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Comparison between long-term and short-term deposit interest rates in a model of adverse selection: a theoretical framework

Abstract

This is a principal-agent model of a bank in a competitive market and depositors. Depositors are either of low or high type which indicates the probability of early withdrawal. The depositors have private information about their type. Therefore, they will consider the long-term and short-term returns in their deposit decision. Banks can offer a menu of contracts with different combinations of short and long-term interest rates to those who withdraw early and wait respectively. This paper investigates the conditions under which the contracts that the banks offer can be sustained as equilibrium – symmetric pooling equilibrium where only one contract is offered and a separating equilibrium where two contracts are offered in order to screen the two types. It is found that early return of more than one can never be sustained (i.e. no short-term interest rate is sustainable). Further, there is no symmetric pooling equilibrium when both types withdraw early with some probability. However, a symmetric pooling equilibrium can be sustained if the proportion of low type agents is high enough and they never withdraw early. There exists a separating equilibrium if the proportion of low type agents is sufficiently high. The problem of establishing equilibrium means, frequent changes in the banks' contract can be expected. Regulators should ensure that information of such changes are communicated clearly and sufficiently in advance to the depositors.

Keywords: adverse selection, bank, interest rates, depositors.
JEL Classifications: D82, G21.

Introduction

Commercial banks choose interest rates they offer for term deposits in order to attract customers and increase profitability. This paper investigates interest rate contracts that can be sustained as an equilibrium in the competitive banking industry. In this paper it is assumed that depositors withdraw early only if they are hit by a liquidity shock. If the depositors are confident of the financial stability of the country and that the banks having access to funds to meet any amount of early demand, they will not withdraw because of self-fulfilling beliefs. We also take into account that depositors have different probabilities of having to withdraw early. The crucial point in this paper is that when making deposits in banks the agents have private information about the probability of being hit by the liquidity shock. This is because early withdrawal becomes necessary because of personal circumstances – their own savings habits; illness of a relative that might incur medical costs; plans to move to a new house; wedding, travel plans etc.

It is possible for the banks to give different returns to those keeping their money with the bank for different lengths of time. Such contracts are common in practice. Banks offer different products such as current accounts with no interests, saving accounts with different interest rates depending on the amount and time length of the deposit. We allow the banks to design contracts which specify the early return and the late return which would be given to those withdrawing early and late respectively.

When making a decision about depositing in a bank, the depositors will consider the short-term and long-term interest rates that are offered by the bank and also their own probability of being early withdrawers. Because the banks operate in competitive environments they will have to offer a good deal and each depositor will choose the contract that gives him the highest expected utility.

We attempt to find contracts that can be sustained as equilibrium if the depositors are of only two types – whose probability of early withdrawal is either high or low. As in the standard literature of similar models, we find that sustaining equilibrium is not easy. This is because of the banks operating in a competitive environment; the risk averseness of agents; and the payoffs of both the principal and the agents being influenced by two variables. This paper contributes to the theoretical aspects of adverse selection, contract theory and its application to the banking industry where the banks face depositors who have private information about their probability of early withdrawal.

Standard literature on screening of two types in adverse selection, Wilson (1977) and Rothschild and Stiglitz (1976), Mas-Colell, Whinston and Green (1995) show there is non-existence of pooling equilibrium but separating equilibrium can be sustained under certain conditions. Their models deal with adverse selection in the insurance market and labor market. In the labor market employees have private information about their types. The choice variables are wages and education. The wage affects the payoffs of both players. However, the other variable, education, is only a screening device which

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affects the payoff of the employee and not the employer. In the insurance market, indemnity and premium are the choice variables, which affect the expected payoffs of both players, and all the agent types are affected by both the variables. This strand of literature is extended where different equilibrium possibilities and efficiency of the outcome are investigated (Bisin and Gottardi, 2006; Diasakos and Koutopoulos, 2011; Dubey and Geanakoplos, 2002; Gale, 1992; Martin, 2007; Rustichini and Siconolfi, 2008).

The model in this current paper is such that both variables – i.e. returns offered to early and late withdrawers – affect the principal's profit function as well as the agents' utility function. It is found that a separating equilibrium where the banks screen two types so that they accept different contracts can be sustained as long as the proportion of low type agents is sufficiently high. The proportion of agents who have low probability of early withdrawal should be sufficiently high to make it worthwhile for the bank to have two contracts. Otherwise the bank would be better off having just one contract for both types of depositors.

It is also found that a symmetric pooling equilibrium where all the banks offer only one contract and both types accept the same contract can be sustained only if the low type agents will never withdraw early and the proportion of such a type is sufficiently high. A reason behind the non-existence of a symmetric pooling equilibrium when both types withdraw with some probability is because the agents are risk averse and the principal is risk neutral. When the low types have zero probability of early withdrawal, only the long-term return affects the agents' utility and, therefore, we are able to sustain an equilibrium.

