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INFLUENCE OF INTRASYSTEM PERTURBATIONS ON THE SPATIAL FREQUENCIES RESOLUTION IN THE CAPON AND MAXIMUM ENTROPY ALGORITHMS

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ВПЛИВ ВНУТРІШНЬОСИСТЕМНИХ ЗБУРЕНЬ НА РОЗДІЛЕННЯ ПРОСТОРОВИХ ЧАСТОТ В АЛГОРИТМАХ КЕЙПОНА І МАКСИМАЛЬНОЇ ЕНТРОПІЇ

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ВЛИЯНИЕ ВНУТРИСИСТЕМНЫХ ВОЗМУЩЕНИЙ НА РАЗРЕШЕНИЕ ПРОСТРАНСТВЕННЫХ ЧАСТОТ В АЛГОРИТМАХ КЕЙПОНА И МАКСИМАЛЬНОЙ ЭНТРОПИИ

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Annotation. *The method of studying the class of inverse problems with the inversion of the estimate of the correlation matrix of observations was further developed. Analytical expressions are obtained that reflect the behavior of the spectral functions for the Capon and maximum entropy algorithms under intrasystem uncertainty conditions. The area of the correct representation of the problem being solved. The spectral reliefs are constructed, allowing to estimate the influence of disturbances of the correlation matrix with different conditionality on the spatial frequencies resolution of the system.*

Keywords: *spatial frequency, spectral function, Capon algorithm, maximum entropy (ME) algorithm, intrasystem perturbations, resolution, correlation matrix, parametric vector, spectral relief.*

Анотація. *Отримав подальший розвиток метод дослідження класу зворотних задач із інверсією оцінки кореляційної матриці спостережень. Отримані аналітичні вирази, що відображають поведінку спектральних функцій для алгоритмів Кейпона і максимальної ентропії в умовах внутрісистемної невизначеності. Визначена область коректного представлення завдання, що розв'язується. Побудовані спектральні рельєфи, що дозволяють оцінити вплив збурень кореляційної матриці з різною обумовленістю на роздільну здатність системи по просторових частотах.*

Ключові слова: *просторова частота, спектральна функція, алгоритм Кейпона, алгоритм максимальної ентропії, внутрісистемні збурення, розподільна здатність, кореляційна матриця, параметричний вектор, спектральний рельєф.*

Аннотация. *Получил дальнейшее развитие метод исследования класса обратных задач с инверсией оценки корреляционной матрицы наблюдений. Получены аналитические выражения, отображающие поведение спектральных функций для алгоритмов Кейпона и максимальной энтропии в условиях внутрисистемной неопределенности. Определена область корректного представления решаемой задачи. Построены спектраль-*

ные рельефы, позволяющие оценить влияние возмущений корреляционной матрицы с различной обусловленностью на разрешающую способность системы по пространственным частотам.

Ключевые слова: *пространственная частота, спектральная функция, алгоритм Кейпона, алгоритм максимальной энтропии, внутрисистемные возмущения, разрешающая способность, корреляционная матрица, параметрический вектор, спектральный рельеф.*

FORMULATION OF THE PROBLEM. ANALYSIS OF STUDIES AND PUBLICATIONS

The solution of a wide class of space-time signal processing problems in applications of optical, radar, sonar, technical diagnostics, communications and radio navigation involves the use of spectral analysis methods to spatial frequencies resolution of the power spectrum of stochastic radiation. Frequency resolution, being a qualitative characteristic of spectral analysis, determines the degree of detail of spectrum studies [1-3].

Under conditions of localization of point radiation sources within the Rayleigh interval, alternatives to classical spectral analysis are algorithms based on calculating the inversion of the maximum likelihood (ML) estimate of the correlation matrix of samples of a multidimensional process. Such a computational procedure is basic for the Capon and maximum entropy (ME) algorithms, which, under ideal conditions, make it possible to obtain asymptotic spectral functions with sharp peaks in the direction to point radiation sources [4-6].

Physically, such restrictions are determined by the level of irremovable intrasystem perturbations generated by internal noise, limited accuracy of the calculations, inadequacy of direct and inverse transformations and the absence of isomorphism in the real system [8, 9]. These factors, regardless of the nature of origin and various kinds of detailed assumptions, generate intrasystem uncertainty, under which stochastic changes of parameters in Capon and ME algorithms are accompanied by non-deterministic variations in the result of solving a computational problem. These circumstances actualize the problem of studying the influence of intrasystem perturbations on the spatial frequencies resolution in Capon and ME algorithms.

The well-known solution of this computational problem [2, 4, 6, 7], based on the methods of adaptive spatial filtering, involves the construction of spectral functions according to Capon $f_C(\Theta)$ or ME algorithms $f_{ME}(\Theta)$, respectively:

$$f_C(\Theta) = \frac{1}{\mathbf{V}(\Theta)^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{V}(\Theta)}, \quad (1)$$

$$f_{ME}(\Theta) = \frac{\mathbf{g}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{g}}{\left|\mathbf{g}^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{V}(\Theta)\right|^2}, \quad (2)$$

where Θ – spatial frequency argument $\omega_x(\Theta)$: $\omega_x(\Theta) = 2\pi\sin\Theta/\lambda$, λ – wavelength; $\mathbf{A}^{-1}$ – inverse correlation matrix of observations; $\mathbf{V}(\Theta)$ – non-random complex *N*-dimensional vector of unit length; $\mathbf{g}^{\mathrm{T}} = [1, 0, 0, ..., 0]$ – vector of dimension N; $[*]^{\mathrm{T}}$ – transpose and complex conjugation symbol.

Practically, spectral functions (1) and (2) are formed by scanning the search vector $\mathbf{V}(\Theta)$ of a certain sector of space, replacing the true correlation matrix of input realizations $\mathbf{A}$ with its ML estimate $\widehat{\mathbf{A}} = \mathbf{A} + \Delta\mathbf{A}$, where $\Delta\mathbf{A}$ is the matrix of intrasystem perturbations. The analysis of the characteristics of spectral functions under intrasystem perturbations is usually carried out by statistical modeling, which, because of a fixed number of experimental results, reflects particular situations, which limits the representation of the problem to be solved at a generalized (asymptotic) level. A natural approach to overcoming this limitation is the use of analytical research methods.

