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ASYMPTOTIC INVESTIGATION OF THE STRIP'S REINFORCED BY RIBS STRESS-STRAIN STATE

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АНОТАЦІЯ: Розглянута розрахункова модель плоскої деформації ребристої смуги. Модель містить швидко змінні коефіцієнти, які відображають вплив ребер. Асимптотичний розв'язок отримано за допомогою узагальнених функцій та теоріі гомогенізаціі.

АННОТАЦИЯ: Рассмотрена расчетная модель плоской деформации ребристой полосы. Модель содержит быстро изменяющиеся коэффициенты, отражающие влияние ребер. Асимптотическое решение получено при помощи теорий обобщенных функций и осреднения.

ABSTRACT: A model describing plane deformation of a reinforced strip is considered. The model contains fast oscillating coefficients due to the ribs. An asymptotic procedure based on distributional approach and homogenization theory is applied to the model.

KEY WORDS: rib, strip, plane theory of elasticity, asymptotic, homogenization.

INTRODUCTION

Let consider the stress-strain state of the strip reinforced by ribs (Fig. 1). This problem is usual for Civil Engineering and for the theory of composite materials (the plane matrix reinforced by fibers). Correct calculation of the stress-strain state of ribbed strip is important as by oneself so from the point of view the correct determination of the pre-buckling state in the stability problems. No wonder that indicated problem attracted draw attention of researches. One dwells on analytical methods because we apply its further.

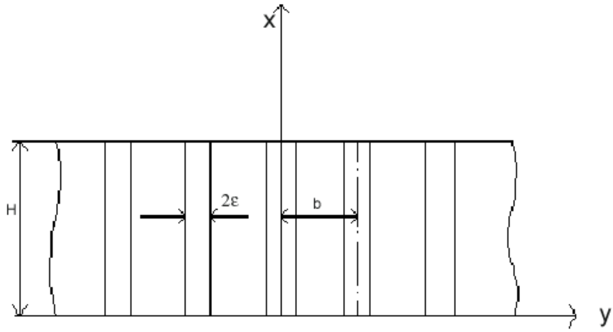

Fig. 1. Reinforced strip

One of the simplest schemes is the next: ribs work only on tension-compression, but the strip (matrix) work only in shear [1]. Such model permitting to catch many characteristic features is nevertheless to rough from the mechanics of solids standpoint.

The method based on expansion with respect to geometrically-rigid parameters [2, 3] for strong anisotropic medium is extremely effective one, but its accuracy is bad in the isotropic case.

The theory of analytical function methods [4] lead to necessity to solve infinite system of coupled algebraic equations. As a rule, the closed form solutions (in combination with the integral transformations and methods of the integral equations theory) succeeded obtains only for the stringer [5].

The exact solution of periodical problems in double trigonometric series is possible only under some special boundary conditions [6].

The homogenization method [6 - 10] permits to obtain a simple analytical solution for one-dimensional ribs, but its generalization for a two-dimensional ribs reserve the sense of dissatisfaction.

The homogenization method in modification [7, 10] is used in this paper for receiving the analytical solution and asymptotic method based on the expansion with using distributions [11] is used for taking into account the ribs width.

The remaining part of the paper is organized as follows. In Section 2, we present governing relations. In Section 3, we investigate influence of rib width. In Section 4, we carry out a homogenization procedure for reinforced strip with one-dimension ribs. In Section 5, we construct boundary layer. Finally, we discuss the results in Section 6.

