declared to Internal Revenue Service and, therefore, not subject to distortionary taxation. Let K be the aggregate capital stock, and θ , $(1 - \theta)$ the percentage allocated

to the regular (underground) sector. The two production levels are defined as follows:

$$Y_M = (\theta K)^a \text{ and } Y_U = ((1 - \theta)K)^b,$$

where Y_M , Y_U denote the regular and the underground production; all firms use the same capital stock K to produce the final production good, but declare to the IRS only a share of it (θ), to re-

¹ ISTAT methodology.

² The use of the minimum between income and corporate tax rates is because we want conservative estimates.

³ This would be extremely welcomed by European countries, which are always struggling for keeping their deficit/GDP ratio below the limit of 3% established by the European Monetary Union.

⁴ The survey has been previously known as Mediocredito Centrale Survey.

duce the effective tax rate. The remaining share is not declared, but used for production. Aggregate production is then computed by linearly aggregate the two quantities $(Y = Y_M + Y_U)^1$.

2.1. Profit maximization. Normalizing to unity the final output price, regular revenues $((\theta K)^a)$ are taxed at the corporate rate τ , while firms do

$$\Pi_{ND} = \left\{ \underbrace{(1-\tau)(\theta K)^a + ((1-\theta)K)^b}_{REVENUES} - \underbrace{I + \alpha(\theta)I - T_0 \frac{(\theta K)^{1+\gamma}}{1+\gamma}}_{COSTS} \right\} \sim 1-\mu, \tag{1}$$

where the net-of-tax revenues are represented by the quantity $(1-\tau)(\theta K)^a + ((1-\theta)K)^b$, in which only the regularly-produced share is subject to corporate statutory tax rate $r \in (0,1)$. Production costs are made of three components: the cost of investing into physical capital (I), transparency costs $(TC)I_0 \frac{(\theta K)^{1+\gamma}}{1+\gamma} = TC(\theta, K)$ in which $T_0 \geq 0$, and $\gamma > 0$. TC are specified such that the bigger a firm is, the larger monitoring (“transparency”) costs are,

$$\Pi_D = \left\{ \underbrace{(1-\tau)(\theta K)^a + (1-\tau s)((1-\theta)K)^b}_{REVENUES} - \underbrace{I(1+\alpha(\theta)) - T_0 \frac{(\theta K)^{1+\gamma}}{1+\gamma} - \frac{P_0}{\xi} \left(\frac{1-\theta}{\theta} K \right)^\xi}_{COSTS} \right\} \sim \mu. \tag{2}$$

The revenue structure is here augmented by the application of the statutory tax rate τ , increased by the surcharge factor $s > 1$, on the underground-produced revenues $(1-\tau s)((1-\theta)K)^a$; the cost structure is also augmented by an additional penalty factor, the quantity $\frac{P_0}{\xi} \left(\frac{1-\theta}{\theta} K \right)^\xi = PN(\theta, K; \xi, P_0)$ with $\xi \geq 1$, which represents a further penalization once a firm is detected evading. The specification implies that $\lim_{\theta \searrow 0} PN(\theta, K; \xi, P_0) = \infty$, and $\lim_{\theta \nearrow 1} PN(\theta, K; \xi, P_0) = 0$.

The penalty for evading is directly proportional to “how much” a firm evaded, and to “how big” a firm is. The idea is that, when a big firm is detected

not pay taxes on underground-produced revenues $((1-\theta)K^b)$. Firms, however, may be discovered evading, with probability $\mu \in (0,1)$, and forced to pay the tax rate, τ , increased by a surcharge factor, $s > 1$, applied to the statutory tax rate. When a firm is *not discovered* evading (with probability $1-\mu$) profits read:

and are an increasing function of θ . Investing costs are mitigated by the quantity $\alpha(\theta)I$, with $\alpha(\theta) < 1$ and $\alpha'(\theta) > 0$, which captures the fact that the larger is the firm, the easier becomes the access to the credit market. In this sense, this quantity represents the benefit from transparency; I is the investment flow (the law of motion of capital stock is detailed below).

On the other hand, when a firm is *discovered* evading (with probability μ), profits read:

evading, it should be fined relatively more, compared to a smaller company.