Objective – application of an analytical approach to the problem of estimating the influence of intrasystem perturbations on the spatial frequencies resolution in Capon and ME algorithms.

To achieve this goal, let us set the initial data model:

− the ML estimate of the $(N \times N)$-dimensional correlation matrix $\widehat{\mathbf{A}}$ is formed by measuring the Gaussian process $\mathbf{U}(N,L)$ by the algorithm $\widehat{\mathbf{A}} = \widehat{\mathbf{A}}(L) = L^{-1}\left[\mathbf{U}(N,L)\cdot \mathbf{U}^{\mathrm{T}}(L,N)\right]$, where L – the number of time discrete of observations;

− estimate of the matrix $\widehat{\mathbf{A}}(L)$ with a limited number of samples L is presented in additive form $\widehat{\mathbf{A}} = \widehat{\mathbf{A}}(L) = \mathbf{A} + \Delta\mathbf{A}(L)$, where $\Delta\mathbf{A}(L)$ – Hermitian random matrix of intrasystem perturbations. Under condition $L >> N$, the estimate of the matrix $\widehat{\mathbf{A}}(L)$ is nondegenerate and has an asymptotic boundary $\lim\limits_{L\to\infty} \widehat{\mathbf{A}}(L) = \mathbf{A}$;

− the statistical moments of the matrix of intrasystem perturbations are equal to

$$\mathrm{M}\{\Delta\mathbf{A}\} = \lim_{L\to\infty}\left\{L^{-1}\Delta\mathbf{A}(L)\right\} = \mathbf{0}(N,N),$$

$$\mathrm{M}\left\{\Delta\mathbf{A}\cdot\Delta\mathbf{A}^{\mathrm{T}}\right\} = \lim_{L\to\infty}\left\{L^{-1}\left[\Delta\mathbf{A}(L)\Delta\mathbf{A}^{\mathrm{T}}(L)\right]\right\} = \sigma_{\mathbf{A}}^2\|\mathbf{A}\|^2\cdot\mathbf{I}(N,N),$$

where $\mathrm{M}\{*\}$ – statistical averaging operator; $\mathbf{0}(*)$ and $\mathbf{I}(*)$ – zero and unit matrices; $\sigma_{\mathbf{A}}^2 = \|\Delta\mathbf{A}\|^2 / \|\mathbf{A}\|^2$ – dispersion of intrasystem perturbations of the matrix, here $\|*\|$ – norm of matrix;

− search vector $\mathbf{V}(\Theta)$ is represented by deterministic complex functions, the argument of which depends on the spatial frequency:

$$\mathbf{V}^{\mathrm{T}}(\Theta) = \left[1 \;\vdots\; \mathbf{r}^{\mathrm{T}}(\Theta)\right],$$

where $\mathbf{r}^{\mathrm{T}}(\Theta) = \left[e^{j\Delta\varphi(\Theta)}\, e^{j2\Delta\varphi(\Theta)} \ldots e^{j(N-1)\Delta\varphi(\Theta)}\right]$. Here $\Delta\varphi(\Theta) = \omega_x(\Theta)\,\Delta X$, ΔX – the distance between the phase centers of isotropic sensors located along the *x* axis of the three-dimensional space $R(x,y,z)$.

Based on the accepted model of statistical moments of intrasystem perturbations, we estimate their influence on the degree of deformation of the spectral functions for the Capon (1) and ME algorithms (2). In this context, we approximate the inverse estimate of the correlation matrix $\widehat{\mathbf{A}}^{-1}$ by the matrix power series:

$$\widehat{\mathbf{A}}^{-1} = (\mathbf{A} + \Delta\mathbf{A})^{-1} = \mathbf{A}^{-1}\sum_{i=0}^{\infty}(-1)^{i}\cdot\left(\Delta\mathbf{A}\cdot\mathbf{A}^{-1}\right)^{i}. \tag{3}$$

To determine the conditions of convergence of the series (3), we use the approximation matrix $\mathbf{E} = \widehat{\mathbf{A}}^{-1} - \mathbf{A}^{-1}$. With a limited number of terms of series $i = \overline{1,\nu}$ and taking into account the Cauchy-Bunyakovsky inequality, the approximation norm $\mathbf{E}$ can be represented as

$$\|\mathbf{E}\| = \left\|\widehat{\mathbf{A}}^{-1} - \mathbf{A}^{-1}\right\| \le \sum_{i=1}^{\nu}\left\|\Delta\mathbf{A}\cdot\mathbf{A}^{-1}\right\|^{i} \le \sum_{i=1}^{\nu}\|\Delta\mathbf{A}\|^{i}\cdot\left\|\mathbf{A}^{-1}\right\|^{i}, \tag{4}$$

where $\sum\limits_{i=1}^{\nu}\|\Delta\mathbf{A}\|^{i}\cdot\left\|\mathbf{A}^{-1}\right\|^{i} = \dfrac{\|\Delta\mathbf{A}\|\cdot\left\|\mathbf{A}^{-1}\right\|\cdot\left(1 - \|\Delta\mathbf{A}\|^{\nu}\cdot\left\|\mathbf{A}^{-1}\right\|^{\nu}\right)}{1 - \|\Delta\mathbf{A}\|\cdot\left\|\mathbf{A}^{-1}\right\|}.$

Using (4), we define the upper limit of the approximation norm for the root-mean-square error of intrasystem perturbations $\sigma_{\mathbf{A}}$:

$$\|\mathbf{E}\| = \left\|\widehat{\mathbf{A}}^{-1} - \mathbf{A}^{-1}\right\| \le \frac{\sigma_{\mathbf{A}}\zeta_{\mathbf{A}}\left[1 - (\sigma_{\mathbf{A}}\zeta_{\mathbf{A}})^{\nu}\right]}{1 - \sigma_{\mathbf{A}}\zeta_{\mathbf{A}}}, \tag{5}$$

where $\zeta_{\mathbf{A}} = \left\| \mathbf{A}^{-1} \right\| \cdot \left\| \mathbf{A} \right\|$ – condition number of matrix $\mathbf{A}$.

