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ANALYSIS OF THE ORTHOGONAL FREQUENCY-DIVISION MULTIPLEXING METHOD FOR STRUGGLE AGAINST INTERSYMBOL INTERFERENCE IN MULTIPATH FADING CHANNELS

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АНАЛІЗ МЕТОДУ ОРТОГОНАЛЬНОГО ЧАСТОТНОГО МУЛЬТИПЛЕКСУВАННЯ ДЛЯ БОРОТЬБИ З МІЖСИМВОЛЬНОЮ ІНТЕРФЕРЕНЦІЄЮ В БАГАТОПРОМЕНЕВИХ КАНАЛАХ ІЗ ЗАВМИРАННЯМИ

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Abstract. *The article gives an estimate of the distortions in the multipath fading channel in the intersymbol interference using the advanced a Kalman digital filter with the algorithm of the adapted distribution of bits and power. The task of allocating resources across subcarriers is a multi-parameter nonlinear programming problem. The method of iterations was applied which gives the following result: if required* $p_b = 10^{-5}$*, the use of adaptation in bits and power allows to increase the information transfer rate by 10 %. It is possible to reduce the sensitivity of an OFDM-signal receiver to phase fluctuations on the path of applying multiple modulation. In channels with multipath and fading the use of m-PSK is more preferable compared to other multiple modulations.*

In addition to these advantages, there are deficiencies in the OFDM system due to the increased peak factor of the emitted signal in comparison with single frequency transmission. Note that the peak factor of the signal increases with increasing number of frequencies N.

In the article shows the block diagrams of non-coherent and coherent demodulators, the block diagram of a Kalman digital filter. The result of analysis are graphs of BER versus signal-to-noise ratio and graphs of frequency detuning and phase fluctuations.

Keywords*: orthogonal frequency multiplexing, intersymbol interference, multipath, fading, quadrature non-coherent demodulator, coherent demodulator, Kalman digital filter.*

Анотація. *У статті дається оцінка спотворень в багатопроменевому каналі із завмираннями при МСІ за допомогою вдосконаленого фільтра Калмана з алгоритмом адаптованого розподілу біт і потужності. Завдання розподілу ресурсів по підносійних коливаннях є багатопараметричним завданням нелінійного програмування. Застосовано метод ітерацій, який дає наступний результат: при вимозі* $p_{пом} = 10^{-5}$ *застосування ада-*

птації по бітам і потужності дозволяє збільшити швидкість передавання інформації на 10 %. Знизити чутливість приймача OFDM-сигналів до фазових флуктуацій можна шляхом застосування багаторазової модуляції. У каналах з багатопроменевістю і завмираннями застосуваннят-PSK більш переважно в порівнянні з іншими багаторазовими видами модуляції.

Окрім цих переваг, у системі OFDM існують недоліки, обумовлені збільшенням коефіцієнта пікового значення випромінюваного сигналу у порівнянні з одночастотним передаванням. Слід зазначити, що піковий коефіцієнт сигналу зростає із зростанням числа частот N.

У статті наведено структурні схеми некогерентних та когерентних демодуляторів, структурна схема цифрового фільтра Калмана. Результатом аналізу є графіки співвідношення BER і сигнал-шум, а також графіки відхилення частоти та коливання фази.

***Ключові слова**: ортогональне частотне мультиплексування, міжсимвольна інтерференція, багатопроменевість, завмирання, квадратурний некогерентний демодулятор, когерентний демодулятор, цифровий фільтр Калмана.*

***Аннотация.** В статье даётся оценка искажений в многолучевом канале с замираниями при МСИ при помощи усовершенствованного фильтра Калмана с алгоритмом адаптированного распределения бит и мощности. Задача распределения ресурсов по поднесущим является многопараметрической задачей нелинейного программирования. Применён метод итераций, который даёт следующий результат: при требовании* $p_{ош} = 10^{-5}$ *применение адаптации по битам и мощности позволяет увеличить скорость передачи информации на 10 %. Понизить чувствительность приёмника OFDM-сигналов к фазовым флуктуациям можно на пути применения многократной модуляции. В каналах с многолучёвостью и замираниями использование m-PSK более предпочтительно по сравнению с другими многократными видами модуляции.*

Кроме этих преимуществ, в системе OFDM существуют дефициты, обусловленные увеличением коэффициента пикового значения излучаемого сигнала по сравнению с одночастотной передачей. Следует отметить, что пиковый коэффициент сигнала возрастает с ростом числа частот N.

