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V.Ya. Gutlyanskiĭ, V.I. Ryazanov, A.S. Yefimushkin

Institute of Applied Mathematics and Mechanics of the NAS of Ukraine, Slovyansk
E-mail: vladimirgut@mail.ru, vl_ryazanov@mail.ru, a.yefimushkin@gmail.com

On a new approach to the study of plane boundary-value problems

Presented by Corresponding Member of the NAS of Ukraine V.Ya. Gutlyanskiĭ

We give a short description of our recent results obtained by a new approach to the boundary-value problems, such as the Dirichlet, Hilbert, Neumann, Poincaré and Riemann problems, for the Beltrami equations and for analogs of the Laplace equation in anisotropic and inhomogeneous media. We show that the approach makes it possible to study many problems of mathematical physics with arbitrary boundary data which are measurable with respect to logarithmic capacity.

Keywords: *Beltrami equation, boundary-value problems, anisotropic media, inhomogeneous media.*

1. Introduction. Here, we give our recent results obtained by a new approach on boundary-value problems for the Beltrami equations and for analogs of the Laplace equation in anisotropic and inhomogeneous media, cf. [8] and [13].

Let D be a domain in the complex plane $\mathbb{C}$ and let $\mu : D \to \mathbb{C}$ be a measurable function with $|\mu(z)| < 1$ a.e. The equation of the form

$$f_{\bar{z}} = \mu(z) \cdot f_z , \tag{1}$$

where $f_{\bar{z}} = \bar{\partial} f = (f_x + i f_y)/2$, $f_z = \partial f = (f_x - i f_y)/2$, $z = x + iy$, f_x and f_y are partial derivatives of the function f with respect to x and y, respectively, is said to be a *Beltrami equation*. The Beltrami equation (1) is said to be *nondegenerate* if $\|\mu\|_\infty < 1$.

Note that a great number of new theorems on the existence and on the boundary behavior of homeomorphic solutions and, on this basis, on the Dirichlet problem for the Beltrami equations with essentially unbounded *distortion quotients* $K_\mu(z) = (1 + |\mu(z)|)/(1 - |\mu(z)|)$ (see, e.g., [6, 7, 10] and references therein) were recently established. However, under the study of other boundary-value problems for (1), we restrict ourselves to the nondegenerate case, because this research leads in the contrary case to a problem on the distortion of boundary measures. Recall that the (continuous, discrete, open) homeomorphic solutions with distributional derivatives of the nondegenerate Beltrami equations are called *quasiconformal (functions) mappings* (see, e.g., [1, 5, 9]).

The first relevant problem is the measurement of sets on boundaries of domains. Recall that the sets of the length measure zero, as well as of the harmonic measure zero, are invariant under conformal mappings. However, they are not invariant under quasiconformal mappings, as it follows from the famous Ahlfors–Beurling example of quasisymmetric mappings of the real axis that are not absolutely continuous (see [2]). Hence, we are forced to apply the so-called absolute harmonic measure by Nevanlinna instead of them, in other words, the logarithmic capacity (see, e.g., [11]), whose zero sets are invariant under quasiconformal mappings.

By the well-known Priwalow uniqueness theorem, the analytic functions in the unit disk $\mathbb{D}=\{z\in\mathbb{C}:|z|<1\}$ coincide if they have the equal boundary values along all nontangential paths to a set E of points in $\partial\mathbb{D}$ of a positive length, see, e.g., Theorem IV.2.5 in [12]. The theorem is valid also for the analytic functions in Jordan domains with rectifiable boundaries (see, e.g., Section IV.2.6 in [12]). However, the examples of Luzin and Priwalow show that there exist nontrivial analytic functions in $\mathbb{D}$, whose radial boundary values are equal to zero on sets $E\subseteq\partial\mathbb{D}$ of a positive measure (see, e.g., Section IV.5 in [12]). Simultaneously, by Theorem IV.6.2 in [12] of Luzin and Priwalow, the uniqueness result is valid if E is of the second category. Theorem 1 in [4] demonstrates that the latter condition is necessary.