Another finding worth highlighting is about the level of insurance offered to early withdrawers by the banks. Equilibrium cannot be sustained if we give early withdrawers an interest. The early withdrawers are never given an early return of more than one in equilibrium (i.e. they withdraw what was deposited with no interest and no penalty fee). This is contrary to Selvaretnam (2007) where early withdrawers had to be penalized and Goldstein and Pauzner (2005) who offered a positive short-term interest. The different outcome is because the depositors withdraw only because they are hit by liquidity shock and not because of self-fulfilling beliefs.

The depositors do get early return of one in certain cases. When we have a symmetric pooling equilibrium, the depositors are offered early return of one, provided the low types will never withdraw early. For a separating equilibrium, the agents who have a high probability of being hit by the liquidity shock get early return of one when the high type agent will definitely withdraw early or the low type agent will never withdraw early. Other than in these cases, the early withdrawer should be given a return that is less than what he actually invested, if equilibrium is to be sustained.

The rest of the paper is organized as follows. In the next section the model is set out. This is followed by the analysis of a pooling equilibrium where all the banks offer just one contract in section 2. The analysis of a separating equilibrium where the banks offer different contracts for the different types of agents is found in section 3 while the final section concludes.

1. The model

There are three periods (t_0, t_1, t_2). A continuum [0, 1] of agents are endowed with one unit at the beginning of t_0 which they can deposit in a bank. There are n number of risk neutral banks which operate in a competitive market. Consumption happens only in periods t_1 and t_2. In this simple set up, the continuum of agents [0, 1] are of two types L and H who have probability λ_L and λ_H of being hit by a liquidity shock in t_1 respectively where $\lambda_L < \lambda_H$.

Once a player receives a liquidity shock he has to withdraw early in t_1 and can derive utility only by consuming in t_1. If the agent does not receive a liquidity shock, he waits till the last period and receives a higher return. In this model there is no withdrawal due to self-fulfilling beliefs.

Each agent has private information as to whether he is type L or H at the beginning of t_0. However it is public knowledge that the proportion of type L and H is p and $(1-p)$ respectively.

The banks are risk neutral. All agents are risk averse with the same utility function which is strictly concave, increasing, twice continuously differentiable, has a relative risk aversion coefficient of $\frac{-cu''(c)}{u'(c)} > 1$ and a functional form $u(c) = c^a$, where $0 < a < 1$.

At the beginning of t_0 each bank j designs and offer contracts which has the pair of returns (r_d^j, R_d^j), where $d \in (L, H)$. Agents who deposited in bank j and accepted contract d receive r_d^j and R_d^j if they withdraw in t_1 and t_2, respectively. We assume that the banks want to survive for many periods and, therefore, will make viable investment decisions. They will fix the depositor returns such that $R_d^j > r_d^j$ so that the patient agents will not want to withdraw early.

In t_0, after observing the contracts offered by the banks (r_d^j, R_d^j) and knowing their own probability of being hit by a liquidity shock (i.e. whether they are type L or H, each agent i will decide to take the contract that gives him the highest expected utility. The bank invests the deposits in a long-term project, keeping just enough as reserves to meet the early withdrawals. The return on the long-term project is realized in t_2. Each unit that is invested in the long-term project in t_0 realizes a fixed amount $\theta > 1$ in t_2.

We can outline the model as follows.

Period t_0:

Stage 1. Agents privately learn their types (type L or H) – i.e. the probability of being hit by the liquidity shock, λ_L or λ_H. Banks simultaneously announce sets of contracts (r_d^j, R_d^j) that are offered.

Stage 2. Given the contracts that are offered, and knowing their own types the agents choose whether to accept a contract, and if so, which one. Banks invest money in a long-term project after keeping just enough to meet the early withdrawals in t_1.

Period t_1: Liquidity shock hits proportion p agents with probability λ_L and $(1 - p)$ proportion of agents with probability λ_H. Those who are hit by it withdraw and receive early return r_d^j.

Period t_2: Banks receive returns θ per unit from their investment. Agents who did not withdraw in t_1 receive a return R_d^j.

The objective of this model is to find contracts (r_d^j, R_d^j) that can be sustained as equilibrium. A contract is an equilibrium if, once all the banks have offered their contracts, no bank can deviate and offer another contract which makes it better off. We check for the existence of a symmetric pooling equilibrium where the bank j offers just the one contract (r^j, R^j), and a separating equilibrium where it offers two contracts $\left\{\left(r_L^j, R_L^j\right), \left(r_H^j, R_H^j\right)\right\}$ to be accepted by the low types and high type respectively.

2. Symmetric pooling equilibrium

This section looks at what happens if all the banks can offer only one contract (r^j, R^j) to the depositors. Can such a contract where all the banks offer just the one and the same contract be sustained as an equilibrium where both types will deposit? Because we assume symmetric pooling equilibrium, all banks offer the same contract (r, R). If the depositors decide to deposit, they will choose one bank with probability $(1/n)$.