В статье приведены структурные схемы некогерентных и когерентных демодуляторов, структурная схема цифрового фильтра Калмана. Результатом анализа являются графики соотношения BER и сигнал-шум, а также графики отклонения частоты и колебаний фаз.

***Ключевые слова**: ортогональное частотное мультиплексирование, межсимвольная интерференция, многолучёвость, замирания, квадратурный некогерентный демодулятор, когерентный демодулятор, цифровой фильтр Калмана.*

One of the effective methods of dealing with intersymbol interference (ISI) is modulation with simultaneous transmission on orthogonal subcarrier frequencies (OFDM). It is known that the influence of ISI can be significantly weakened by increasing the duration of the symbol, but this leads to decrease information bit rate [1]. If the stream of symbols on one carrier frequency is divided into a large number *N* of more slowly transmitted substreams, each on a separate subcarrier frequency, this will not reduce the information bit rate.

The aim of the work is to estimate the distortions in the multipath fading channel at ISI using the advanced a Kalman digital filter with the algorithm of the adapted distribution of bits and power.

In OFDM, transmission is carried out by using a large number of subcarrier frequencies – high frequency harmonic oscillations. Figure 1 shows a block diagram of a non-coherent demodulator based on correlators. It can be seen from the scheme that the in-phase (I) and quadrature (Q) channels are used for non-coherent detection of two adjacent signals, each of which has its own carrier frequency. The two upper branches of the circuit are configured to detect the signal at frequency ω_1, and the reference signal for the in-phase branch is $\cos\omega_2 t$, and for quadrature signal is $\sin\omega_1 t$. The two lower branches of the circuit are configured to detect a signal with frequency ω_2, and the reference signal for the in-phase branch is $\cos\omega_2 t$, and for the quadrature signal is $\sin\omega_2 t$. If the received signal corresponds exactly to the phase of the reference signal, then maximum power signal will be received at the output of the integrator. If the signal is orthogonal to the reference signal, then the integrator output will have a “zero” value.

The device, after integrators of the products of the input and reference signals, performs the squaring operation, which prevents the occurrence of possible negative values. Then, for each signal set the values s_1^2 are added from the in-phase channel s_2^2 and from the quadrature channel. In the decision making scheme, a signal with a frequency ω_1 or ω_2 by the criterion of higher power (amplitude) is selected.