There exist a trade-off between absconding capital stock from the tax base and evading. A firm completely operating in the market economy has access to favorable credit conditions on one hand but, on the other hand, it has to pay large taxes and suffers from the transparency costs. On the contrary, a firm that chooses to evade taxes has a reduced tax base, and therefore pays less taxes but, on the other hand, it cannot have so favorable conditions when renting capital stock. Our results depend on which one of these effects dominate.

To compute expected profits (EΠ) we apply a linear projection:

$$\begin{aligned} E\Pi &= \mu\Pi_{ND} + (1-\mu)\Pi_D = \\ &= (1-\tau)(\theta K)^a + (K(1-\theta))^b (s\tau(\mu-1)-1) - I(1+\alpha(\theta)) - T_0 \frac{(\theta K)^{1+\gamma}}{1+\gamma} - \frac{P_0}{\xi} \left(\frac{1-\theta}{\theta} K \right)^\xi (1-\mu). \end{aligned}$$

¹ For simplicity, in a first stage, we set aside labor services; this assumption allows, indeed, to use a linear aggregator between the two produced quantities.

2.1.1. *The value of a firm.* Each instant a firm decides how much capital input allocate to the regular production θ and to the underground production $1-\theta$, and it also decides, maximizing the inter-temporal cash-flow function, how much of the revenue to invest, I . Investment cannot be absconded from the IRS, and depends on the relative magnitude among

transparency costs, credit market tightness, and penalties for evading. The value of a firm is the expected present value of its revenues minus expenditures on capital, and it includes all these trade-offs, which will be analyzed in details in what follows. The representative firm maximizes expected cash flow V subject to a series of constraints:

$$\max_{\{I, \mu\}} V = \int_{t=0}^{\infty} e^{-rt} \Pi dt$$

$$s.to: E\Pi = (1-\tau)(\theta K)^a + (K(1-\theta))^b (s\tau(\mu-1)-1) - I(1+\alpha(\theta)) - TC(\theta K) - PN(\theta K)(1-\mu),$$

$$K = I - \delta K,$$

$$TC = T_0 \frac{(\theta K)^{1+\gamma}}{1+\gamma}; PN = \frac{P_0}{\xi} \left(\frac{1-\theta}{\theta} K \right)^\xi,$$

$$0 \leq \theta \leq 1, \bar{K}_0 < 0$$

Forming the Hamiltonian, the optimization problem reads:

$$H = \max_{\{I, \theta\}} \int_{t=0}^{\infty} e^{-rt} \left\{ \begin{aligned} & (1-\tau)(\theta K)^a + (K(1-\theta))^b (s\tau(\mu-1)-1) - I(1+\alpha(\theta)) - T_0 \frac{(\theta K)^{1+\gamma}}{1+\gamma} + \\ & - \frac{P_0}{\xi} \left(\frac{1-\theta}{\theta} K \right)^\xi (1-\mu) + \lambda_0 (I - \omega K) + \lambda_1 \theta - \lambda_2 (\theta - 1) \end{aligned} \right\},$$

where λ_0 is the Hamilton-Jacobi-Euler multiplier and λ_1, λ_2 denote the Kuhn Tucker multipliers. Next, derive the necessary and sufficient conditions. It is convenient to define the following auxiliary functions, with the corresponding first partial derivatives.

$$\text{Definition 1} \left\{ \begin{aligned} & TC(\theta, K) = T_0 \frac{(\theta K)^{1+\gamma}}{1+\gamma}; TC_K(\theta, K) = \theta T_0 (K\theta)^\gamma; TC_\theta(\theta, K) = K T_0 (K\theta)^\gamma, \\ & PN(\theta, K) = \frac{P_0}{\xi} \left(\frac{1-\theta}{\theta} K \right)^\xi; PN_K(\theta, K) = \frac{1}{\theta} (P_0 - \theta P_0) \left(\frac{1}{\theta} (K - K\theta) \right)^{\xi-1}, \\ & PN_\theta(\theta, K) = -\frac{K}{\theta^2} P_0 \left(\frac{1}{\theta} (K - K\theta) \right)^{\xi-1}. \end{aligned} \right.$$

Given these quantities we can rewrite the Hamiltonian:

$$H = \max_{\{I, \theta\}} \left\{ \begin{aligned} & (1-\tau)(\theta K)^a - I(1+\alpha(\theta)) - TC(\theta, K) + (K(1-\theta))^b (s\tau(\mu-1)-1) + \\ & - PN(\theta, K)(1-\mu) + \lambda_0 (I - \omega K) + \lambda_1 \theta - \lambda_2 (\theta - 1) \end{aligned} \right\}$$

and derive the necessary and sufficient conditions¹.