First of all, the depositors should find it worthwhile to accept the contract. The participation constraints for the low types and the high types are given by PC_L and PC_H, respectively:

$$\lambda_L u(r) + (1 - \lambda_L) u(R) \geq u(1). \qquad (PC_L)$$

Because $\lambda_L < \lambda_H$ and $u(r) < u(R)$, PC_H is steeper than PC_L.

The indifference curves for the low type and the high type can be given as follows:

$$\lambda_L u(r) + (1 - \lambda_L) u(R) = k_L. \qquad (IC_L)$$

$$\lambda_H u(r) + (1 - \lambda_H) u(R) = k_H. \qquad (IC_H)$$

When $k_L = k_H = 1$, these become the participation constraints. As has been illustrated in Figure 1, the indifference curve of the type H, IC_H is steeper than that of the type L, IC_L, for any given level of utility.

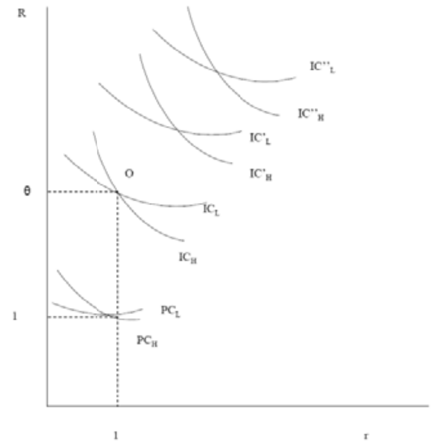

Fig. 1. The indifference curves with single crossing

If only the type L deposits, the proportion of deposits will be p and the profit to the banks will be π_L. The banks will keep $\lambda_L r$ as reserves, which is paid out in t_1. It invests the balance in a long-term project which earns θ per unit. In the last period, R is paid out to those who withdraw late.

$$\pi_L = \frac{p\{(1 - \lambda_L r)\theta - (1 - \lambda_L) R\}}{n} \qquad (1)$$

Likewise, if only the type H deposits the proportion of deposits will be $(1 - p)$ and the profit to the banks will be π_H,

$$\pi_H = \frac{(1 - p)\{(1 - \lambda_H r)\theta - (1 - \lambda_H) R\}}{n}. \qquad (2)$$

If both types decide to deposit, the profit π to one bank is given by

$$\pi = \frac{(p\{(1 - \lambda_L r)\theta - (1 - \lambda_L) R\} + (1 - p)\{(1 - \lambda_H r)\theta - (1 - \lambda_H R\}}{n}. \qquad (3)$$

If both the participation constraints do not hold, then the profit will be zero because there will be no deposits.

Lemma 1. *Any contract* (r, R) *is not a symmetric pooling equilibrium if the payoff* $\pi \neq 0$.

Proof. If $\pi > 0$ the banks will have incentive to increase either of the returns slightly and attract all the depositors from the other banks to increase their profits. If $\pi < 0$, the banks will reduce the deposit rates so that there will be no deposits and the bank makes zero profits. Therefore (r, R) such that $\pi > 0$ or $\pi < 0$ cannot be an equilibrium.

Lemma 2. *Any contract* (r, R) *which gives* $\pi = 0$ *other than* $(1, \theta)$ *is not an equilibrium.*

Proof. All the combinations of (r, R) where the banks break-even are shown in Figure 2. The break-even line when both types would deposit is given by line BE_A.

$$p\{(1-\lambda_L r)\theta-(1-\lambda_L)R\}+ \\ +(1-p)\{(1-\lambda_H r)\theta-(1-\lambda_H)R\}=0. \qquad (BE_A)$$

If only one type prefers to deposit in equilibrium and the banks have to make zero profits, the combinations of (r, R) will give different break-even lines. These lines are called the low-type break-even line, BE_L and the high-type break-even line, BE_H given by the following:

$$(1-\lambda_L r)\theta-(1-\lambda_L)R=0$$

$$\Leftrightarrow \frac{\theta}{1-\lambda_L}-\frac{\lambda_L\theta}{1-\lambda_L}r=R. \qquad (BE_L)$$

$$\left(1-\lambda_H r\right)\theta-\left(1-\lambda_H\right)R=0$$

$$\Leftrightarrow \frac{\theta}{1-\lambda_H}-\frac{\lambda_H\theta}{1-\lambda_H}r=R. \qquad (BE_H)$$

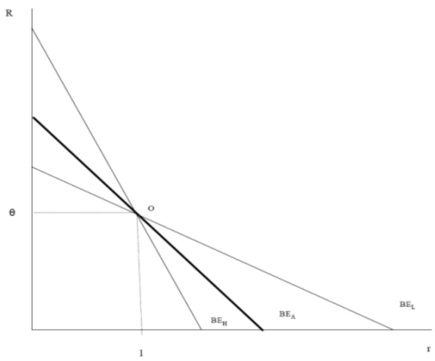

Fig. 2. The break-even lines

Line BE_H is steeper than line BE_L while line BE_A is between BE_H and BE_L.