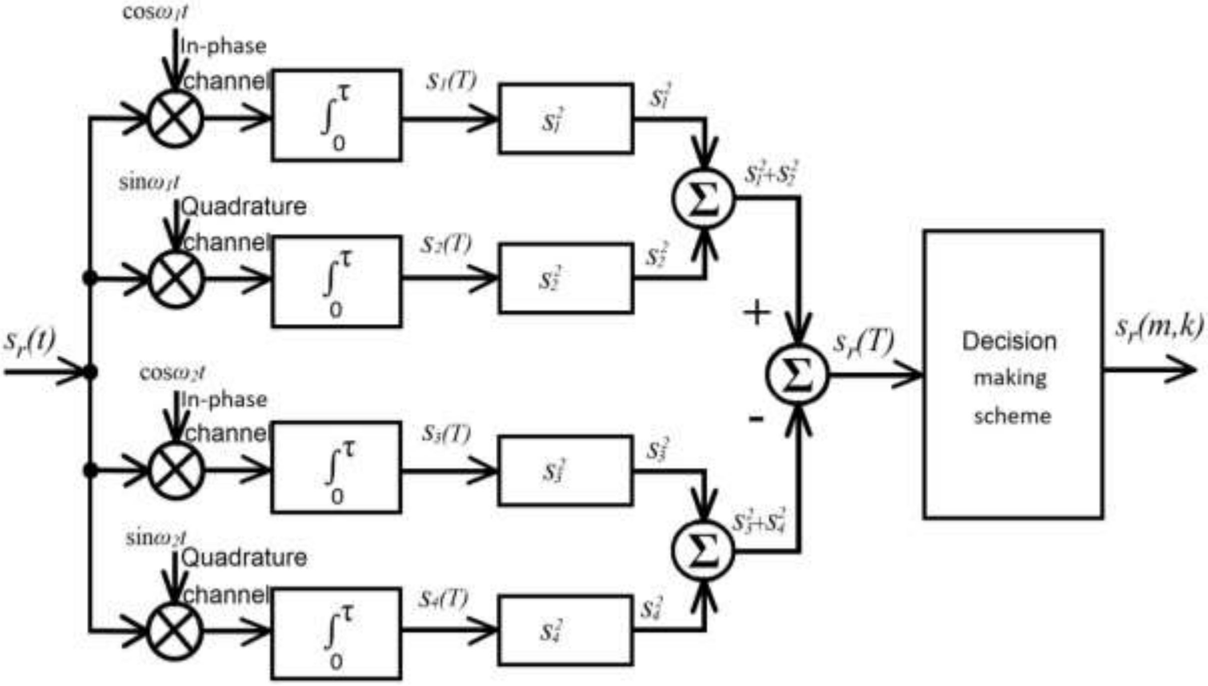

Figure 1 – The block diagram of quadrature non-coherent demodulator

Figure 2 shows the block diagram of a coherent demodulator.

Unlike the non-coherent demodulator, in a coherent demodulator there are no two branches for quadrature channels with frequencies ω_1 or ω_2, since the reference signals $\cos\omega_1 t$ and $\cos\omega_2 t$ are also phased into the received signal.

To estimate the channel distortions, various algorithms are used, for example, based on the use of a Wiener or a Kalman digital filters, parametric estimation of the impulse response (IR) and frequency response (FR) of the channel.

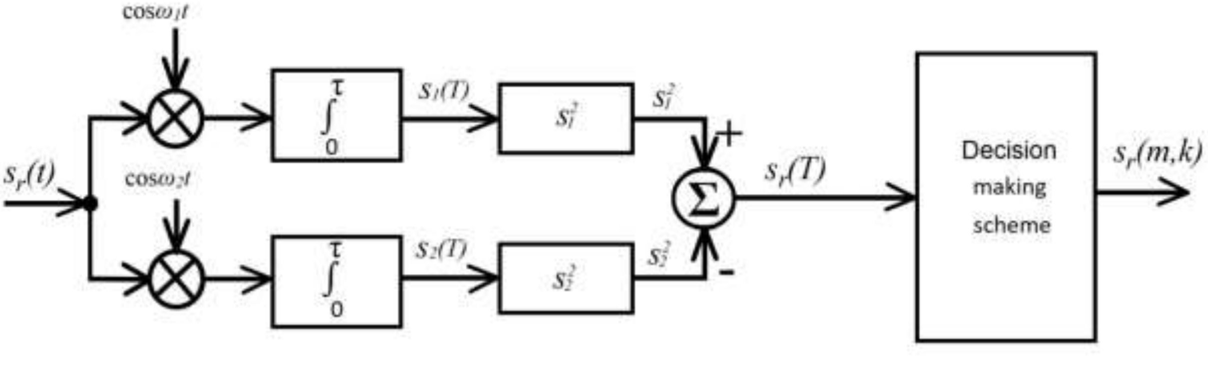

Figure 2 – The block diagram of coherent demodulator

The transmitted signal $s_t(t)$ is a sequence of OFDM symbols of duration T, represented by a sum N of subcarriers and separated by a guard interval of duration T_g. The signal $s_t(t)$ propagates through a multipath fading channel IR $h(t,\tau)$. The received signal is:

$$s_r(t)=\int_0^{\infty} h(t,\tau)S_t(t-\tau)d\tau .$$

The received *m*-symbol is:

$$s_r(m,t)=\int_0^{\infty} h(t,\tau)s_t(m,t-\tau)d\tau=\sum_{k=1}^{N} H(m,k,t)s_r(m,k)\exp(-j\omega\tau),$$

where is $s_r(m,k)$ the *k*-subcarrier amplitude in the *m*-symbol.

The channel frequency response for the *k*-subcarrier:

$$H(m,k,t)=\int_0^{\infty} h(mT_S+t,\tau)\exp(-j\omega\tau)d\tau ,$$

where $T_S=T+T_g, T$ is the duration of OFDM symbols, T_g is the duration of the guard interval.