Recall Baire's terminology for categories of sets and functions. Namely, given a topological space X, a set $E\subseteq X$ is of the *first category* if it can be written as a countable union of nowhere dense sets, and is of the *second category* if E is not of first category. Given topological spaces X and X_*, $f:X\to X_*$ is said to be a *function of Baire class 1* if $f^{-1}(U)$ for every open set U in X_* is an F_σ set in X, where F_σ is the union of a sequence of closed sets.

Theorem 1 in [4] can be formulated in the following way:

Theorem A. *Let D be a bounded domain in $\mathbb{C}$, whose boundary consists of a finite number of mutually disjoint Jordan curves, and let $\{\gamma_\zeta\}_{\zeta\in\partial D}$ be a family of Jordan arcs of the Bagemihl–Seidel class in D.*

Suppose M is an F_σ set of the first category on ∂D and $\Phi(\zeta)$ is a complex-valued function of Baire class 1 on M. Then there is a nonconstant single-valued analytic function $f:D\to\mathbb{C}$ such that, for all $\zeta\in M$ along γ_ζ,

$$\lim_{z\to\zeta} f(z)=\Phi(\zeta). \tag{2}$$

We say that a family of Jordan arcs $\{J_\zeta\}_{\zeta\in C}$ is of the *Bagemihl–Seidel class*, abbr. *class $\mathcal{BS}$*, if all J_ζ lie in a ring $\mathfrak{R}$ generated by C and a Jordan curve C_* in $\mathbb{C}$, $C_*\cap C=\varnothing$, J_ζ is joining C_*, $\zeta\in C$, every $z\in\mathfrak{R}$ belongs to a single arc J_ζ, and, for a sequence of mutually disjoint Jordan curves C_n in $\mathfrak{R}$ such that $C_n\to C$ as $n\to\infty$, $J_\zeta\cap C_n$ consists of a single point for each $\zeta\in C$ and $n=1,2,\ldots$, cf. [4, pp. 740–741].

In particular, a family of Jordan arcs $\{J_\zeta\}_{\zeta\in C}$ is of class $\mathcal{BS}$ if J_ζ is generated by an isotopy of C. For instance, every curvilinear ring $\mathfrak{R}$, one of whose boundary components is C, can be mapped with a conformal mapping g onto a circular ring R and the inverse mapping $g^{-1}:R\to\mathfrak{R}$ maps radial lines in R onto suitable Jordan arcs J_ζ and centered circles in R onto Jordan curves giving the corresponding isotopy of C to other boundary component of $\mathfrak{R}$. We may also choose a curve in R, which is tangent to its boundary components and which intersects every centered circle in R only one time, and to obtain the rest lines by its rotation.

Finally, if $\Omega\subset\mathbb{C}$ is an open set bounded by a finite collection of mutually disjoint Jordan curves, then we say that a family of Jordan arcs $\{J_\zeta\}_{\zeta\in\partial\Omega}$ is of class $\mathcal{BS}$ if its restriction to each component of $\partial\Omega$ is so.

On the basis of Theorem A, in the case of domains D whose boundaries consist of rectifiable Jordan curves, Theorem 2 in [4] on the existence of analytic functions $f: D \to \mathbb{C}$ such that (2) holds a.e. on ∂D with respect to the natural parameter for each prescribed measurable function $\Phi: \partial D \to \mathbb{C}$ was formulated.

2. Boundary-value problems for quasiconformal functions. The following statement is similar to Theorem 2 in [4] but formulated in terms of a logarithmic capacity instead of a natural parameter. It is worth nothing that this theorem does not assume that boundary Jordan curves are rectifiable. This fact is key.

Theorem 1. *Let D be a bounded domain in $\mathbb{C}$, whose boundary consists of a finite number of mutually disjoint Jordan curves, and let a function $\Phi: \partial D \to \mathbb{C}$ be measurable with respect to the logarithmic capacity.*

Suppose that $\{\gamma_\zeta\}_{\zeta\in\partial D}$ is a family of Jordan arcs of class $\mathcal{BS}$ in D. Then there is a nonconstant single-valued analytic function $f: D \to \mathbb{C}$ such that (2) *holds along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the logarithmic capacity.*

Theorem 1 allows one to solve the Dirichlet, Hilbert, Riemann, Neumann, and Poincaré boundary-value problems for analytic functions, as well as quasiconformal functions with an arbitrary prescribed complex dilatation μ.