It is the condition number that determines the distance to the degenerate matrix and the sensitivity of the algorithms with the inversion of the estimate of the correlation matrix to intrasystem perturbations of the original data [10, 11]. Physically, characterizing the contrast of the spectral decomposition of the correlation matrix, the condition number depends on the energy of the observed process, the sample size of the input realization, the number of radiation sources and their relative position in space [3, 4].

Subject to condition $\sigma_{\mathbf{A}}\zeta_{\mathbf{A}} << 1$ and an unlimited number of terms of series ν, expression (5) has a limit

$$\lim_{\nu \to \infty} \left\| \widehat{\mathbf{A}}^{-1} - \mathbf{A}^{-1} \right\| = \frac{\sigma_{\mathbf{A}}\zeta_{\mathbf{A}}}{1 - \sigma_{\mathbf{A}}\zeta_{\mathbf{A}}} \cong \sigma_{\mathbf{A}}\zeta_{\mathbf{A}}. \qquad (6)$$

Consequently, the permissible level of perturbations of the elements of the correlation matrix $\mathbf{A}$ is determined by inequality $\sigma_{\mathbf{A}} << 1/\zeta_{\mathbf{A}}$, which allows us to limit the matrix decomposition of inverse estimate $\widehat{\mathbf{A}}^{-1}$ in the Capon and ME algorithms to terms of the second order of smallness:

$$\widehat{\mathbf{A}}^{-1} = (\mathbf{A} + \Delta\mathbf{A})^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\Delta\mathbf{A}\mathbf{A}^{-1} + \mathbf{A}^{-1}\Delta\mathbf{A}\mathbf{A}^{-1}\Delta\mathbf{A}\mathbf{A}^{-1}. \qquad (7)$$

Let us first estimate the influence of the perturbations $\Delta\mathbf{A}$ of the correlation matrix $\mathbf{A}$ on the deformation of the Capon spectral function (1). To do this, we take into account the initial statistics of the perturbation matrix $\Delta\mathbf{A}$ and, averaging expression (7), we obtain:

$$\mathrm{M}\left\{\widehat{\mathbf{A}}^{-1}\right\} = \mathbf{A}^{-1}\left[\mathbf{I} + \left(\sigma_{\mathbf{A}}^{2} \cdot \left\|\mathbf{A}\right\|^{2} \cdot \mathrm{tr}\,\mathbf{A}^{-1}\right) \cdot \mathbf{A}^{-1}\right]. \qquad (8)$$

where $\mathrm{tr}\,\mathbf{A}^{-1}$ – spur of matrix $\mathbf{A}^{-1}$.

Dependence (8) makes it possible to write the expression for the Capon spectral function (1) with a perturbed inverse correlation matrix in the generalized form

$$\widehat{f}_C(\Theta) = \frac{1}{\mathbf{V}(\Theta)^{\mathrm{T}}\mathrm{M}\left\{\widehat{\mathbf{A}}^{-1}\right\}\mathbf{V}(\Theta)}. \qquad (9)$$

To clarify the degree of deformation of the Capon spectral function (9) under the influence of intrasystem perturbations, we define its relationship with the condition number $\zeta_{\mathbf{A}}$ of matrix $\mathbf{A}$. Given the result of averaging (8) and using the matrix inequality $\left\| \mathbf{A}^{-1}\mathbf{V}(\Theta) \right\| \le \left\| \mathbf{A}^{-1} \right\| \cdot \left\| \mathbf{V}(\Theta) \right\|$, we detail expression (9) as follows:

$$\widehat{f}_C(\Theta) = \frac{1}{\mathbf{V}(\Theta)^{\mathrm{T}}\mathbf{A}^{-1}\mathbf{V}(\Theta) + \sigma_{\mathbf{A}}^{2}\zeta_{\mathbf{A}}^{2}\, N\, \mathrm{tr}\mathbf{A}^{-1}}, \qquad (10)$$

where $N = \left\| \mathbf{V}(\Theta) \right\|^{2}$ – dimension of the adaptive system.

The form of representation (10) allows you to set the interval of allowable values of a perturbed Capon spectral function. In particular, the upper limit of function (10) is determined from condition $\sigma_{\mathbf{A}}^{2} \to 0$, and its lower limit satisfies the condition when $\mathbf{A} = P_0\,\mathbf{I}(N,N)$, where P_0 – the level of internal (thermal) system noise. Therefore, the correct values of the Capon spectral function (10) belong to interval $P_0 N^{-1} \le \widehat{f}_{\mathrm{C}}(\Theta) \le f_{\mathrm{C}}(\Theta)$ provided that $\sigma_{\mathbf{A}}^{2}\zeta_{\mathbf{A}}^{2} << 1$.

Further, following the accepted logic of reasoning, we estimate the influence of perturbations $\Delta\mathbf{A}$ of the correlation matrix $\mathbf{A}$ on the deformation of the spectral function for the ME algorithm (2). For this purpose, we divide the correlation matrix into blocks:

$$\mathbf{A}=\left[\begin{array}{c:c} A_{11} & \boldsymbol{\alpha}^{\mathrm{T}} \\ \hdashline \boldsymbol{\alpha} & \mathbf{B} \end{array}\right], \tag{11}$$

where A_{11} – element of the matrix $\mathbf{A}$; $\boldsymbol{\alpha}=\mathbf{A}(2:N,1)$ – column vector of the matrix $\mathbf{A}$ of dimension $(N-1)\times 1$; $\mathbf{B}=\mathbf{A}(2:N,2:N)$ – block of the matrix $\mathbf{A}$ of dimension $(N-1)\times(N-1)$ with condition number $\zeta_{\mathbf{B}}=\|\mathbf{B}^{-1}\|\cdot\|\mathbf{B}\|$.