However, all three lines go through $(1, \theta)$.

At O, both types of agents will participate because $(1, \theta) > (1, 1)$.

Any point on the break-even line BE_A which is not $(1, \theta)$ means that some depositors are creating profits and others are creating losses for the bank. Therefore, it would be better for the bank to move to a point where the loss creators are worse off and, therefore, not take the contract.

We analyze different situations that can occur as shown in the following diagrams. Recall that at any point IC_H is steeper than IC_L.

Consider a contract X which is on the left of O on line BE_A (where you are below line BE_H and above line BE_L) in Figure 3. Because all the banks are offering this same contract, both types would be depositing. One bank can move to a point that is higher than IC_H and lower than IC_L (any point in the shaded area) so that the high types who will create profits are attracted and the low types who are creating losses are better off leaving to other banks. After the deviation, because only the type Hs are attracted to this bank and the contract is below line BE_H, the bank can make positive payoff.

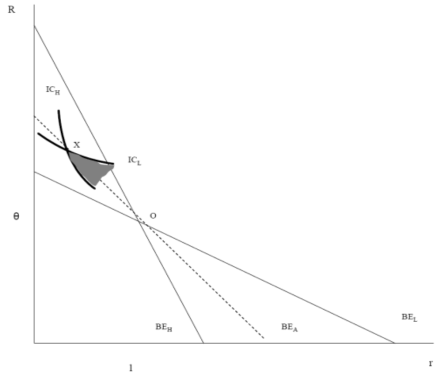

Fig. 3. ***X*** **is not an equilibrium**

Now consider a contract Y which is on the right of O on line BE_A (where you are above line BE_H and below line BE_L) in Figure 4.

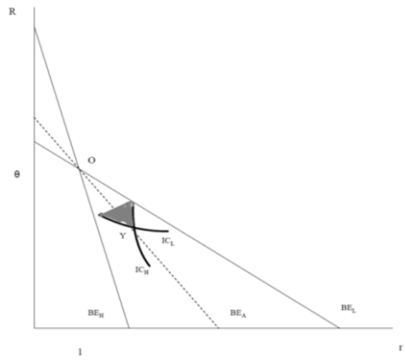

Fig. 4. ***Y*** **is not an equilibrium**

One bank can move to a point that is higher than IC_L and lower than IC_H so that the low types who will create profits are attracted to you and the high types who are creating losses are better off leaving to other banks. Now because only the type Ls deposit, and the contract is below line BE_L, the bank can make a positive profit. Therefore, any point other than $(1, \theta)$ where $\pi = 0$ is not an equilibrium.

Proposition 1. *There does not exist a symmetric pooling equilibrium as long as* $\lambda_L > 0$.

Proof. It has been shown in Lemma 1 and Lemma 2 that a contract cannot be an equilibrium if $\pi \neq 0$ $\pi \neq 0$ and when it is not $(1, \theta)$. If at all an equilibrium exists, it has to be at the point $(1, \theta)$ through which all three break-even lines pass and the bank makes zero profit. Recall the break-even lines BE_L and BE_H,

$$\left(1-\lambda_d r_d\right)\theta-\left(1-\lambda_d\right)R_d=0, \quad (4)$$

where $d \in (L, H)$.

So the slope of the break-even line is fixed at $-\frac{\lambda_d\theta}{1-\lambda_d}$.

Recall the indifference curves given by IC_d.

$$\lambda_d u\left(r_d\right)+\left(1-\lambda_d\right)u\left(R_d\right)=d. \quad (5)$$

The slope is given by

$$\frac{dR_d}{dr_d}=-\frac{\lambda_d u'\left(r_d\right)}{\left(1-\lambda_d\right)u'\left(R_d\right)}. \quad (6)$$

Now recall that the utility functions are of a specific form $u(c) = c^a$, where $0 < a < 1$.

Therefore, the slopes of IC_H, IC_L, break-even lines BE_L and BE_H at the point $(1,\theta)$ are $-\frac{\lambda_L}{1-\lambda_L}\theta^{1-\alpha}$, $-\frac{\lambda_H}{1-\lambda_H}\theta^{1-\alpha}, -\frac{\lambda_L}{1-\lambda_L}\theta, -\frac{\lambda_H}{1-\lambda_H}$, respectively.

We know that $\theta > \theta^{1-\alpha}$.

Therefore $\frac{\lambda_L}{1-\lambda_L}\theta > \frac{\lambda_L}{1-\lambda_L}\theta^{1-\alpha}; \frac{\lambda_H}{1-\lambda_H} > \frac{\lambda_H}{1-\lambda_H}\theta^{1-\alpha}$.

From this we can see that at $(1,\theta)$, break-even line BE_L is steeper than the indifference curve IC_L; break-even line BE_H is steeper than the indifference curve IC_H.

We already know that IC_H is steeper than IC_L and BE_H is steeper than BE_L.