Theorem 2. *Let D be a bounded domain in $\mathbb{C}$, whose boundary consists of a finite number of mutually disjoint Jordan curves, let $\mu: D \to \mathbb{C}$ be a (Lebesgue) measurable function with $\|\mu\|_\infty < 1$, and let a function $\Phi: \partial D \to \mathbb{C}$ be measurable with respect to the logarithmic capacity. Suppose that $\{\gamma_\zeta\}_{\zeta\in\partial D}$ is a family of Jordan arcs of class $\mathcal{BS}$ in D. Then the Beltrami equation* (1) *has a regular solution $f: D \to \mathbb{C}$ such that* (2) *holds along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the logarithmic capacity.*

Recall that the classical statement of the *Hilbert (Riemann—Hilbert) boundary-value problem* is to find analytic functions *f* in a domain $D \subset \mathbb{C}$ bounded by a rectifiable Jordan curve with the boundary condition

$$\lim_{z\to\zeta} \mathrm{Re}\{\overline{\lambda(\zeta)} \cdot f(z)\} = \varphi(\zeta), \quad \forall \zeta \in \partial D, \tag{3}$$

where the functions λ and φ were continuously differentiable with respect to the natural parameter *s* on ∂D and, moreover, $|\lambda| \neq 0$ everywhere on ∂D. Hence, without loss of generality, one can assume that $|\lambda| \equiv 1$ on ∂D.

Theorem 3. *Let D be a bounded domain in $\mathbb{C}$, whose boundary consists of a finite number of mutually disjoint Jordan curves, let $\mu: D \to \mathbb{C}$ be a (Lebesgue) measurable function with $\|\mu\|_\infty < 1$, and let $\lambda: \partial D \to \mathbb{C}$, $|\lambda(\zeta)| \equiv 1$, $\varphi: \partial D \to \mathbb{R}$ and $\psi: \partial D \to \mathbb{R}$ be functions that are measurable with respect to the logarithmic capacity.*

Suppose that $\{\gamma_\zeta\}_{\zeta\in\partial D}$ is a family of Jordan arcs of class $\mathcal{BS}$ in D. Then the Beltrami equation (1) *has a regular solution $f: D \to \mathbb{C}$ such that*

$$\lim_{z\to\zeta} \mathrm{Re}\{\overline{\lambda(\zeta)} \cdot f(z)\} = \varphi(\zeta), \tag{4}$$

$$\lim_{z\to\zeta} \mathrm{Im}\{\overline{\lambda(\zeta)} \cdot f(z)\} = \psi(\zeta) \tag{5}$$

along γ_ζ for a.e. $\zeta \in \partial D$ with respect to the logarithmic capacity.

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Remark 1. Thus, the space of all solutions f of the Hilbert problem (4) in the given sense has the infinite dimension for any such prescribed φ, λ and $\{\gamma_\zeta\}_{\zeta\in D}$, because the space of all functions $\psi:\partial D\to\mathbb{R}$, which are measurable with respect to the logarithmic capacity (note that continuous functions are so), has the infinite dimension. The same is valid for all other boundary-value problems.

Theorem 4. *Let D be a Jordan domain in $\mathbb{C}$, let $\mu: D\to\mathbb{C}$ be a function of the Hölder class C^α with $\alpha\in(0,1)$ and $|\mu(z)|\leqslant k<1$, $z\in D$, $\nu:\partial D\to\mathbb{C}$, $|\nu(\zeta)|\equiv 1$, and let $\Phi:\partial D\to\mathbb{C}$ be measurable with respect to the logarithmic capacity.*

Suppose that $\{\gamma_\zeta\}_{\zeta\in\partial D}$ is a family of Jordan arcs of class $\mathcal{BS}$ in D. Then the Beltrami equation (1) *has a regular solution $f: D\to\mathbb{C}$ of the class $C^{1+\alpha}$ such that*

$$\lim_{z\to\zeta}\frac{\partial f}{\partial\nu}(z)=\Phi(\zeta) \tag{6}$$

along γ_ζ for a.e. $\zeta\in\partial D$ with respect to the logarithmic capacity.