Using the block entry of the matrix $\mathbf{A}$ (11) and performing the corresponding vector-matrix transformations [8, 11], we obtain the expression for the spectral function of the ME algorithm

$$f_{ME}(\Theta)=\frac{\det\ \mathbf{A}\cdot(\det\ \mathbf{B})^{-1}}{\left|\left[1\ \vdots\ \mathbf{W}^{\mathrm{T}}\right]\cdot\mathbf{V}(\Theta)\right|^{2}}, \tag{12}$$

where det – matrix determinant; $\mathbf{W}$ – parametric vector that has dimension $(N-1)\times 1$ and satisfies Wiener's solution [2-4]:

$$\mathbf{W}=-\mathbf{B}^{-1}\boldsymbol{\alpha}. \tag{13}$$

Under condition (13), the ratio of determinants $\det\mathbf{A}\cdot(\det\mathbf{B})^{-1}$ is

$$J_0=\frac{\det\mathbf{A}}{\det\mathbf{B}}=A_{11}-\boldsymbol{\alpha}^{\mathrm{T}}\mathbf{B}^{-1}\boldsymbol{\alpha},$$

which corresponds to the minimum of a positive definite quadratic form

$$J(\mathbf{W},\mathbf{W})=\left[1\ \vdots\ \mathbf{W}^{\mathrm{T}}\right]\mathbf{A}\left[1\ \vdots\ \mathbf{W}^{\mathrm{T}}\right]^{\mathrm{T}}.$$

Then, taking into account the introduced notation, the spectral function (12) can be represented as follows:

$$f_{ME}(\Theta)=\frac{J_0}{|G(\Theta)|^{2}},$$

where $G(\Theta)=\left[1\ \vdots\ \mathbf{W}^{\mathrm{T}}\right]\ \mathbf{V}(\Theta)$ – spatial transfer function of the adaptive system.

Note that, under condition (13), the zero values of function $G(\Theta)$ will be oriented towards radiation sources, and function $f_{ME}(\Theta)$ will have maximum values in these directions.

Let us study the behavior of the spectral function for the ME algorithm $f_{ME}(\Theta)$ under conditions of intrasystem uncertainty, when parametric vector $\mathbf{W}$ differs from Wiener's solution (13) by a random value $\Delta\mathbf{W}=\Delta\mathbf{W}(\Delta\mathbf{B},\Delta\boldsymbol{\alpha})$:

$$\widehat{\mathbf{W}}=\mathbf{W}+\Delta\mathbf{W}=(\mathbf{B}+\Delta\mathbf{B})^{-1}(\boldsymbol{\alpha}+\Delta\boldsymbol{\alpha})\neq\mathbf{W}. \tag{14}$$

Block elements $\Delta\mathbf{B}$ and $\Delta\boldsymbol{\alpha}$ of intrasystem perturbations matrix

$$\Delta\mathbf{A}=\left[\begin{array}{c:c} \Delta A_{11} & \Delta\boldsymbol{\alpha}^{\mathrm{T}} \\ \hdashline \Delta\boldsymbol{\alpha} & \Delta\mathbf{B} \end{array}\right]$$

retain the characteristics of the statistical moments inherent in the original matrix $\Delta\mathbf{A}$, namely: $\mathrm{M}\{\Delta\boldsymbol{\alpha}\}=\mathbf{0}(N-1,1)$; $\mathrm{M}\{\Delta\mathbf{B}\,\Delta\boldsymbol{\alpha}\}=\mathbf{0}(N-1,1)$; $\mathrm{M}\{\Delta\boldsymbol{\alpha}\,\Delta\boldsymbol{\alpha}^{\mathrm{T}}\}=\sigma^2_{\boldsymbol{\alpha}}\|\boldsymbol{\alpha}\|^2\cdot\mathbf{I}(N-1,N-1)$; $\mathrm{M}\{\Delta\mathbf{B}\cdot\Delta\mathbf{B}^{\mathrm{T}}\}=\sigma^2_{\mathbf{B}}\|\mathbf{B}\|^2\cdot\mathbf{I}(N-1,N-1)$, here $\sigma^2_{\boldsymbol{\alpha}}=\|\Delta\boldsymbol{\alpha}\|^2/\|\boldsymbol{\alpha}\|^2$ и $\sigma^2_{\mathbf{B}}=\|\Delta\mathbf{B}\|^2/\|\mathbf{B}\|^2$.

We define the random displacement of the parametric vector $\Delta\mathbf{W}$ in expression (14). When condition $\sigma_{\mathbf{B}}\zeta_{\mathbf{B}}\ll 1$ is fulfilled, vector $\Delta\mathbf{W}$ can be approximated by a linear form [8, 9]:

$$\Delta\mathbf{W}=\sum_{i=1}^{N-1}\sum_{j=1}^{N-1}\frac{\partial\mathbf{W}(\mathbf{B},\boldsymbol{\alpha})}{\partial B_{ij}}\cdot\Delta B_{ij}+\sum_{i=1}^{N-1}\frac{\partial\mathbf{W}(\mathbf{B},\boldsymbol{\alpha})}{\partial\alpha_i}\cdot\Delta\alpha_i\,, \tag{15}$$

where B_{ij}, α_i – elements of blocks $(\mathbf{B},\boldsymbol{\alpha})\in\mathbf{A}$; $i,j\in\overline{1,N-1}$.

Having performed a series of transformations [8, 10], we will reduce the expression (15) to the matrix form $\Delta\mathbf{W}=-\mathbf{B}^{-1}(\Delta\mathbf{B}\mathbf{W}-\Delta\boldsymbol{\alpha})$ and represent the perturbed parametric vector (14) in the final form:

$$\widehat{\mathbf{W}}=\mathbf{W}-\mathbf{B}^{-1}(\Delta\mathbf{B}\mathbf{W}-\Delta\boldsymbol{\alpha})\neq\mathbf{W}. \tag{16}$$

Taking into account (16), we rewrite the expression of the spectral function (12) as follows:

$$\widehat{f}_{ME}(\Theta)=\frac{J(\widehat{\mathbf{W}},\widehat{\mathbf{W}})}{\left|\left[1\,\vdots\,\widehat{\mathbf{W}}^{\mathrm{T}}\right]\cdot\mathbf{V}(\Theta)\right|^2}=\frac{J(\widehat{\mathbf{W}},\widehat{\mathbf{W}})}{\left|\widehat{G}(\Theta)\right|^2}, \tag{17}$$

where $J(\widehat{\mathbf{W}},\widehat{\mathbf{W}})=\left[1\,\vdots\,\widehat{\mathbf{W}}^{\mathrm{T}}\right]\mathbf{A}\left[1\,\vdots\,\widehat{\mathbf{W}}^{\mathrm{T}}\right]^{\mathrm{T}}$ – quadratic form with perturbed argument; $\widehat{G}(\Theta)=\left[1\,\vdots\,\widehat{\mathbf{W}}^{\mathrm{T}}\right]\mathbf{V}(\Theta)$ – spatial transfer function of an adaptive system with a perturbed parametric vector.