GOVERNING RELATIONS

Input equations for matrix anisotropy, whose main directions coincide with Cartesian co-ordinates may be written in the following form (when mass forces are absent)

$$[B_{11}+B_0\Phi_0(y)]\frac{\partial^2 u}{\partial x^2} + B_{33}\frac{\partial^2 u}{\partial y^2}+(B_{33} + B_{12})\frac{\partial^2 v}{\partial x\partial y} = 0$$

$$B_{22}\frac{\partial^2 v}{\partial x^2} + B_{33}\frac{\partial^2 v}{\partial y^2}+(B_{33} + B_{12})\frac{\partial^2 u}{\partial x\partial y} = 0 \qquad (1)$$

Here B_{ij} are rigid parameters those characterized the matrix, i = 1, 2; B_0 is ribs rigidity on tension – compression;

$\Phi_0(y) = \sum_{k=-\infty}^{\infty}[H(y + kb - \varepsilon) - H(y - kb + \varepsilon)]$, H(…) is the Heaviside function.

Bending ribs rigidity does not take into account when equations (1) are deriving because this magnitude is small in comparison with rigidities on tension-compression. The boundary conditions may be written as follows

$$\text{For } x = 0,\ H \quad v = 0 \qquad (2)$$

$$[B_{11}+B_0\Phi_0(y)]\frac{\partial u}{\partial x} = P(y)\Phi_0(y) \qquad (3)$$

In this scheme solution's singularities in the place of contact between the ribs and the strip are absent [5].

PASSING TO THE MODEL WITH ONE-DIMENSIONAL RIBS

Let's first note the ribs width. One can use parameter 2ε as small one and expand the function $\Phi_0(y)$ in powers of ε considering that ribs are thin. We assume that B_{11}, B_{22}, B_{12}, B_{33} are O (1). Let's achieve this expansion on example of function $\varphi(y) = H(x + \varepsilon) – H(x - \varepsilon)$ [11].

Using double-side Laplace transformation one obtains

$$\bar{\varphi}(p) = \frac{\exp(-\varepsilon p) - \exp(\varepsilon p)}{p}$$

Here p is Laplace transformation parameter.

After expansion function $\bar{\varphi}(p)$ in powers of ε one has

$$\bar{\varphi}(p) = 2\varepsilon - \sum_{n=1}^{\infty}(-1)^n \varepsilon^{n+1} p^n /(n+1)!$$

As result of inverse Laplace transformation (justifying in the framework of distributions theory) one obtains

$$\varphi(y) = 2\varepsilon\delta(y) - \sum_{n=1}^{\infty} \varepsilon^{n+1}\delta^{(n)}(y)$$

Here $\delta(x)$ is Dirac delta-function, $\delta^{(n)}(x)$ is its n-th derivative.

Hence function $\Phi_0(y)$ may be written as following expression

$$\Phi_0(y) = 2\varepsilon\Phi(y) + 2\varepsilon \sum_{n=1}^{\infty} \varepsilon^{n}\Phi^{(n)}$$

Here $\Phi(y) = \sum_{k=-\infty}^{\infty} \delta(y-kb)$.

Now the input boundary value problem (1) – (3) solution may be presented in the following form

$$u = u_0 + \varepsilon u_1 + \ldots$$
$$v = v_0 + \varepsilon v_1 + \ldots \qquad (4)$$

After splitting one obtains following recurrent system of boundary value problems

$$L_1(u_0, v_0) = 0 \qquad (5)$$
$$L_2(u_0, v_0) = 0 \qquad (6)$$

$$L_1(u_1, v_1) = -0.5B_1b\frac{\partial^2 u_0}{\partial x^2}\Phi'(\eta_1)$$

$$L_2(u_1, v_1) = 0$$

$$L_1(u_n, v_n) = -0.5\, B_1b\, \frac{\partial^2 u_j}{\partial x^2}\Phi^{(n-1)}(\eta_1)$$

$$L_2(u_n, v_n) = 0$$

$$\text{for } x = 0, H \quad v_i = 0, 1, 2, \ldots \qquad (7)$$

$$L_3(u_1) = 0.5[-bB_1\frac{\partial u_0}{\partial x} + P_1b^{-1}]\Phi'(\eta_1) \qquad (8)$$

$$L_3(u_n) = -0.5bB_1 \sum_{i=0}^{n-1} \frac{\partial u_j}{\partial x}\Phi^{(n-1)}(\eta_1) + P_1b^{-n}]\Phi^{(n)}(\eta_1)$$