To estimate the amplitudes of subcarriers on the receiving side discrete Fourier transforms are used.

Then at the output of the demodulator for the *k*-subcarrier we have:

$$s_r(m,k)=\frac{1}{N}\sum_{i=0}^{N-1} s_r\left(m,i\frac{T}{N}\right)\exp\left(-j\omega i\frac{T}{N}\right).$$

The use of a Kalman digital filter for estimating the channel is known [2]. The implementation of a Kalman digital filter for estimating the FR channel in OFDM systems is computationally complex, especially with a large number of subcarriers. Compensation of the influence of ISI is the most costly fragment in the computational procedure of signal processing. Direct implementation of this stage of the computational process is proportional to $O(N^3)$. However, if only the elements of the main diagonal are stored in the channel matrix, the complexity of the computational process will become proportional to $O(N)$.

A Kalman filter algorithm is based on a two-parameter probability density distribution for estimating distortions in the ISI channel:

$$\omega(\mu,\nu)=\frac{\nu}{2\Gamma(1/\nu)\sigma}\left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}\right]^{1/2}\exp\left\{-\frac{|\mu|^{\nu}}{\sigma^{\nu}}\left[\frac{\Gamma(3/\nu)}{\Gamma(1/\nu)}\right]^{\nu/2}\right\},$$

where μ,ν –the distribution parameters, when $\nu=2$ we have the Gaussian distribution.

In this case, a first order differential equation describing the filtering process is as follows:

$$\frac{dy}{dt}=-\gamma y(t)+\sigma^2 A(\nu)\left|x(t)-y(t)\right|^{\nu-1}\ \text{at}\ \mathrm{y}(0)=\mathrm{y}_0, \qquad (1)$$

where $A(\nu)=\left[\Gamma(3/\nu)/\Gamma(1/\nu)\right]^{\frac{\nu}{2}}\frac{\nu}{\sigma^{\nu}}$, $\gamma>0$.

The block diagram of a Kalman digital filter that implements algorithm (1) is shown in Figure 3. Note that the IEEE 802.11 standard is valid for uniform power distribution with the same types of modulation and coding on all subcarriers of the OFDM signal. However, under conditions of frequency selective fading, this is not the best option. It is necessary that the best mode of modulation and coding provide equal p_b on each subcarrier and in this case the maximum transmission rate in the multipath fading channel can be ensured.

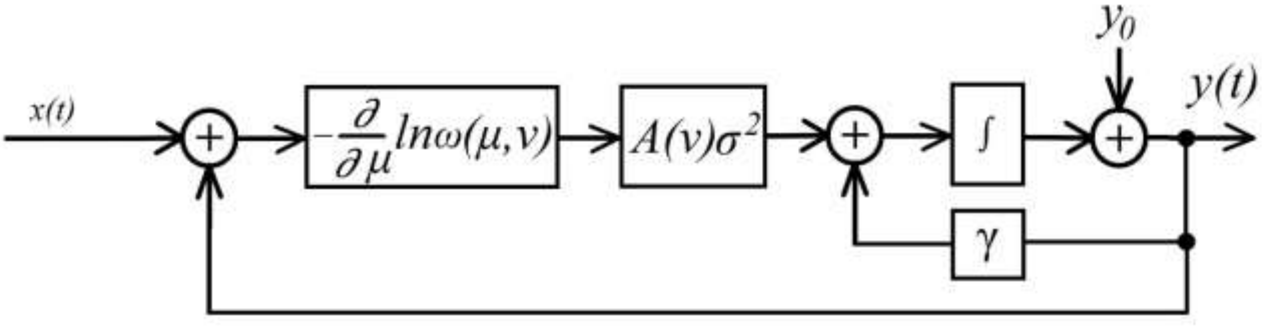

Figure 3 – The block diagram of a Kalman digital filter

The following procedure is proposed for the adaptive allocation of information bits and power across subcarriers.

Step 1. The noise power values P_{noise} on the subcarriers are sorted so that $P_{noise1} = P_{noise\,\min}$ and $P_{noise\,N} = P_{noise\,\max}$. From this set, the main subcarriers are selected by the iteration method. The comparison is carried out with a predetermined threshold value of the signal-to-noise (SNR) ratio providing the required value p_b. At the current step of the iterative procedure, the number of main subcarriers can be reduced by one. The procedure stops at that step when the power of the main subcarriers exceeds a predetermined threshold value. This distribution of power allows you to maximize the transmission rate for a given value p_b.