In Figure 5, note that to the left of O (where $r < 1$), IC_L is below line BE_L; while IC_H is above IC_L; line BE_H is above all the curves.

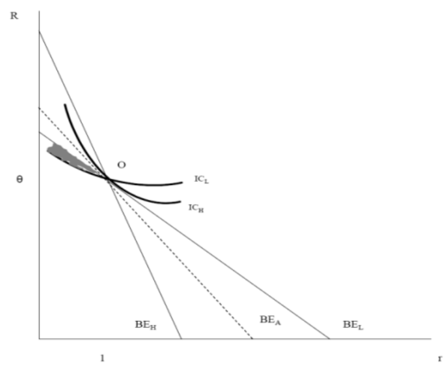

Fig. 5. *O* is not an equilibrium

Therefore, we can find a point to the left of O, above IC_L and below IC_H and line BE_L (any point in the shaded area).

This would mean that the high types will leave for other banks while the low types will be attracted to the deviant bank.

This will give positive profits to the deviant bank.

Therefore, point O cannot be sustained as an equilibrium so long as $\lambda_L > 0$ (note that only because $\lambda_L > 0$, we have IC_L is below line BE_L).

The driving force behind Proposition 1 which ruled out the existence of a symmetric pooling equilibrium is that the line BE_L is steeper than IC_L which is because the agents are risk averse while the banks are risk neutral. This makes it possible for a bank to deviate profitably.

If the probability of early withdrawal is zero ($\lambda_L = 0$), only late return R will affect the break-even line and the indifference curves, so that both are horizontal. The next proposition says that a pooling equilibrium can be sustained at $(1,\theta)$ if $\lambda_L = 0$ as long as there is sufficient proportion of the low type agents.

Proposition 2. *A symmetric pooling equilibrium* $(1,\theta)$ *can be sustained if* $\lambda_L = 0$ *and the proportion of type L is sufficiently high.*

Proof. When $\lambda_L = 0$, the indifference curve IC_L and break-even line BE_L are horizontal where early return r does not affect them.

Now break-even line BE_L is given by a horizontal line $R=\theta$. Agents' indifference curves, IC_L are also horizontal lines given by $u(R)=k_L$.

If the proportion of low type agents, p, is too low so that line BE_A is steeper than IC_H at O, then the bank can find a point above both IC_L and IC_H, but below line BE_H.

Therefore, a bank can deviate and offer a contract that is any point in the shaded area in Figure 6 to attract both types of agents and make a profit.

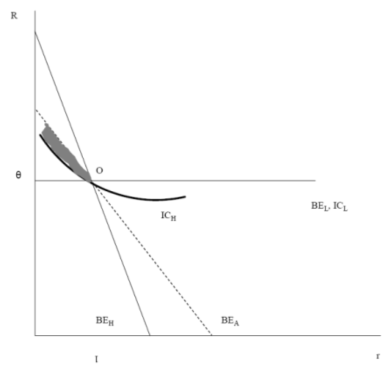

Fig. 6. No symmetric pooling equilibrium

However, if p is sufficiently high so that line BE_A is flatter than IC_H at O (Figure 7), the bank cannot profitably deviate either to the left or right of O.

Therefore, if the proportion of low types is high enough so that line BE_A is sufficiently flat, a symmetric pooling equilibrium can be sustained at $(1, \theta)$ where all the banks can offer just one contract and they offer $r = 1; R = \theta$.

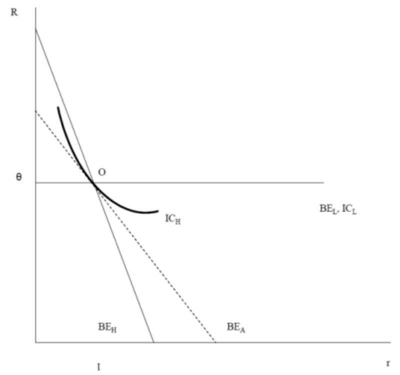

Fig. 7. Existence of symmetric pooling equilibrium

3. Separating equilibrium

Keeping in mind that the banks cannot observe the types, can two different contracts $\{(r_L, R_L), (r_H, R_H)\}$ be designed so that the low types and the high types would accept the different contracts? First of all, the banks have to offer sufficient returns for the agents to decide that it is worthwhile depositing rather than not depositing in the bank. In addition to that, they have to offer enough for one type of agent to prefer one contract over the other. Accordingly, the participation constraints, PC_d, and the individual rationality constraints, IR_d, are derived below.