Proposition 1: Equilibrium is characterized by the following necessary and sufficient conditions.

$$I : (1+\alpha(\theta)) = \lambda_0$$

$$\theta : (1-\tau)Ka((K\theta)^{a-1}) - I\alpha'(\theta) - TC_\theta(\theta, K) - ((K(1-\theta))^{b-1})bK(s\tau(\mu-1)-1) + PN_\theta(\theta, K)(1-\mu) + \lambda_1 - \lambda_2 = 0$$

$$a\theta(1-\tau)(K\theta)^{a-1} - TC_K(\theta, K) + (1-\theta)(K(1-\theta))^{b-1}(s\tau(\mu-1)-1) + PN_K(\theta, K)(1-\mu) - \lambda_{0,\omega} = \dot{\lambda}_0 = 0$$

¹ It is trivial to show that the Hamiltonian is convex.

$$\dot{K} = I - \omega K$$

$$\dot{\lambda}_0 = \alpha'(\theta)\dot{\theta}$$

2.2. The 2x2 dynamical system. The next step assumes that we have an interior solution ($\lambda_1 = \lambda_2 = 0$), and eliminates I by using the capital accumulation constraint ($\dot{K} + \omega K = I$). The previous FOCs now read:

$$(1 - \tau)Ka((K\theta)^{a-1}) - (\dot{K} + \omega K)\alpha b4(\theta) - TC_\theta(\theta, K) - ((K(1 - \theta))^{b-1})bK(s\tau(\mu - 1) - 1) - PN_\theta(\theta, K)(1 - \mu),$$

$$a\theta(1 - \tau)(K\theta)^{a-1} - TC_K(\theta, K) + (1 - \theta)(K(1 - \theta))^{b-1}(s\tau(\mu - 1) - 1) - PN_K(\theta, K)(1 - \mu) =$$

$$= \alpha b4(\theta)\dot{\theta} - (1 + \alpha(\theta))(r - \omega).$$

Manipulating these two equations, we obtain a 2x2 dynamical system, which is the object of our analysis:

$$c^{-1}(1 - \tau)Ka((K\theta)^{a-1}) + \omega Kc - TC_\theta(\theta, K) - ((K(1 - \theta))^{b-1})bK(s\tau(\mu - 1) - 1) + -PN_\theta(\theta, K)(1 - \mu) = K,$$

$$c^{-1}a\theta(1 - \tau)Ka((K\theta)^{a-1}) - \frac{TC_K(\theta, K)}{c} + (1 - \theta)(K(1 - \theta))^{b-1}(s\tau(\mu - 1) - 1) + -PN_K(\theta, K)(1 - \mu(\theta)) +$$

$$+ (1 + c\theta)(r - \omega) = \dot{\theta}.$$

The steady state is defined as the locus where $\dot{K} = \dot{\theta} = 0$, and it is defined below.

3. Stationary state

Definiton 2: A steady state for this economy is a pair $(\bar{K}; \bar{\theta})$ that satisfies the following conditions.

$$\left\{ \begin{array}{l} (1 - \tau)Ka((K\theta)^{a-1}) + \omega Kc - TC_\theta(\theta, K) - ((K(1 - \theta))^{b-1})bK(s\tau(\mu - 1) - 1) - PN_\theta(\theta, K)(1 - \mu) = 0 \\ a\theta(1 - \tau)Ka((K\theta)^{a-1}) - \frac{TC_K(\theta, K)}{c} + (1 - \theta)(K(1 - \theta))^{b-1}(s\tau(\mu - 1) - 1) - PN_K(\theta, K)(1 - \mu(\theta)) + \\ + (1 + c\theta)(r - \omega) = 0 \end{array} \right\}.$$

The model has no closed form solution and, therefore, we derive the steady state numerically. This means that it is necessary to pin down parameters' values. Our calibration is detailed below.