Applying the averaging operation to expression (17) and taking into account the statistical moments of the intrasystem perturbations $\Delta\mathbf{B}$ and $\Delta\boldsymbol{\alpha}$, we obtain the expression for the spectral function for the ME algorithm

$$\widehat{f}_{ME}(\Theta)=\frac{J_0+\left(\sigma_{\mathbf{B}}^2\|\mathbf{B}\|^2\|\mathbf{W}\|^2+\sigma_{\boldsymbol{\alpha}}^2\|\boldsymbol{\alpha}\|^2\right)\cdot\mathrm{tr}\,\mathbf{B}^{-1}}{|G(\Theta)|^2+\left(\sigma_{\mathbf{B}}^2\|\mathbf{B}\|^2\|\mathbf{W}\|^2+\sigma_{\boldsymbol{\alpha}}^2\|\boldsymbol{\alpha}\|^2\right)\cdot\left\|\mathbf{B}^{-1}\mathbf{r}(\Theta)\right\|^2}. \tag{18}$$

Let us establish the dependence of function (18) on the condition number of the correlation matrix $\mathbf{B}\in\mathbf{A}$ based on the representation of the matrix norms $\left\|\mathbf{B}^{-1}\mathbf{r}(\Theta)\right\|\le\left\|\mathbf{B}^{-1}\right\|\cdot\left\|\mathbf{r}(\Theta)\right\|$, $\|\mathbf{W}\|\le\left\|\mathbf{B}^{-1}\right\|\cdot\|\boldsymbol{\alpha}\|$ and $\|\mathbf{r}(\Theta)\|^2=N-1$. As a result, we obtain:

$$\widehat{f}_{ME}(\Theta)=\frac{J_0+\left(\sigma_{\mathbf{B}}^2\zeta_{\mathbf{B}}^2+\sigma_{\boldsymbol{\alpha}}^2\right)\cdot\|\boldsymbol{\alpha}\|^2\cdot\mathrm{tr}\,\mathbf{B}^{-1}}{|G(\Theta)|^2+\left(\sigma_{\mathbf{B}}^2\zeta_{\mathbf{B}}^2+\sigma_{\boldsymbol{\alpha}}^2\right)\cdot(N-1)\cdot\|\mathbf{W}\|^2}. \tag{19}$$

Let us determine the interval within which the values of the perturbed spectral function are found for the ME algorithm (19). Based on the logic of the previous arguments regarding the Capon algorithm, we assume that the upper and lower limits of the function (19) correspond to independent conditions: $(\sigma_{\mathbf{B}}^2,\sigma_{\boldsymbol{\alpha}}^2)\to 0$, $\mathbf{A}=P_0\,\mathbf{I}(N,N)$. In this case, the values of the spectral function (19) will belong to the interval $P_0\le\widehat{f}_{ME}(\Theta)\le f_{ME}(\Theta)$ under condition $\sigma_{\mathbf{B}}^2\zeta_{\mathbf{B}}^2<<1$.

In a situation with poor conditionality of the correlation matrix $\mathbf{B}$, when $\zeta_{\mathbf{B}}>>1$, and equality of the values of variances $\sigma_{\mathbf{B}}^2,\sigma_{\boldsymbol{\alpha}}^2$, the intrasystem perturbations in (19) will satisfy inequality $\sigma_{\mathbf{B}}^2\zeta_{\mathbf{B}}^2>>\sigma_{\boldsymbol{\alpha}}^2$. It follows that the weight of perturbations of matrix $\Delta\mathbf{B}$ prevails over the weight of perturbations of absolute terms $\Delta\boldsymbol{\alpha}$, that is, the spectral function of the ME algorithm in different degrees is critical to the perturbations of the Wiener parametric vector (16).

The results of the analysis of the spectral functions of the Capon (10) and ME algorithms (19) make it possible to place accents in the considered analytical approach:

− first, with an increase in the level of intrasystem perturbations $\sigma_{\mathbf{A}}^2,\sigma_{\mathbf{B}}^2$ and $\sigma_{\boldsymbol{\alpha}}^2$, the values of the maxima of the functions, reflecting the position of radiation sources in space, decrease;

− secondly, the influence of intrasystem perturbations on the contrast of spectral functions increases with an increase in the conditional number of the correlation matrix of observations and the dimension of the adaptive system. Accordingly, the contrast of spectral functions at the output of the *N*-dimensional system deteriorates and, as a result, the resolution of spectral lines closely located in frequency decreases. However, ignoring intrasystem perturbations, an increase in the condition number of the correlation matrix and the dimension of the adaptive system leads to the opposite effect in the theory – an increase in the contrast of spectral functions (10) and (19), and, consequently, an increase in the spatial frequencies resolution of the system. This contradiction is characteristic of a fairly wide class of inverse problems whose solution is based on the inversion of the estimate of the correlation matrix of observations with a limited sample size [6, 10, 11, 12];

− thirdly, the neglect of intrasystem perturbations reduces the adequacy of the analytical model of the system to real conditions, which in practice leads to a significant increase in information losses [4, 6, 9, 12].