Here, $\dfrac{\partial f}{\partial\nu}$ denotes the derivative of u at ζ in the direction $\nu=\nu(\zeta)$:

$$\frac{\partial f}{\partial\nu}:=\lim_{t\to 0}\frac{f(\zeta+t\cdot\nu)-f(\zeta)}{t}. \tag{7}$$

Corollary 1. *For every measurable function $\Phi:\partial\mathbb{D}\to\mathbb{C}$, one can find a nonconstant single-valued analytic function $\Phi:\partial\mathbb{D}\to\mathbb{C}$ such that, for a.e. point $\zeta\in\partial\mathbb{D}$, there exist:*

1) the finite radial limit

$$f(\zeta):=\lim_{r\to 1}f(r\zeta), \tag{8}$$

2) the normal derivative

$$\frac{\partial f}{\partial n}(\zeta):=\lim_{t\to 0}\frac{f(\zeta+t\cdot n)-f(\zeta)}{t}=\Phi(\zeta), \tag{9}$$

3) the radial limit

$$\lim_{r\to 1}\frac{\partial f}{\partial n}(r\zeta)=\frac{\partial f}{\partial n}(\zeta), \tag{10}$$

where $n=n(\zeta)$ denotes the unit interior normal to $\partial\mathbb{D}$ at the point ζ.

Recall that the classical statement of the *Riemann problem* in a smooth Jordan domain D of the complex plane $\mathbb{C}$ is to find analytic functions $f^+: D\to\mathbb{C}$ and $f^-:\mathbb{C}\setminus\overline{D}\to\mathbb{C}$ that admit continuous extensions to ∂D and satisfy the boundary condition

$$f^+(\zeta)=A(\zeta)\cdot f^-(\zeta)+B(\zeta),\qquad \forall\zeta\in\partial D, \tag{11}$$

with prescribed Hölder continuous functions $A:\partial D\to\mathbb{C}$ and $B:\partial D\to\mathbb{C}$.

Theorem 5. *Let D be a domain in $\overline{\mathbb{C}}$, whose boundary consists of a finite number of mutually disjoint Jordan curves, $\mu:\mathbb{C}\to\mathbb{C}$ be a (Lebesgue) measurable function with $\|\mu\|_\infty<1$, $A:\partial D\to\mathbb{C}$, and $B:\partial D\to\mathbb{C}$ be functions that are measurable with respect to the logarithmic capacity. Suppose that $\{\gamma_\zeta^+\}_{\zeta\in\partial D}$ and $\{\gamma_\zeta^-\}_{\zeta\in\partial D}$ are families of Jordan arcs of class $\mathcal{BS}$ in D and $\mathbb{C}\setminus\overline{D}$, correspondingly.*

Then the Beltrami equation (1) *has regular solutions* $f^+ : D \to \mathbb{C}$ *and* $f^- : \overline{\mathbb{C}} \setminus \overline{D} \to \mathbb{C}$ *that satisfy* (11) *for a.e.* $\zeta \in \partial D$ *with respect to the logarithmic capacity, where* $f^+(\zeta)$ *and* $f^-(\zeta)$ *are limits of* $f^+(z)$ *and* $f^-(z)$ *as* $z \to \zeta$ *along* γ_ζ^+ *and* γ_ζ^-, *correspondingly.*

Furthermore, the space of all such couples (f^+, f^-) *has the infinite dimension for every couple* (A, B) *and any collections* γ_ζ^+ *and* γ_ζ^-, $\zeta \in \partial D$.

Remark 2. One can show that this approach makes it possible to solve other mixed and nonlinear boundary-value problems for the Beltrami equations and, in particular, for analytic functions.

3. Boundary-value problems for A-harmonic functions. Here, we give our results on the boundary-value problems for the Laplace equation and its generalizations corresponding to problems of mathematical physics in inhomogeneous and anisotropic media.