4. Calibration

This section pins down numerical values for the parameters of the model; in particular the corporate tax rate τ , the credit easiness c , the depreciation capital rate ω , the surcharge factor s , the probability of being detected μ , the constant part of the transparency cost T_0 , the constant part of the penalty cost P_0 and the interest rate r . The elasticity of the transparency cost with respect to the regular production γ and the elasticity of the penalty cost with respect to the underground size of the economy ξ are calibrated consistently to have an internal solution in the space $K * \theta$. Moreover, the production function is supposed to be linear in capital, i.e., $a = b = 1$. The theoretical model is calibrated for two countries: Italy, as a proxy for fully developed countries with a significant share of underground sector, and Turkey, as a proxy for a transition country, still with a large underground sector.

4.1. Parametrization for the Italian economy. We first focus on the Italian economy, having in mind that this analysis can be qualitatively addressed to other European countries like Belgium, Denmark, Greece, Portugal and Spain, and to the United States as well. Calibration is based on seasonally adjusted ISTAT series from 1970: 1 to 1996: 4, expressed in constant 1995 prices¹. The Italian parametrization value set is summarized in Table 1.

Table 1. Italian parametrization value set

τ	c	ω	s	μ^*	T_0	P_0	γ	ξ	a	b	r^*
0.33	5.5	0.025	1.3	0.03	0.5	0.5	2.1	1.1	1	1	0.08

The corporate tax rate is set equal to the legal corporate tax rate nowadays in Italy, 33%, the fixed components of the transparency cost T_0 and of the penalty cost P_0 are set equal to 0.5, to give them the same weight on the determination of the profits and to generate an internal solution in the space $K * \theta$. The parameter s represents the surcharge on the standard tax rate that a firm, detected employing workers in non-market sector, must pay. We rely on

¹ The data source is the ISTAT (Italy's National Statistical Institute) at <http://www.istat.it/English/The-Insttit/index.htm>.

Busato and Chiarini (2004) calibration that set this parameter, following the Italian Tax Law (Legislative Decree 471/97, Section 13, Paragraph 1) at 30% of the statutory tax rate; $s^* = 1.30$. The interest rate, set equal to 0.08, represents an average of the short-term interest rates for the analyzed sample; the parameter c , which indicates the coefficient of the advantages on the credit market with respect to the declared size of the economy, is set equal to 5.5 in order to have an internal solution in the space $K * \theta$. The depreciation capital rate ω is set equal to 0.025, a standard value for the Italian economy.

4.2. Parameterizations for the Turkish economy.

We next set the parameters space for the Turkish economy; due to the lack of data, the calibration for this country is restricted to the three quantities for which data are either available or can be indirectly proxied: the corporate tax rate τ , the short-term interest rate r and the probability of being detected evading μ . The other parameters are set to have an internal solution in the space $K * \theta$. The parametrization value set is summarized in Table 2.

Table 2. Turkish parametrization value set

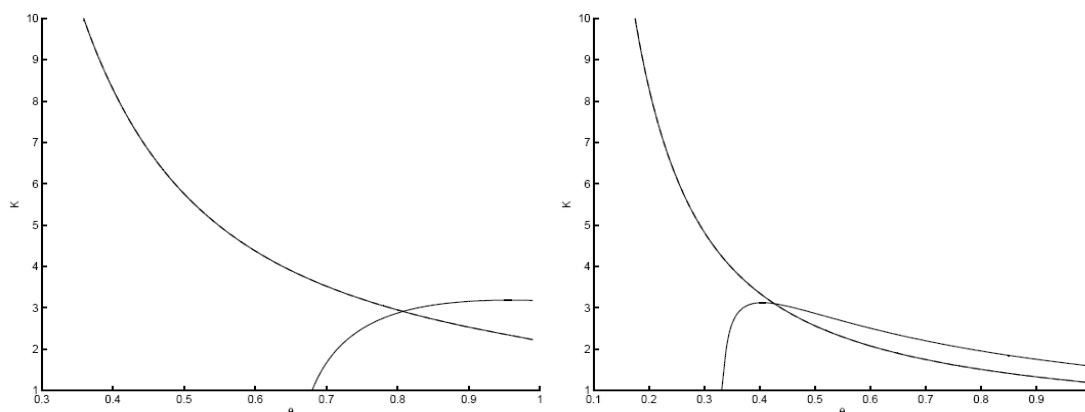
τ	c	ω	s	μ^*	T_0	P_0	γ	ξ	a	b	r^*
0.3	3	0.025	1.3	0.001	0.5	5	1.1	1.1	1	1	0.7

The corporate tax rate is set equal to the legal corporate tax rate nowadays in Turkey, 30%¹; the short-term interest rate is the average of the short-term interest rates over the sample 1970 :1-1996: 4 (OECD quarterly data)². For the value of μ , we have looked at the “Tax inspectors board results” over the period of 1995-2003³: tax inspectors in the analyzed sample determined a 202% discrepancy in the tax base; we can consequently think that, in a context

where tax evasion has a so high dimension, the probability of being detected μ is very small; hence we have set the value of μ equal to 0.001.