We will demonstrate the behavior of the spectral functions for the Capon and ME algorithms and estimate the spatial frequencies resolution of the system under the following conditions:

− the relative distance between the phase centers of spatially separated recording isotropic sensors $\Delta X/\lambda = 0{,}5$;

− independent noise emissions of the same intensity from the directions $\Theta_1 = -1°$, $\Theta_2 = 1°$ or $\Theta_1 = -2°$, $\Theta_2 = -1°$, $\Theta_3 = 2°$ act on the system input;

− the excess of the noise emissions of each source above the internal noise level of the system is $30\,dB$;

− the levels of intrasystem perturbations of the matrices $\mathbf{A}$, $\mathbf{B}$ and vector $\boldsymbol{\alpha}$ are equivalent $\sigma^2 = \sigma^2_{\mathbf{A}} = \sigma^2_{\mathbf{B}} = \sigma^2_{\boldsymbol{\alpha}}$;

− correctness of the results obtained are guaranteed by constraints $\sigma^2_{\mathbf{A}}\zeta^2_{\mathbf{A}} << 1$ and $\sigma^2_{\mathbf{B}}\zeta^2_{\mathbf{B}} << 1$, determined by the perturbation theory of matrix forms.

The behavior of the spectral functions (10) and (19) under intrasystem perturbation conditions is reflected in the direction-finding reliefs $\widehat{f}_C(\Theta, \sigma^2)$ and $\widehat{f}_{ME}(\Theta, \sigma^2)$, shown in Fig. 1-6 for systems of dimension $N = 5$ (Fig. 1, 2) and $N = 15$ (Fig. 3-6), respectively. The direction of arrival of signals from radiation sources in Fig. 1-6 are indicated by arrows.

The contrast of the spectral relief and the distance between its peaks determine the frequency resolution of radiation sources.

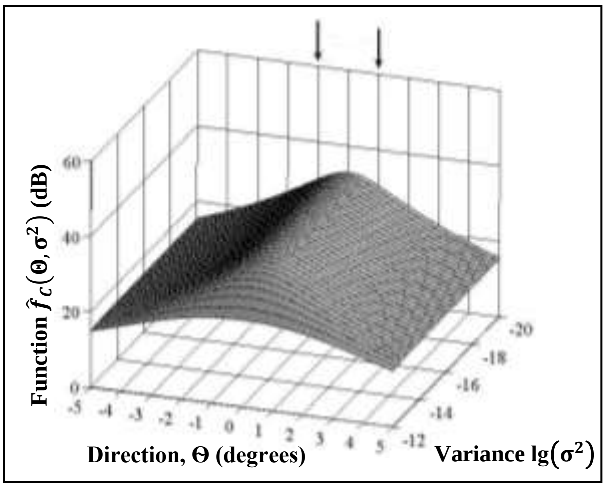

Figure 1 – Direction-finding relief $\widehat{f}_C(\Theta, \sigma^2)$ for: $N = 5$, $\Theta_1 = -1°$, $\Theta_2 = 1°$

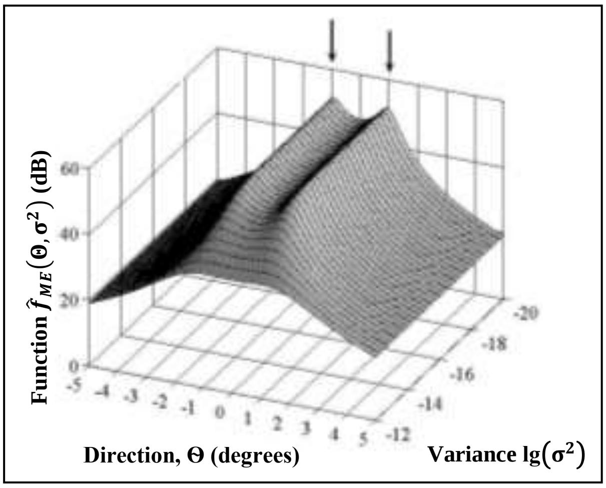

Figure 2 – Direction-finding relief $\widehat{f}_{ME}(\Theta, \sigma^2)$ for: $N = 5$, $\Theta_1 = -1°$, $\Theta_2 = 1°$

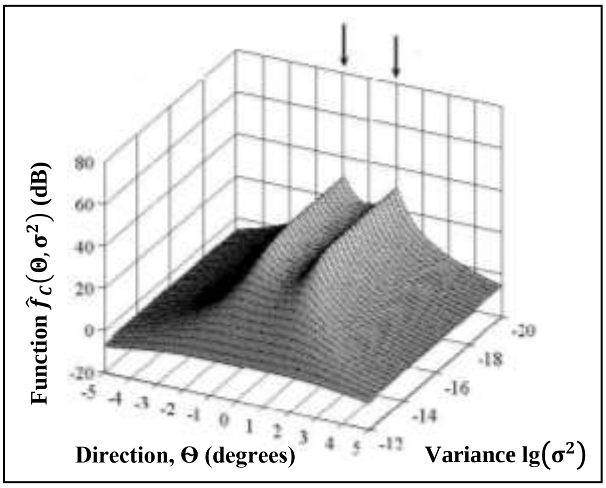

Figure 3 – Direction-finding relief $\widehat{f}_C(\Theta,\sigma^2)$ for: $N = 15$, $\Theta_1 = -1°$, $\Theta_2 = 1°$

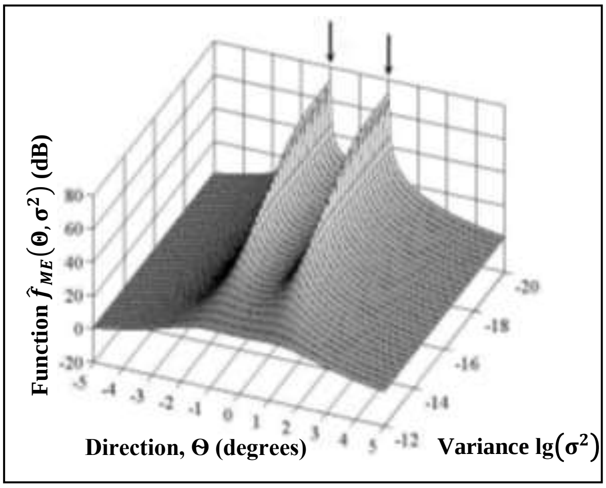

Figure 4 – Direction-finding relief $\widehat{f}_{ME}(\Theta,\sigma^2)$ for: $N = 15$, $\Theta_1 = -1°$, $\Theta_2 = 1°$