The type L agent will accept the contract (r_L, R_L) if and only if

$$\lambda_L u(r_L) + (1-\lambda_L)u(R_L) \geq 1, \qquad (PC_L)$$

and

$$\lambda_L u(r_L) + (1-\lambda_L)u(R_L) \geq$$
$$\geq \lambda_L u(r_H) + (1-\lambda_L)u(R_H). \qquad (IR_L)$$

Type H agent will accept the (r_H, R_H) contract if and only if,

$$\lambda_H u(r_H) + (1-\lambda_H)u(R_H) \geq 1, (PC_H)$$

and

$$\lambda_H u(r_H) + (1-\lambda_H)u(R_H) \geq \lambda_H u(r_L) + (1-\lambda_H)u(R_L). \quad (IR_H)$$

For the participation and individual rationality constraints to hold, because $\lambda_L < \lambda_H$, we can deduce that $r_L \leq r_H \leq R_H \leq R_L$[1].

As explained earlier the indifference curve IC_H is steeper than IC_H for any given (r, R). Therefore at any point that they cross each other, IC_L will be below IC_H to the left of that point, and IC_L will be above IC_H to the right of that point (refer to Figure 1).

Also recall that if only one group invests, the profit being zero from that group is line BE_L for the low types and line BE_H for the high types, with line BE_H being steeper than line BE_L and both going through $(1, \theta)$ which was depicted in Figure 2.

If all the banks offer two contracts $\{(r_L, R_L), (r_H, R_H)\}$ which are taken by the low types and the high types respectively can it be sustained as an equilibrium?

Lemma 3. *The points* (r_d, R_d) *where the break-even line is tangent to the utility function are given by:*

$$\left(\frac{1}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}}, \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}} \right)$$ *for the type L agent, and*

$$\left(\frac{1}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}}, \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}} \right)$$ *for the type H agent.*

Proof. It has already been shown in the proof of Proposition 1 that the slope of the break-even lines BE_L and BE_H is $-\frac{\lambda_d \theta}{1-\lambda_d}$, and the slope of the indifference curves is $-\frac{\lambda_d u'(r_d)}{(1-\lambda_d)u'(R_d)}$, where $d \in (L, H)$.

At the point of tangency,

$$-\frac{\lambda_d \theta}{1-\lambda_d} = -\frac{\lambda_d u'(r_d)}{(1-\lambda_d)u'(R_d)}. \qquad (7)$$

This gives,

$$R_d = \sqrt[(1-\alpha)]{\theta} * (r_d) \qquad (8)$$

Substituting this in the break-even line we get the tangency points:

$$r_d = \frac{1}{\lambda_d + (1-\lambda_d)\theta^{\frac{\alpha}{1-\alpha}}}; R_d = \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_d + (1-\lambda_d)\theta^{\frac{\alpha}{1-\alpha}}} \qquad (9)$$

Therefore, the tangency points are

$$\left\{ \left(\frac{1}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}}, \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}} \right); \left(\frac{1}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}}, \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}} \right) \right\}. \qquad (10)$$

[1] For those not hit by the liquidity shock to wait till t_2, it should be that $r_L \leq R_L$ and $r_H \leq R_H$. If $r_L \geq r_H$ the high types will prefer the low type contract.

Next we have Lemma 4, which is about the tangency points.

Lemma 4. *Both the tangency points would be to the left of O.*

Proof. Since $\theta > 1$, $\lambda_d + (1-\lambda_d)\theta^{\frac{\alpha}{1-\alpha}} > 1$, where $d \in (L, H)$.

Therefore,

$$\frac{1}{\lambda_d + (1-\lambda_d)\theta^{\frac{\alpha}{1-\alpha}}} < 1, \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_d + (1-\lambda_d)\theta^{\frac{\alpha}{1-\alpha}}} > \theta.$$

Therefore, the slopes of the indifference curves and break-even lines are such that the tangency points will be to the left of *O* where $r_L, r_H < 1$ and $R_L, R_H > \theta$.

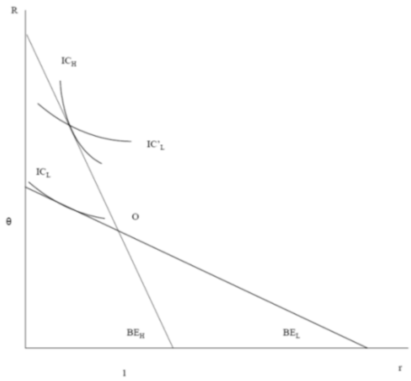

Fig. 8. Tangency points

Figure 8 and Lemma 5 below show that the tangency points cannot be sustained as a separating equilibrium. This is because the low type agents will be better off pretending to be high types.

Lemma 5. *The tangency points are such that compared to type L, type H has higher early return as well as higher late returns.*

Proof. Recall the tangency points for type *L* $\left(\frac{1}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}}, \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}}\right)$ and for type *H* $\left(\frac{1}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}}, \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}}\right)$.

Even though $\lambda_H > \lambda_L$ and $(1-\lambda_L) > (1-\lambda_H)$ we know that $\theta^{\frac{\alpha}{1-\alpha}} > 1$.

So $\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}} > \lambda\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}$.