4.3. Phase diagrams and steady states.

Figure 3 presents the phase diagram for the two countries for the parametrization above. The long-run equilibrium of the model is represented at the intersection of the two loci. The Italian economy (left panel) is characterized by a relative larger share of regular economy (θ_{ITA} is above 80%), and the average firm size is approximately equal to 3.00 unit of capital stock. We do not interpret this measure quantitatively, but we consider it as a benchmark value, from a qualitative perspective. For the Turkish economy, on the other hand, there exists a quite large portion of underground economy (the regular share of the economy θ_{TUR} equals approximately 40%), and the average firm size is comparable to the Italian figures (around 3.00). Hence the key difference between these two countries is in the share of declared revenues to the Tax Offices. The declared size of a representative Italian firm equals $\theta_{ITA} * K_{ITA}^* = 0.80 * 3.00 = 2.4$ units of capital stock, while the corresponding measure for the Turkish firm equals $\theta_{TUR} * K_{TUR}^* = 0.40 * 3.00 = 1.4$. As a result, the average size of firms operating within the Turkish economy will only appear smaller, while being qualitatively comparable with the Italian one. The main differences between the two economies are in the probability of being detected and in the easiness for acceding to the credit market, which is much smaller for the Turkish economy, in the tax rate, which is also slightly smaller for the Turkish economy, and in the penalty cost, which is instead larger for the Turkish economy.



Notes: Left panel: Italy; right panel: Turkey.

Fig. 3. Caption

¹ As suggested by the IMF.

² We have chosen this sample to be able to compare the Turkish parametrization with the Italian one. Moreover the “true” average short interest rate is equal to 111%, however we have chosen $r = 0.70$ in order to have an internal solution in the space $(K * \theta)$.

³ Data are available at the web-site <http://www.huk.gov.tr/results.html>.

5. Policy analysis

This section presents selected policy experiments, to evaluate the impact of transparency costs, anti-evasion policies and credit market tightness on the long run equilibrium of the model, mainly focusing on the average firms' size and on the share of

regular activities. The analysis is here restricted to the Italian economy, as a representative European economy characterized by a relatively large share of underground economy. The model is solved numerically, and results of simulations are included in Table 3 below.

Table 3. Simulation results

		K^*	θ^*	$\theta^* \times K^*$	Ann. welf.	Delta (%)	Act. welf.	Delta (%)
Baseline		2.91	0.81	2.36	100.00		100.00	
Transparency costs (\uparrow)	$T_0 = 0.6$	2.67	0.81	2.16	91.75	-8.25%	91.75	-8.25%
	$\gamma = 2.2$	2.80	0.81	2.27	96.22	-3.78%	96.22	-3.78%
	$T_0 = 0.6$ e $\gamma = 2.2$	2.58	0.81	2.09	88.66	-11.34%	88.66	-11.34%
Transparency costs (\downarrow)	$T_0 = 0.4$	3.24	0.81	2.62	111.34	-11.34%	111.34	-11.34%
	$\gamma = 2.0$	3.00	0.81	2.43	103.09	3.09%	103.09	3.09%
	$T_0 = 0.4$ e $\gamma = 2.0$	3.40	0.81	2.75	116.84	16.84%	116.84	16.84%
Low returns to capital	$r = 0.025$	2.46	0.92	2.26	96.02	-3.98%	84.54	-15.46%
Higher returns to capital	$r = 0.10$	3.10	0.78	2.42	102.58	2.58%	106.53	6.53%
Tighter credit access	$c = 5$	2.81	0.83	2.33	98.95	-1.05%	96.56	-3.44%
Easy credit access	$c = 6$	3.00	0.76	2.28	96.73	-3.27%	103.09	3.09%
Higher penalty costs	$P_0 = 0.6$	2.91	0.82	2.39	101.23	1.23%	100.00	0.00%
	$\xi = 1.2$	2.91	0.81	2.36	100.00	0.00%	100.00	0.00%
	$P_0 = 0.6$ e $\xi = 1.2$	2.91	0.82	2.39	101.23	1.23%	100.00	0.00%
Low penalty costs	$P_0 = 0.4$	2.91	0.79	2.30	97.53	-2.47%	100.00	0.00%
	$\xi = 1.0$	2.91	0.81	2.36	100.00	0.00%	100.00	0.00%
	$P_0 = 0.4$ e $\xi = 1.0$	2.91	0.79	2.30	97.53	-2.47%	100.00	0.00%
Detection prob. (\downarrow)	$\mu = 0.01$	2.91	0.80	2.33	98.77	-1.23%	100.00	0.00%
Detection prob. (\uparrow)	$\mu = 0.1$	2.92	0.78	2.28	96.63	-3.37%	100.34	0.34%
Surcharge factor (\uparrow)	$s = 2$	2.91	0.82	2.39	100.89	0.89%	99.66	-0.34%
Surcharge factor (\downarrow)	$s = 2$	2.93	0.79	2.31	97.86	-2.14%	100.34	0.34%