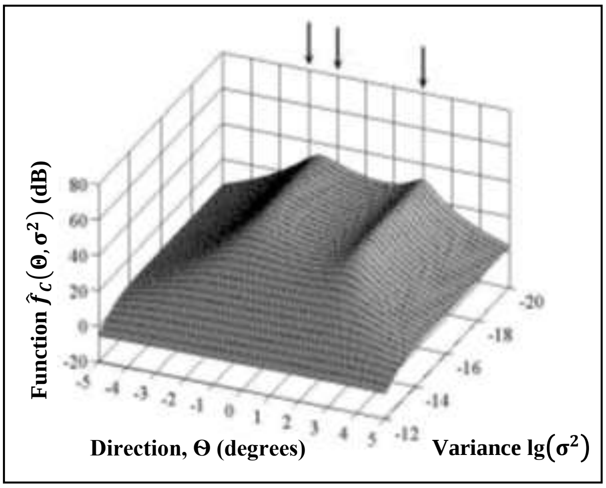

Figure 5 – Direction-finding relief $\widehat{f}_C(\Theta,\sigma^2)$ for: $N = 15$, $\Theta_1 = -2°$, $\Theta_2 = -1°$, $\Theta_3 = 2°$

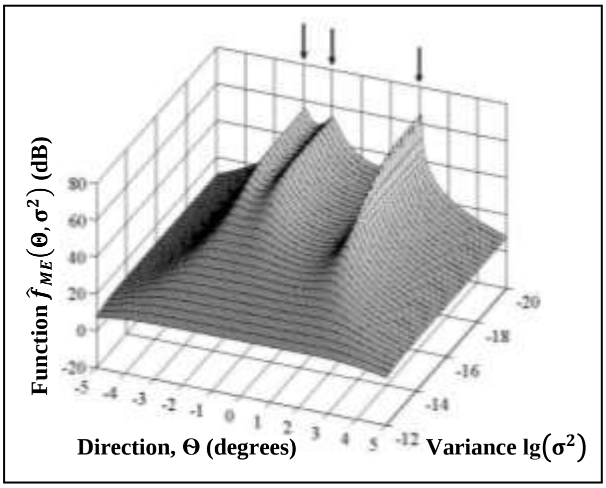

Figure 6 – Direction-finding relief $\widehat{f}_{ME}(\Theta,\sigma^2)$ for: $N = 15$, $\Theta_1 = -2°$, $\Theta_2 = -1°$, $\Theta_3 = 2°$

Table 1 presents the peaks' coordinates $\widehat{\Theta}$ of the spectral reliefs $\widehat{f}_C(\Theta,\sigma^2)$ and $\widehat{f}_{ME}(\Theta,\sigma^2)$, depending on the dimension of the system *N* and the conditioning numbers of the correlation matrices $\zeta_{\mathbf{A}}$, $\zeta_{\mathbf{B}}$ at fixed levels of intrasystem perturbations σ^2.

Table 1– Coordinates of the peaks of spectral reliefs

Function	N	Θ, degrees	$\zeta_{\mathbf{A}}$	$\zeta_{\mathbf{B}}$	$\widehat{\Theta}$, degrees		Fig. №
					$\sigma^2 = 10^{-16}$	$\sigma^2 = 10^{-12}$	
$\widehat{f}_C(\Theta,\sigma^2)$	5	-1; 1	3×10^8		0	0	1
	15	-1; 1	105×10^8		-0,98; 0,98	no local maxima	3
		-2; -1; 2	183×10^8		-1,57; 1,93		5
$\widehat{f}_{ME}(\Theta,\sigma^2)$	5	-1; 1	–	$1,3\times10^8$	-0,93; 0,93		2
	15	-1; 1	–	85×10^8	-1; 1		4
		-2; -1; 2	–	151×10^8	-1,99; -1,05; 2,0		6

The analysis of the spectral reliefs $\widehat{f}_C(\Theta,\sigma^2)$, $\widehat{f}_{ME}(\Theta,\sigma^2)$, and the data given in Tab. 1, indicates a steady tendency of deformation of the functions' peaks of the Capon (Fig. 1, 3, 5) and ME algorithms (Fig. 2, 4, 6), deterioration of the reliefs' contrast and decrease spectral resolution in space under conditions of intrasystem uncertainty. At the same time, with an admissible level of intrasystem perturbations, the ME algorithm retains the ability to resolve the frequencies of the spatial power spectrum of stochastic radiations, which is not characteristic of the Capon algorithm.

CONCLUSIONS

In a certain sense, the presented material can be considered as a generalizing commentary on the development of an analytical approach for the study of spatial spectral analysis algorithms, which are based on the inversion of perturbed estimates of the correlation matrices of the observed processes. This approach allows us to link the spatial frequencies resolution, the level of intrasystem perturbations, and the values of spectral functions for both the Capon and the ME algorithms with a single analytical dependence. Such dependence allows:

− first, to assess the degree of influence of the perturbations' level of the correlation matrix on the deformation of the spectral reliefs for Capon and ME algorithms;

− secondly, it is theoretically to determine the permissible limits for changes in the values of spectral functions at which there is a frequencies resolution of the spatial power spectrum under conditions of intrasystem uncertainty.