Here

$$L_1(u_0, v_0) = [B_{11} + B_1\Phi(\eta_1)]\frac{\partial^2 u_0}{\partial x^2} + B_{33}\frac{\partial^2 u_0}{\partial y^2} + (B_{33} + B_{12})\frac{\partial^2 v_0}{\partial x \partial y}$$

$$L_2(u_0, v_0) = B_{22}\frac{\partial^2 v_0}{\partial x^2} + B_{33}\frac{\partial^2 v_0}{\partial y^2} + (B_{33} + B_{12})\frac{\partial^2 u_0}{\partial x \partial y}$$

$$L_3(u_0) \equiv [B_{11} + B_1\Phi(\eta_1)]$$

$B_1 = 2\varepsilon B_0/b$; $\eta_1 = y/b$; $P_1 = P(y)/b$

We assume $B_1 \sim B_{11}$.

HOMOGENIZATION PROCEDURE

Now let's pass to construction of homogenized relations. One shall use the two-scale method [8, 9]. We introduce some evaluation. We consider the strip is quite wide (H >>2b) and the external loading is changing slow from rib to rib. We use the ratio $\varepsilon_1 = b/H$ as a small parameter and suppose $\varepsilon_1 >> \varepsilon$. Let's write the system of equilibrium equation (5), (6) in form:

$$B_{11}\frac{\partial^2 u_0}{\partial x^2} + B_{33}\frac{\partial^2 u_0}{\partial y^2} + (B_{33} + B_{12})\frac{\partial^2 v_0}{\partial x \partial y} = 0$$

$$L_2(u_0,v_0) = 0 \tag{9}$$

$$bk < y < b(k + 1), \quad k=0, \pm 1, \ldots$$

$$\{u_0^+, v_0^+, \frac{\partial v_0^+}{\partial y}\} = \{u_0^-, v_0^-, \frac{\partial v_0^-}{\partial y}\}$$

$$B_{11}(\frac{\partial u_0^+}{\partial y} - \frac{\partial u_0^-}{\partial y}) = bB_1\frac{\partial^2 u_0^+}{\partial x^2} \tag{10}$$

where $(\ldots)^{\pm} = \lim\limits_{y \to bk \pm 0}(\ldots)$

We introduce non-dimensional variables $\xi = x/H$; $\eta = y/H$. Then the relations (9), (10) may be written as follows

$$L_{11}(u_0,v_0) = 0$$

$$L_{12}(u_0,v_0) = 0 \tag{11}$$

$$\varepsilon_1 k < y < \varepsilon_1(k + 1)$$

$$\{u_0^+, v_0^+, \frac{\partial v_0^+}{\partial y}\} = \{u_0^-, v_0^-, \frac{\partial v_0^-}{\partial y}\} \tag{12}$$

$$B_{11}(\frac{\partial u_0^+}{\partial \eta} - \frac{\partial u_0^-}{\partial \eta}) = \varepsilon B_1\frac{\partial^2 u_0}{\partial \xi^2} \tag{13}$$

where $(\ldots)^{\pm} = \lim\limits_{\eta \to \varepsilon_1 k \pm 0}(\ldots)$

$$L_{11}(u_0,v_0) \equiv B_{11}\frac{\partial^2 u_o}{\partial \xi^2} + B_{33}\frac{\partial^2 u_o}{\partial \eta^2} + (B_{33} + B_{12})\frac{\partial^2 v_o}{\partial \xi \partial \eta}$$

$$L_{12}(u_0,v_0) \equiv B_{22}\frac{\partial^2 v_o}{\partial \xi^2} + B_{33}\frac{\partial^2 v_o}{\partial \eta^2} + (B_{33} + B_{12})\frac{\partial^2 u_o}{\partial \xi \partial \eta}$$