Notes: K is the true average size of the firm; θ is a share of regular (declared) capital stock; θK is a value of declared capital stock; Ann. welf. (i.e., announced welfare) is a welfare measure constructed over the declared capital stock, and we interpret it as a measure of what a firm would declare; Act. welf. (i.e., actual welfare) is a welfare measure constructed over the actual capital stock, and we interpret it as a measure of what is the actual welfare of a firm. The remaining parameters are set to the following values: $t = 0.33$; $c = 5.5$; $\omega = 0.025$; $s = 1.53$; $\mu = 0.03$; $T_0 = 0.5$; $\gamma = 2.1$; $\xi = 1.1$; $P_0 = 0.5$; $r = 0.08$.

The following sections offer more detailed interpretation of the results.

5.1. Anti evasion policies. As noted in the previous section the theoretical and empirical literature on the issue here investigated, agree on the existence of an inverse relation between size and tax evasion and, consequently, on possible effects deriving from fiscal policy design and tax enforcement. In our theoretical model we have introduced these variables considering a surcharge factor applied in case of detection of capital evaded, or a tightening in the penalties applied to evading firms.

5.1.1. Surcharge factor effect. The exercise has been done considering an increase (reduction) of the surcharge factor s applied to the corporate tax rate τ when a firm is discovered evading. A reduction in the surcharge factor increases the average size of firms with respect to the baseline (from 2.91 to 2.93) while it is unaffected by an

increase of s . As we expect, a higher (lower) surcharge factor increases (decreases) the share of declared capital stock and consequently the regular share of the economy. The two welfare measures computed instead, are asymmetrically influenced by changes in the surcharge factor. The announced welfare grows as s increases and lowers if s decreases. On the other side, the actual welfare decreases when the surcharge factor increases and vice-versa. The strengthening of the tax enforcement, has a slightly depressing effect on capital accumulation, and this is clearly witnessed by the contrasting effects on the announced and actual welfare, with the first one slightly increasing and the true one going in the opposite direction. This is confirmed by the exactly reversed effects of a reduction of s . Policy makers should take into account these results when projecting interventions aiming at reducing the underground sector.

Before going to analyze the tightening of penalties it is worthwhile to note the results of our experiment when increasing/reducing the rate of return r . A decrease (increase) in the short-term interest rate generates a decrease (increase) in the equilibrium capital value and an increase (decrease) in the regular share of the economy. It is important to notice that a lower cost of capital ($r = 0.025$) reduces the equilibrium capital value, but increases the regular share of the economy.

5.1.2. Tighter penalties. It is, then, interesting to investigate the relationships between policies contrasting tax evasion with tighter penalties and the average firms' size. We consider an increase in P_0 , in ξ and in both parameters. In this case the average firm size does not change, as the K column suggests (K stock is constant at 2.91). The share of declared revenues, however, increases with P_0 but it is not affected by the variation of the elasticity component of the penalty costs. The welfare computed on the declared capital stock increases by approximately 1 percent. It is interestingly to notice that the "true" welfare, which is the welfare computed on the total capital stock (declared and not declared to the IRS) is not affected. That is because an increase in penalty costs just affects the share of output reported while leaving unaffected the firms' capital stock.