REFERENCES

1. Пространственно-временная обработка сигналов / И.Я. Кремер, А.И. Кремер, В.М. Петров и др.; под ред. И.Я. Кремера. – М.: Радио и связь, 1984. – 224 с.
2. Даджион Д. Цифровая обработка многомерных сигналов / Д. Даджион, Р. Мерсеро; пер. с англ. В.А. Григорьева, К.Г. Финотенова; под ред. Л.П. Ярославского. – М.: Мир, 1988. – 488 с.
3. Монзиго Р. А. Адаптивные антенные решётки. Введение в теорию / Р.А. Монзиго, Т.У. Миллер; пер. с англ. под ред. В.А. Лексаченко. – М.: Радио и связь, 1986.– 448 с.
4. Ратынский М.В. Адаптация и сверхразрешение в антенных решетках / М.В. Ратынский. – М.: Радио и связь, 2003. – 200 с.
5. Гун С. Спектральный анализ: от обычных методов к методам с высокой разрешающей способностью / С. Гун, Д Рао, К Арун // Сверхбольшие интегральные схемы и современная обработка сигналов: Пер. с англ. / Под ред. С. Гуна, Х. Уайтхауса, Т. Кайлата. – М.: Радио и связь, 1989. – С. 45–64.
6. Леховицкий Д.И. Новые «сверхразрешающие» пеленгаторы источников шумовых излучений с высоким быстродействием / Д.И. Леховицкий, Д.В. Атаманский., Д.С. Рачков, А.В. Семеняка // Современные информационные и электронные технологии: XV-я Междунар. науч.-практ. конф. «СИЕТ-2014», Одесса, 26 – 30 мая 2014 г. – Одесса: ОНПУ, 2014. – С. 174-180.
7. Егоров В.А. Исследование свойств алгоритма Кейпона в задачах пеленгации стационарных активных помех / В.А. Егоров, Н.С. Кондыбаев // Компьютерные инструменты в образовании. – 2011. – №3. – С. 10-17.
8. Скачков В.В. Анализ эффективности адаптивной обработки сигналов в условиях дестабилизирующих воздействий / В.В. Скачков // Радиотехника.– 1998.– № 11. – С. 10-15.
9. Скачков В.В. Энтропийный подход к исследованию информационных возможностей адаптивной радиотехнической системы при внутрисистемной неопределенности / В.В. Скачков, В.В. Чепкий, Г.Д. Братченко, А.Н. Ефимчиков // Известия вузов. Радиоэлектроника. – 2015. – т.58, №6. – С. 3-12.
10. Деммель Дж. Вычислительная линейная алгебра. Теория и приложения: пер. с англ. / Дж. Деммель. – М.: Мир, 2001. – 430 с.
11. Голуб Дж. Матричные вычисления / Голуб Дж., Ван Лоун Ч.; пер. с англ. Ю.М. Нечепуренко, А.Ю. Романова, А.В. Собянина, Е.Е. Тыртышникова; под ред. В.В. Воеводина. – М.: Мир, 1999. – 548 с.
12. Skachkov V. V. Evaluation the Influence of Intrasystem Perturbations of the Covariance Matrix of Observations on the Df Response of Adaptive Antenna Array / V.V. Skachkov, V.V. Chepkyi, A.N. Efymchykov, H.D. Bratchenko, V.I. Pavlovich // XI Int. Conf. of Antenna Theory and Techniques: ICATT'17, Igor Sikorsky Kyiv Polytechnic Institute, 24–27May 2017, Kyiv, Ukraine: proc. – IEEE, 2017. – p.p. 174–179. – DOI: 10.1109/ICATT.2017.7972614.

REFERENCES

1. Space-time Signal Processing / I.Ya. Kremer, A.I. Kremer, V.M. Petrov et al.; edited by I.Ya. Kremer. – Moscow: Radio and communication, 1984. – 224 p.
2. Dudgeon, Dan. E. and Mersereau, Russell M. Multidimensional Digital Signal Processing. Prentice-Hall,

1984, 400 p.

3. Monzingo, Robert A., Haupt, Randy L. and Miller, Thomas W. Introduction to Adaptive Arrays. 2nd ed. SciTech Publishing, 2011, 541 p.

4. Ratynsky, M.V. Adaptation and superresolution in antenna arrays / M.V. Ratynsky. – Moscow: Radio and communications, 2003. – 200 p.

5. Kung, S.Y., Rao, Bhaskar D.V. and Arun, K.S. Spectral analysis: from conventional methods to high-resolution methods // VLSI and Modern Signal Processing / Editors: S.Y. Kung, H.J. Whitehouse and T. Kailath. Englewood Cliffs, New Jersey 070632: Prentice-hall, 1985.

6. Lekhovitsky, D.I. New "superresolution" direction finders for noise sources with high speed / D.I. Lekhovitsky, D.V. Atamansky, D.S. Rachkov, A.V. Semenyaka // Modern Information and Electronic Technologies: XVth International scientific-practical conf. "MIET-2014", Odessa, May 26 – 30, 2014. – Odessa: ONPU, 2014. – p. 174–180.

7. Egorov, V.A. Study of the properties of the Capon algorithm in problems of direction finding stationary active interference / V.A. Egorov, N.S. Kondybaev // Computer tools in education, 2011. – №3. – p. 10–17.

8. Skachkov, V.V. Analysis of the effectiveness of adaptive signal processing under destabilizing effects / V.V. Skachkov // Radio engineering, 1998. – № 11. – p. 10–15.

9. Skachkov, V.V. Entropy approach to the investigation of information capabilities of adaptive radio engineering system in conditions of intrasystem uncertainty / V.V. Skachkov, V.V. Chepkyi, H.D. Bratchenko, A.N. Efymchykov // Radioelectronics and Communications Systems, 2015. – Vol. 58, Issue 6. – p. 241–249. – https://doi.org/10.3103/S0735272715060011.

10. Demmel, James. W. Applied Numerical Linear Algebra. Philadelphia: SIAM, 1997, 419 p.

11. Golub, Gene H. and Van Loan, Charles F. Matrix Computations. 3rd ed. Baltimore and London: The John Hopkins University Press, 1996, 694 p.

12. Skachkov, V.V. Evaluation the Influence of Intrasystem Perturbations of the Covariance Matrix of Observations on the Df Response of Adaptive Antenna Array / V.V. Skachkov, V.V. Chepkyi, A.N. Efymchykov, H.D. Bratchenko, V.I. Pavlovich // XI Int. Conf. of Antenna Theory and Techniques: ICATT'17, Igor Sikorsky Kyiv Polytechnic Institute, 24–27 May 2017, Kyiv, Ukraine: proc. – IEEE, 2017. – p. 174–179. – DOI: 10.1109/ICATT.2017.7972614.