According to the two-scale method, one introduces the "fast" variable $\eta_1 = y/b$, notation for the "slow" one remainsη. Then

$$\frac{\partial}{\partial \eta} = \frac{\partial}{\partial \eta} + \varepsilon_1^{-1}\frac{\partial}{\partial \eta_1} \tag{14}$$

Further the solution will be looked for in the form of the following expansions:

$$u_0 = u^{(0)} + \varepsilon_1^2 u^{(1)} + \varepsilon_1^3 u^{(2)} + \ldots$$

$$v_0 = v^{(0)} + \varepsilon_1^2 v^{(1)} + \varepsilon_1^3 v^{(2)} + \dots \tag{15}$$

for $u^{(0)} \equiv u^{(0)}(\xi,\eta)$, $v^{(0)} \equiv v^{(0)}(\xi,\eta)$

$u^{(i)} \equiv u^{(i)}(\xi,\eta,\eta_1)$

$$v^{(i)} \equiv v^{(i)}(\xi,\eta,\eta_1),\ i = 1,2,\dots \tag{16}$$

$u^{(i)}(\xi,\eta,\eta_1+1) = u^{(i)}(\xi,\eta,\eta_1)$

$$v^{(i)}(\xi,\eta,\eta_1+1) = v^{(i)}(\xi,\eta,\eta_1) \tag{17}$$

Using the relations (11) – (17), after splitting with respect to ε_1 one obtains:

$$\frac{\partial^2 u^{(1)}}{\partial \eta_1^2} = -L_{11}(u^{(0)}, v^{(0)}) \tag{18}$$

$$\frac{\partial^2 v^{(1)}}{\partial \eta_1^2} = -L_{12}(u^{(0)}, v^{(0)}) \tag{19}$$

Equations (18), (19) are easily integrated leading to

$$u^{(1)} = -\frac{\eta_1^2}{2} L_{11}(u^{(0)}, v^{(0)}) + C_1(\xi,\eta)\eta_1 + C_2(\xi,\eta)$$

$$v^{(1)} = -0.5\eta_1^2 L_{12}(u^{(0)}, v^{(0)}) + C_3(\xi,\eta)\eta_1 + C_4(\xi,\eta)$$

Conditions (12), (13) may be written as follows:

$$\{u^{(1)}, v^{(1)}, \frac{\partial v^{(1)}}{\partial \eta_1}\}\Big|_{\eta_1=0} = \{u^{(1)}, v^{(1)}, \frac{\partial v^{(1)}}{\partial \eta_1}\}\Big|_{\eta_1=1} \tag{20}$$

$$B_{11}\left(\frac{\partial u^{(1)}}{\partial \eta_1}\Big|_{\eta_1=1} - \frac{\partial u^{(1)}}{\partial \eta_1}\Big|_{\eta_1=0}\right) = B_1 \frac{\partial^2 u^{(o)}}{\partial \xi^2} \tag{21}$$

Constants C_1, C_2 are not determined from the conditions (20), (21) (they have to be related to the next approximation). One obtains the following equation from conditions (20)

$$L_{12}(u^{(0)}, v^{(0)}) = 0 \tag{22}$$

and $C_3 = 0$, $C_1 = L_{11}(u^{(0)}, v^{(0)})$.

One obtains the homogenized equation satisfying the conditions (21):

$$L_{11}(u^{(0)}, v^{(0)}) + B_1 \frac{\partial^2 u^{(o)}}{\partial \xi^2} = 0 \tag{23}$$

Equations (22), (23) constitute the looked for homogenized system of equations.

The first "fast" correction may be written as follows:

$$u_1 = \eta_1(0.5 - \eta_1) B_1 \frac{\partial^2 u^{(o)}}{\partial \xi^2}$$

$$v_1 = 0$$

Let us analyze the boundary conditions (7), (8). From boundary conditions (7) one can easily obtained

$$\text{for } \xi=0,1 \quad v^{(0)} = 0 \tag{24}$$

After homogenization relations (8) one obtains

$$\text{for } \xi=0,1, \quad (B_{11} + B_1)\frac{\partial u^{(o)}}{\partial \xi} = P_1(\eta) \tag{25}$$

where $P_1(\eta) = P_1H/b$.