An easier policy against tax evasion (i.e., lower penalty costs when detected) has qualitatively symmetric implications. From a quantitative perspective, however, it has a relative larger impact than a tighter policy (tax evasion increases by 2 percentage points) and the welfare computed on declared capital stock falls by approximately the same figure.

Also the probability of being detected affects the average firms' size K and the share of regular economy θ . Higher (lower) detection probability, $\mu = 0.1$ ($\mu = 0.01$) increases (equals) the average capital stock but reduces the share of regular economy. It is interesting to notice, however, that when the probability of being detected is higher, we observe a loss in the announced welfare while the true one slightly increases (-3.37% and +0.34%). This means that, from the policy maker point of view, the strengthening of tax enforcement seems to go in the right direction given that it is welfare reducing only when measuring the welfare on the regular part of the capital stock.

5.2. Costs and benefit from transparency. Consider, first, the consequences of an increase in transparency costs. This can be done by increasing either T_0 or γ , or both. In all cases the average size of firm is reduced, while the share of declared revenues is unaffected. Overall, we observe a reduction in the declared capital stock ($\theta * K$). This is consistent with

our expectations, because the higher transparency cost reduces the firm incentives to grow, while leaving unchanged their incentives to evade taxes. The main (and negative) consequence we observe for the economy is the reduction in firms' size. We consider this being a negative impact for the economy because it turns out to be welfare reducing. The symmetrical reduction of transparency costs, on the other hand, incentivates an expansion of the average firm size (K increases from 2.91 in the benchmark model to 3.24, 3.00 and 3.40) without affecting, also in this case, the share of revenues declared to internal revenue service. Quantitatively speaking, a reduction in transparency costs has an asymmetric effect compared to the symmetric increase. A 0.1 point reduction in T_0 , γ , and both produces a percentage welfare increase equal to 11.34%, 3.09% and 16.84%, respectively, while the symmetric increase lowers the welfare by 8.25%, 3.78% and 11.34%.

5.3. Credit market. A tighter credit market ($c = 5$) reduces the average firm's size (from 2.91 to 2.81) but rises the share of the regular economy (it goes from 0.81 to 0.83). An easier access to the credit market ($c = 6$), on the other hand, increases the average size of firms (from 2.91 to 3.00), but incentivates the corporate to abscond more revenues to the IRS (θ falls from 0.81 to 0.76). Both policy experiments are welfare reducing, when measuring it on the declared capital stock; that is because the value of not evaded capital ($K * \theta$) falls in both cases (from 2.36 to 2.33 and 2.28). The firm's size component (K) dominates when credit market gets tighter, while the evasion component (θ) is the key quantity when credit market access becomes easier. Notice that the true welfare measure presents a different picture. A tighter credit market has a negative impact on welfare (-3.44%) while an easier access makes it growing (+3.09%). That is because the true welfare measure is computed on the true capital stock, without considering how much of it is not declared to IRS. Moreover, the negative variation of the true welfare measure, in case of a tighter credit market, is wider than the one on the announced welfare. That is, again, because the former measures K as it is. This is also in line with the fact that, in case of an easier access to credit, the true welfare grows instead of lowering as the announced does. This means, overall considered, that from the policy perspective, the "actual" welfare is the correct indicator to consider. In summary, it is interesting to notice that monetary policy has an asymmetric effect on firm's size and tax evasion, and the overall consequences in terms of declared share of capital depends on whether the size component (K) or the evasion component ($1 - \theta$) dominates.

5.4. Comparison with existing literature. After having discussed the results of our calibration exercise it is worthwhile to spend some words on previous contributions to the theme. It is important to note, in fact, that the model here developed tries to reconcile three different perspectives: the relationship between firms size and tax evasion, given that we model the declaration of being small as an option to avoid transparency costs; the relationship between tax evasion and fiscal policy, considering that we explicitly include in the analysis taxes and penalties, and the relationship between firms size and institutions such as, from one side public authorities, which have to detect tax-evaders, and from the other financial institutions, which operate in the credit market. These themes have been mostly deepened in the field of Public Economy and in the so-called NIE (New Institutional Economics) which study the connections within firms and all other economic agents¹. The idea of this last strand of research is that the strategic choices of firms rely crucially on the economic and institutional environment they work in².