Therefore,

$$\frac{1}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}} > \frac{1}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}};$$

$$\frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_H + (1-\lambda_H)\theta^{\frac{\alpha}{1-\alpha}}} > \frac{\theta^{\frac{1}{1-\alpha}}}{\lambda_L + (1-\lambda_L)\theta^{\frac{\alpha}{1-\alpha}}}.$$

The tangency points, therefore, cannot constitute a separating equilibrium. However in the next Proposition, we prove the existence of a separating equilibrium as long as the proportion of low type agents is sufficiently high.

Proposition 3. *If banks offer two different contracts, there exists an equilibrium* $\left\{\left(r_L^*, R_L^*\right), \left(r_H^*, R_H^*\right)\right\}$, *where the two types accept different contracts so long as there is a sufficient proportion of low type agents.*

Proof. First of all, the contracts should be such that from each type the bank makes zero profit. Otherwise any bank can offer a slightly better deal and attract all the customers. So, (r_L^*, R_L^*) and $\left(r_H^*, R_H^*\right)$ should be on the break-even lines BE_L and BE_H, respectively.

For the individual rationality constraints to hold, the indifference curves should be such that, $IC_L(r_H, R_H) \leq IC_L(r_L, R_L)$ and $IC_H(r_L, R_L) \leq IC_H(r_H, R_H)$.

This means not only should we have $r_L \leq r_H \leq R_H \leq R_L$, but also, $\frac{1-\lambda_H}{\lambda_H} \leq \frac{u(r_H) - u(r_L)}{u(R_L) - u(R_H)} \leq \frac{1-\lambda_L}{\lambda_L}$ [1].

So, for the individual rationality constraints to be satisfied, the contract points $\{(r_L^*, R_L^*), (r_H^*, R_H^*)\}$ should be sufficiently far apart, but not too much.

The existence of a separating equilibrium is illustrated in Figure 9.

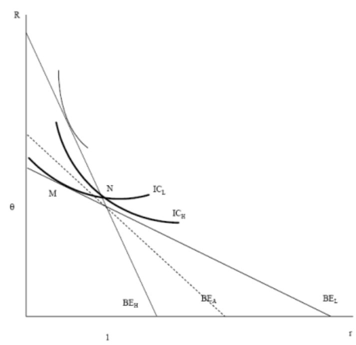

Fig. 9. Existence of a separating equilibrium

We know that the tangency points are such that the type *L*s both returns are lower. Therefore, we fix one of the contracts (r_L^*, R_L^*) as the low type's tangency point, *M*. The other contract $\left(r_H^*, R_H^*\right)$ is the point *N*, where the IC_L which is tangent to BE_L, cuts the BE_H line.

Then there is no incentive for a bank to deviate as long as BE_A is always below the IC_N that goes through *N*. Now, if you move to a point that makes the high types

[1] $\lambda_L u(r_L) + (1-\lambda_L)u(R_L) \geq \lambda_L u(r_H) + (1-\lambda_L)u(R_H)$;
$\lambda_H u(r_H) + (1-\lambda_H)u(R_H) \geq \lambda_H u(r_L) + (1-\lambda_H)u(R_L) - \lambda\{H\})u(R\{L\})$.

better off, the low types will also be attracted. Since any such point is above BE_A, it giving a loss to the bank. This would be so if the low type agents are high enough so that BE_A is sufficiently flat.

Figure 10 below shows that when the low types are not sufficiently high, BE_A is steeper so that a bank can deviate to make profit. If it offers a contract in the shaded area above IC_H, but below BE_A, both types will take that contract, making the deviant bank better off.

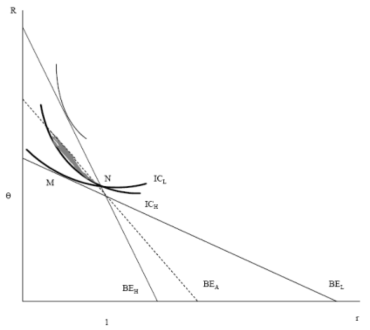

Fig. 10. Non-existence of a separating equilibrium

Therefore, we can sustain a separating equilibrium if we have a sufficient proportion of the low types.

This finding is in line with that in the standard literature on screening two types of agents where separating equilibrium can be sustained under certain conditions. Wilson (1977), Rothschild and Stiglitz (1976), Mas-Colell, Whinston and Green (1995). It is crucial in our model of bank returns that both the variables r and R affect the payoff functions of the agents and principal (the depositors and the banks). In the labor market models one of the variables (education) is just a screening devise that affects only the agents' indifference curves. We are also able to discuss what happens when only one variable affects the payoff functions.

We go further in this model to find something interesting for the banking industry. Because depositors face the possibility of being hit by liquidity shocks, the bank gives them insurance in the form of early returns. Sharing of risk mean that the early withdrawers get some interest which is shared by those who were not hit by the liquidity shock. It is worth noting that the equilibrium contracts never give early returns more than one – i.e., no interest for early withdrawers.