Step 2. The modulation type m is determined from a given set of modulation types. This is done by comparing the maximum signal-to-noise ratio on the “best” subcarrier with a set of threshold signal-to-noise ratios. The maximum threshold signal-to-noise ratio, which does not exceed the maximum signal-to-noise ratio on the “best” subcarrier, corresponds to the desired modulation type.

Then a set of subcarriers is determined on which information it can be transmitted by m-type of modulation at the required value p_b. For the found subcarrier set m, the distribution of information bits is performed. After certain values of transmitted power P_m for m a set of subcarriers, you can determine the excess power ΔP_m that can be distributed over the remaining subcarriers, which will increase the transmission rate.

Step 3. The set of $(m-1)$ subcarriers on which the modulation of the modulation will be transmitted $(m-1)$ at the required value p_b is also found by the iteration method. For a set of $(m-1)$ subcarriers, the power distribution and excess power P_{m-1} is calculated ΔP_{m-1}. The iterative process continues until all N-subcarriers are analyzed. The excess power ΔP_{m-N} remaining after processing the last set of subcarriers can be distributed over the main subcarriers, which reduces the value p_b. This shows that if $N \geq 256$, then a fast Fourier transform is needed to process orthogonal subcarriers.

The graphs in Figure 4 are built according to this algorithm. Curves 1,2 correspond to the reception without compensation of the ISI, curves 3,4 correspond to the reception with compensation of the ISI. Curves 1, 3 are constructed for $f > 0.05/\mathrm{T}$, curves 2, 4 – for $f > 0.1/\mathrm{T}$, $T_g = T/4$. The proposed algorithm makes it possible with satisfactory accuracy to estimate the distortion of the OFDM signal in a multipath fading channel including with ISI.

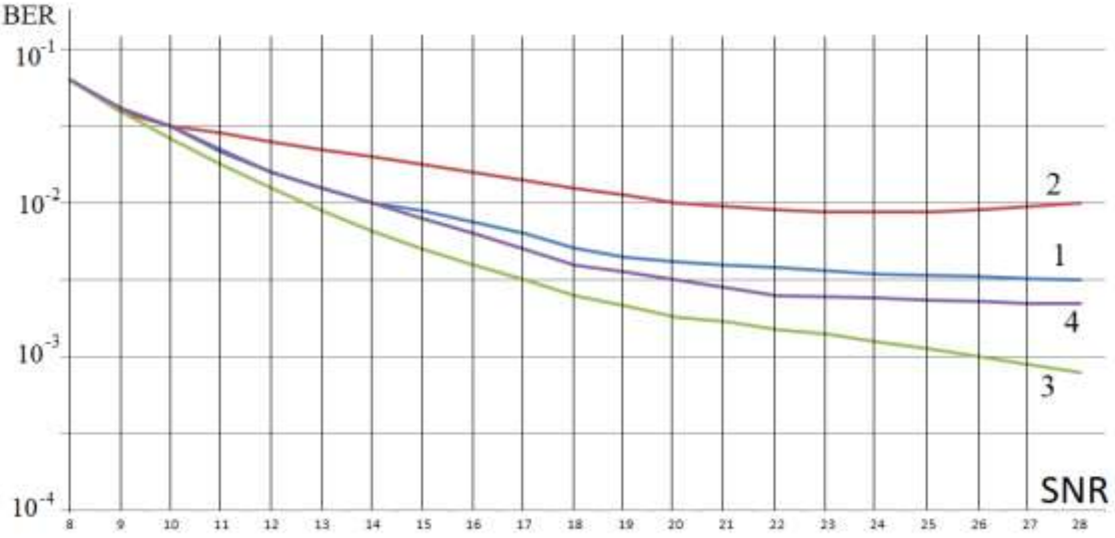

Figure 4 – Graphs of BER versus signal-to-noise ratio

The use of an algorithm for adaptive distribution of bits and power over subcarriers leads to the use of fast Fourier transform. But the fast Fourier transform leads to additional crosstalk on subcarriers, manifested as phase fluctuations. The signal at the output of the OFDM transmitter is generated using the inverse fast Fourier transform and in the OFDM receiver the signal is subjected to forward fast Fourier transform. Phase fluctuations are due to frequency mismatch between transmitter and receiver due to non-ideal synchronization, as well as phase noise caused by frequency instability of the oscillators. For example, the use of high-order *m*-QAM makes the OFDM signal even more sensitive to fluctuations.