Anyway it should be said that the main difficulty in analyzing these phenomena is linked to the research of the exact relation between institutional elements and firms dimensional choices.

In general, the role played by economic and financial institutions has been deepened in the context of dynamic macroeconomic models which study the determinants of economic growth. Among the others, Bassanini and Scarpetta (2001), studying a sample of OECD countries, find out that a well developed financial system and a fair fiscal regulation, contribute to economic growth. From a microeconomic point of view, Davis and Henrikson (1999) analyze the size distribution of Swedish firms, characterized by a majority of big firms, and demonstrate that this is due to the impact of institutional factors such as a good system of taxation and a well functioning credit market.

Regarding the issue of the effectiveness of penalties, Kumar, Rajan and Zingales (1999), find out that countries with well functioning legal systems present a greater percentage of big firms compared to countries in which the effectiveness of penalties is not proved. The easiness to access credit market has finally been investigated by de Caprariis and Guiso (2004), finding out that it is a fundamental condition for the growth of firms and by Castelli, Dwyer and Hasan (2006) who report that small firms have more difficulties in getting financial sustain from banks.

¹ The linkage between tax evasion and fiscal policy as been studied starting from the work of Allingham and Sadmo (1972) who first interpreted evasion as a risky choice.

² For a detailed reconstruction of the literature see Ercoli (2006).

Going back to the relation between firm size and tax evasion, which is the focus of this work, most of the empirical results have been obtained on the Italian economy, which presents a high percentage of tax evasion. For example Di Nicola and Santoro (2000) highlight an inverse relation between firms size and tax evasion; CER (2001) deepens the issue of the relation between tax evasion and investment decisions, finding out that the investment option is critically linked to the size. Ercoli (2006), using a sample of Italian firms during the period of 1991-1996, find out that: (1) small firms have a higher probability of being detected; (2) there exist an inverse relation between size and tax evasion and this relation is stronger for small firms; (3) the effective dimension of firms is smaller than the declared one. This last result is consistent with our hypothesis on the opportunity, represented by tax evasion, for reducing transparency costs.

Given these considerations, the choice on how much capital abscond from the tax base seems to be strictly connected to the amount of output the firm decide to produce. The issue of the separability of these two decisions deserve to be investigated for at least two reasons: first of all it is fundamental to explain the size distribution of firms, and second, by supposing that these two choices could influence each other, we introduce the possibility of some kind of intervention by public authorities. The literature on the separability issue starts from the work of Marrelli (1984). Some years later Yaniv (1994; 1995) and Lee (1998) conclude that if the probability of being detected is considered as a function of the output produced, the separability is not proved while it is admissible if the same probability is a function of profits. All these works suggest that there exist a trade-off between costs and benefits associated to different productive decisions. In a context in which investment choices have to be done under uncertainty, tax evasion could be interpreted as a risky investment and firms reactions to tax enforcement have to be taken into account. As noted by Bagella (1997; 1998) there exist incentives and disincentives connected to the transparency issue. In case of detection in fact, the existence of different type of costs (which could be monetary and non monetary) could influence firms strategic decisions.

Conclusions

This paper discusses relations between “transparency costs” and tax evasion within the context of an optimal investment framework. Tax evasion is explicitly incorporated into the analysis, as an opportunity that firms have for allocating resources into a relative more flexible market and for reducing costs associated to be transparent. Indeed, the bigger is a firm, the larger are the monitoring

(“transparency”) costs¹; on the contrary, the smaller is a firm, the smaller is the probability of being detected evading and/or eluding taxes². In summary, we obtain the following results. If benefits from evasion increase (there is an increase in tax rate, or an increase in the penalty factor when a firm is detected) there is an increase in the regular share of the economy, but a reduction of the aver-

age size of firms. The average firm, therefore, gets smaller and more honest. If the costs of transparency lower (credit conditions get better for the average firm), then there is an increase in the regular share of the economy, and of firms’ average size. Better conditions for accessing the credit market increase the transparent part of the economy, without crowding out private capital stock.

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¹ The size of a firm depends on the announced dividends and on its equilibrium capital stock.

² In the model presented we also allow for the possibility that managers abscond dividends away from the shareholders. These flows are, of course, not subject to corporate taxation, and therefore represent an additional source of tax evasion.