Equations (22), (23) and the boundary conditions (24) are formed the homogenized boundary value problem.

BOUNDARY LAYER

If we use only solution (15) the boundary conditions (8) are satisfied in average. As a result, the discrepancies are self-balanced on parts $k-1 < \eta_1 < 1$, $k = 0, \pm1,\ldots$. Hence, the stress-strain state corresponding to this fictitious loading is concentrated near the strip edges. In other words, we have to take into account that the stress-state near the boundary is of the boundary layer type.

Let's introduce the new "fast" variable $\xi_1 = \xi/\varepsilon_1$, the "slow" one being still ξ. Then

$$\frac{\partial}{\partial \xi} = \frac{\partial}{\partial \xi} + \varepsilon_1^{-1}\frac{\partial}{\partial \xi_1} \tag{26}$$

We will look for the solution as the boundary layer in the form:

$$u_b = \varepsilon_1 u_b^{(0)}(\xi,\xi_1,\eta,\eta_1) + \varepsilon_1^2 u_b^{(2)}(\xi,\xi_1,\eta,\eta_1) + \ldots \tag{27}$$

$$v_b = \varepsilon_1 v_b^{(0)}(\xi,\xi_1,\eta,\eta_1) + \varepsilon_1^2 v_b^{(2)}(\xi,\xi_1,\eta,\eta_1) + \ldots$$

Substituting the relation (26) and the expansions (27) into the relations (9), (10) after splitting with respect to powers of ε, one obtains in the first approaching

$$B_{11}\frac{\partial^2 u_b^{(o)}}{\partial \xi_1^2} + B_{33}\frac{\partial^2 u_b^{(o)}}{\partial \eta_1^2} + (B_{33} + B_{12})\frac{\partial^2 v_b^{(o)}}{\partial \xi_1 \partial \eta_1} = 0$$

$$B_{22}\frac{\partial^2 v_b^{(o)}}{\partial \xi_1^2} + B_{33}\frac{\partial^2 v_b^{(o)}}{\partial \eta_1^2} + (B_{33} + B_{12})\frac{\partial^2 u_b^{(o)}}{\partial \xi_1 \partial \eta_1} = 0 \tag{28}$$

for $\eta_1 = k$, $k = 0, \pm1,\ldots.$

$$B_{11}\left(\frac{\partial u_b^{(o)+}}{\partial \eta_1} - \frac{\partial u_b^{(o)-}}{\partial \eta_1}\right) = B_c\frac{\partial^2 u_b^{(o)}}{\partial \xi^2}\, v_b^{(0)} = 0 \tag{29}$$

$$\text{for } \xi_1 = 0, \varepsilon_1^{-1} \quad v_b^{(0)} = 0 \tag{30}$$

$$B_{11}\frac{\partial u_b^{(0)}}{\partial \xi_1} = -\varepsilon_1 P_1(\eta) \tag{31}$$

$$\text{for } \xi_1 \to \infty \quad u_b^{(0)},\ v_b^{(0)} \to 0 \tag{32}$$

$$\text{for } \xi_1 \to \infty \quad u_b^{(0)},\ v_b^{(0)} \to 0 \tag{33}$$

It is clear that one can solve the problem for one period (for k = 0, 1) and not go belong the boundary conditions (29) - (33).

So one obtains strip clamped on the long sides. Quantity of references devoted to this problem are too large that why one not goes belong enumeration of approaching which can be used in this case.

CONCLUDING REMARKS

We developed analytical methodologies for computing of the stress-strain state in the reinforced strip. Homogenization procedure allowed us to obtain global solution with local correctors. Proposed asymptotic procedure can be applied for plane composites with fiber inclusions.

On the other hand, one of the important problems is the accuracy of the proposed approach. Sometimes for this aim one can use numerical methods or experimental results.

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