Therefore, for the OFDM signal the SNR is written as follows:

$$SNR = \frac{P_S}{P_{noise} + P_{fl\ noise}},$$

where P_S is the power of the useful signal, P_{noise} is the power of additive noise in the frequency band of the signal $P_{fl\ noise} = P_{f\ noise} + P_{\varphi\ noise}$, is the power of phase fluctuations caused by frequency detuning of power $P_{f\ noise}$ and phase noise by power $P_{\varphi\ noise}$.

In the OFDM signal, there are algorithms for estimating changes in the signal-to-noise ratio. As a rule, for a signal-to-noise ratio of 30 dB, the reduction due to magnitude $P_{\varphi\ noise}$ should not exceed 1 dB.

The formation of OFDM signals with $N \geq 256$ is achieved by applying a fast Fourier transform. However, the use of multi-modulation types, especially *m*-QAM, leads to the appearance of phase fluctuations and frequency detunings. The presence, for example, of phase fluctuations leads to a violation of the orthogonality of the subcarriers and a decrease in the rate of information transfer. In the OFDM signal, it is customary to estimate the phase distortion on each subcarrier.

So the noise variance due to frequency detuning can be estimated approximately:

$$\sigma_f^2 = \left(\pi \Delta F T N\right)^2 \big/ 3,$$

where ΔF is the frequency mismatch.

The noise variance of phase fluctuations is roughly estimated as follows:

$$\sigma_\varphi^2 = \frac{2}{3} \pi F_\varphi T N,$$

where F_φ – fluctuations in the frequency of the generator.

The combined effect of frequency detuning and phase fluctuations is defined as:

$$\sigma^2 = \sigma_f^2 + \sigma_\varphi^2.$$

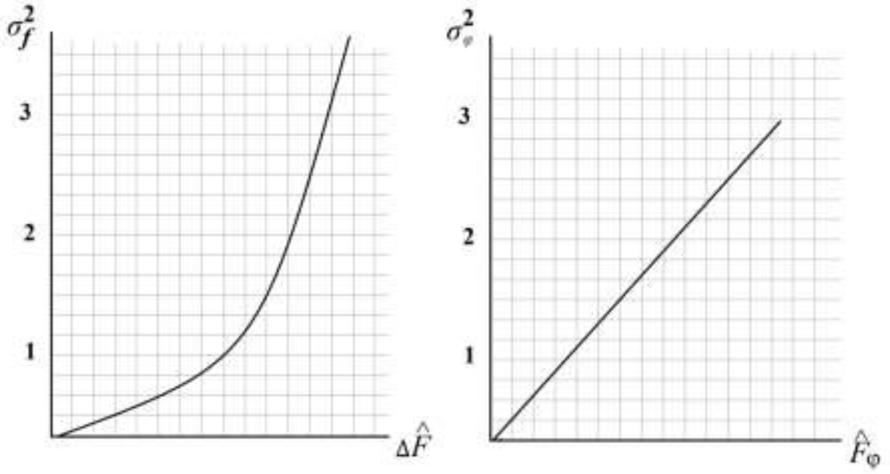

Figure 5 – Graphs of frequency detuning and phase fluctuations

The desire to increase the rate in the multipath channel with fading requires an increase in the number of subcarriers $N \geq 256$. This in turn leads to an increase in the peak factor of the signal.

To apply the adaptation algorithm, it is necessary that the modulation on each subcarrier provides an equal probability of error p_b. Given a number of advantages of phase shift keying (PSK) for channels with multipath [1, 2], we will analyze the change in peak factor when using a PSK signal. The probability of error in this case is:

$$p_b = \left(1 + \operatorname{erf}\left(A \big/ \sqrt{2}\sigma\right)\right) \big/ 2, \tag{2}$$

where σ is the noise power at the input of the resolver.