We already know from Proposition 2, for a symmetric pooling equilibrium to be sustained, early return of $r = 1$ is given, provided the low types will never withdraw early. In a separating equilibrium, the low types will always receive less than what he invested (i.e. $r_L < 1$). However, early return of one, is given to the high types $(r_H = 1)$ provided they will definitely withdraw early or when the low types will never withdraw early.

Higher the probability of being early withdrawers, higher the early return. Also when the low type's probability of early withdrawal is lower, it is easier to give a higher early return to the high type without having to worry about the low type preferring the high type's contract. This is summarized in the next Proposition.

Proposition 4. *Early return of* $r_H^* = 1$ *is given to the high types, only when they are sure to withdraw early* $\lambda_H = 1$ *or when the low types will never withdraw early* $(\lambda_L = 0)$.

Proof. This is shown diagrammatically.

If $\lambda_H = 1$ where the high type agents will withdraw early for sure, we have only the early return r affecting the high type functions. The line BE_H and the IC_H would then be vertical. This is illustrated in Figure 11. Then we can have $\left(r_L^*, R_L^*\right)$, where the IC_L is tangent to line BE_L at point M'. The high types should be offered contract $\left(r_H^*, R_H^*\right)$ given by any point on line BE_H that is below N' where the tangent IC_L intersects the vertical BE_H (any point on the dark line). Then the bank cannot profitably deviate. Therefore, we can sustain a separating equilibrium if $\lambda_H = 1$ with $r_H^* = 1$.

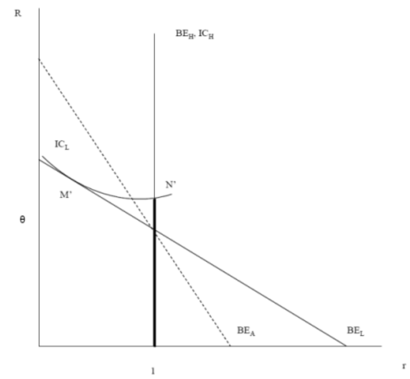

Fig. 11. Existence of separating equilibrium when $\lambda_H = 1$

In Figure 12 we show the existence of a separating equilibrium with $r_H^* = 1$ for the high types when the low types will definitely not withdraw early $\lambda_H = 0$.

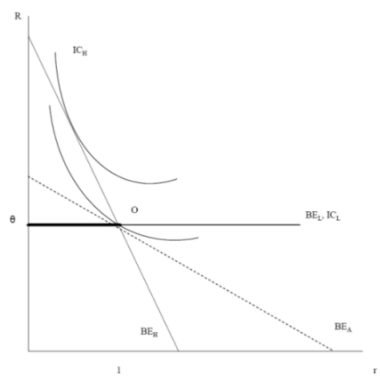

Fig. 12. Separating equilibrium when $\lambda_H = 0$

This means that the IC_L and BE_L will be horizontal. So we can sustain an equilibrium where $\left(r_H^*, R_H^*\right)$ is at point O, $(1, \theta)$ and $\left(r_L^*, R_L^*\right)$ is at any point on the horizontal BE_L to the left of O – i.e. $r_L^* < 1$, $R_L^* = \theta$.

In this case, giving a higher early return is not going to lure the low type of agents to the high type contract. This makes it possible for the banks to offer $r_H^* = 1$ to the high types.

Conclusion

This paper looked at a principal-agent model where we have two types of agents. The types are distinguished by whether the agents, who are the depositors of banks, have high or low probability of being hit by a liquidity shock and withdraw early. The agents have private information of their type and the banks which are in competition, design contracts with short-term and long-term interest rates which can be chosen by the agents.

The risk averseness of the agents, together with the competition in the market and having both the variables affecting the payoffs make it difficult to sustain equilibrium.

We have established the existence of separating equilibrium where the two types would take two different contracts offered by the bank, provided the proportion of the low type agents is large enough.

A symmetric pooling equilibrium where all the banks offer just the one contract can be sustained as an equilibrium only when the low type agents have probability of zero of withdrawing early and that proportion is large enough.

This paper contributes to the strand of literature which explores applications of contracts to be offered when the principal faces an adverse selection problem, and the conditions under which pooling and separating equilibria can be established.

From the findings of this paper some policy implications can be drawn for the banking industry. Long-term investments earn more money for the bank, and thus it would be fair if patient depositors are rewarded, especially if they can indicate their type by choosing the appropriate contract. Policy makers should encourage different contracts being offered. However, our findings show hard it is for an equilibrium outcome. If the proportion of low types is not high enough, one can expect these changes to be more frequent. The banking industry therefore, should be prepared for menus of such contracts to be updated by banks from time to time. This means depositors should be alert to changing contracts and to respond by transferring to the bank that offers the best deal for them. The regulator should be aware of this problem and make sure depositors are notified in a clear and timely fashion. Furthermore, the government can earn some revenue by making a charge on each change in contract. Even if it is a small charge to an individual, it will be high on the whole.

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