With multiple modulations, a multilevel signal is processed in the resolver. Then the probability of error p_b in a multilevel system is as follows:

$$p_b = \frac{1}{\log_2 N} \frac{N-1}{N} \operatorname{erfc}\left[\frac{U}{(N-1)\sqrt{2}\sigma}\right], \tag{3}$$

where $\operatorname{erfc}(X) = 1 - \operatorname{erf}(X)$, U^2 is the peak signal power.

Unlike expression (2), the expression (3) relates the probability of error to the peak signal U^2 power. The average signal power $U_{cp}{}^2$ in the N-level system is as follows:

$$U_{cp}{}^2=\frac{2}{N}\left[\left(\frac{U}{N-1}\right)^2+\left(\frac{3U}{N-1}\right)^2+\ldots+U^2\right]=\frac{2U^2}{N(N-1)^2}\sum_{i=1}^{N/2}(2i-1)^2\,. \tag{4}$$

In expressions (2) – (4), the value of U can be written as follows:

$$U=\pm\left(N-1\right)\cdot A, \tag{5}$$

where A is the normalization factor.

Taking into account expressions (4)–(5), the average power of the OFDM signal is determined by the formula:

$$U_{cp}{}^2=\frac{2\left(\sqrt{2}\left(N-1\right)A\right)^2}{N\left(N-1\right)^2}\sum_{i=1}^{N/2}\left(2i-1\right)^2=\frac{4A^2}{N}\sum_{i=1}^{N/2}\left(2i-1\right)^2\,. \tag{6}$$

To modulate QPSK from expression (6) we have:

$$U_{cp}{}^2=\frac{4A^2}{N}\sum_{i=1}^{N/2}(2i-1)^2=2A^2\,. \tag{7}$$

If the multiple of the modulation is change, the average signal power of the transmitter should remain unchanged and from formula (7) it follows:

$$A=\frac{1}{\sqrt{2}}\,. \tag{8}$$

Expression (8) is the normalization factor for QPSK modulation. When increasing the modulation frequency to maintain the average OFDM-signal power unchanged, it is necessary to apply the appropriate normalization factors. For example, for $N=16$ coefficient $A=1\big/\sqrt{10}$, for $N=64$ coefficient $A=1\big/\sqrt{42}$, for $N=256$ coefficient $A=1\big/\sqrt{195}$.

In conclusion, we note that a large number of subcarriers can be used in OFDM signals, for example, 2048. Therefore, the task of allocating resources across subcarriers is a multi-parameter nonlinear programming problem. The most simplified, in our opinion, approach is the use of the iteration method, which gives the following result: if required $p_b=10^{-5}$, the use of adaptation in bits and power allows you to increase the information transfer rate by 10 %. It is possible to reduce the sensitivity of an OFDM-signal receiver to phase fluctuations on the path of applying multiple modulation. In channels with multipath and fading, the use of m-PSK is more preferable compared to other multiple modulations.

In addition to these advantages, there are deficiencies in the OFDM system due to the increased peak factor of the emitted signal in comparison with single frequency transmission. Note that the peak factor of the signal increases with increasing number of frequencies N.

Thus, compared with other methods of dealing with ISI, as for example, given in [3] the OFDM method does not give a clear advantage and in each case depending on the system requirements the developer needs to look for a “hybrid” solution.

ЛІТЕРАТУРА

1 Варгаузин В.А., Цикин И.А. Методы повышения энергетической и спектральной эффективности цифровой радиосвязи. – СПб.: БХВ-Петербург, 2013. – 352 с.

2 Проакис Дж. Г. Цифровая связь. – М.: Радио и связь, 2000. – 800 с.

3 Почерняев В.Н. Борьба с межсимвольной интерференцией в мобильных системах связи// Зв’язок, 2006. – №4. – С. 18–20.

REFERENCES

1 Vargauzin V.A., Tsikin I.A. Methods for improving the energy and spectral efficiency of digital radio communications. – St. Petersburg: BHV-Petersburg, 2013. – 352 p.

2 Proakis G. Digital communication. – M.: Radio and communication, 2000. – 800 p.

3 Pochernyaev V.N. Combating intersymbol interference in mobile communication systems // Zv'jazok, 2006. - №4. – P.18–20.