As known (see, e.g., [3]), if $f = u + i \cdot v$ is a regular solution of the Beltrami equation (1), then the function u is a continuous generalized solution of the divergence-type equation

$$\operatorname{div} A(z) \nabla u = 0 \tag{12}$$

called *A-harmonic function*, i.e. $u \in C \cap W^{1,1}$ and

$$\int_D \langle A(z) \nabla u, \nabla \varphi \rangle = 0 \quad \forall \varphi \in C_0^\infty(D), \tag{13}$$

where $A(z)$ is the matrix function:

$$A = \begin{pmatrix} |1-\mu|^2 / (1-|\mu|^2) & -2\operatorname{Im}\mu / (1-|\mu|^2) \\ -2\operatorname{Im}\mu / (1-|\mu|^2) & |1+\mu|^2 / (1-|\mu|^2) \end{pmatrix}. \tag{14}$$

As we see, the matrix $A(z)$ is symmetric, $\det A(z) \equiv 1$, and its entries $a_{ij} = a_{ij}(z)$ are dominated by $K_\mu(z)$, i.e., they are bounded if the Beltrami equation (1) is not degenerate.

Vice versa, the uniformly elliptic equations (12) with symmetric $A(z)$ and $\det A(z) \equiv 1$ just correspond to the nondegenerate Beltrami equations (1) with the coefficient

$$\mu = \frac{1}{\det(I + A)} (a_{22} - a_{11} - 2ia_{21}) = \frac{a_{22} - a_{11} - 2ia_{21}}{1 + \operatorname{Tr} A + \det A}, \tag{15}$$

where I denotes the identity 2×2 matrix, $\operatorname{Tr} A = a_{22} + a_{11}$ (see, e.g., theorem 16.1.6 in [3]). Following [8], call all such matrix functions $A(z)$ as those of *class* $\mathcal{B}$. Recall that (12) is said to be *uniformly elliptic* if $a_{ij} \in L^\infty$ and $\langle A(z)\eta, \eta \rangle \geqslant \varepsilon |\eta|^2$ for some $\varepsilon > 0$ and for all $\eta \in \mathbb{R}^2$.

The following statement on a potential boundary behavior of A-harmonic functions of the Dirichlet type is a direct consequence of Theorem 2.

Corollary 2. *Let* D *be a bounded domain in* $\mathbb{C}$, *whose boundary consists of a finite number of mutually disjoint Jordan curves,* $A(z)$, $z \in D$, *be a matrix function of class* $\mathcal{B}$, *and let a function* $\varphi : \partial D \to \mathbb{R}$ *be measurable with respect to the logarithmic capacity.*

Suppose that $\{\gamma_\zeta\}_{\zeta \in \partial D}$ *is a family of Jordan arcs of class* $\mathcal{BS}$ *in* D. *Then there exist A-harmonic functions* $u : D \to \mathbb{R}$ *such that*

$$\lim_{z \to \zeta} u(z) = \varphi(\zeta) \tag{16}$$

along γ_ζ *for a.e.* $\zeta \in \partial D$ *with respect to the logarithmic capacity.*

Furthermore, the space of all such A-harmonic functions u has the infinite dimension for any such prescribed A, φ, *and* $\{\gamma_\zeta\}_{\zeta\in\partial D}$.

The next conclusion in the particular case of the Poincaré problem on directional derivatives follows directly from Theorem 3.

Corollary 3. *Let* D *be a Jordan domain in* $\mathbb{C}$, $A(z)$, $z\in D$, *be a matrix function of class* $\mathcal{B}\cap C^{a}$, $\alpha\in(0,1)$, *and let* $\nu:\partial D\to\mathbb{C}$, $|\nu(\zeta)|\equiv 1$, *and* $\varphi:\partial D\to\mathbb{R}$ *be measurable with respect to the logarithmic capacity.*

Suppose that $\{\gamma_\zeta\}_{\zeta\in\partial D}$ *is a family of Jordan arcs of class* $\mathcal{BS}$ *in* D. *Then there exist A-harmonic functions* $u:D\to\mathbb{R}$ *of the class* $C^{1+\alpha}$ *such that*

$$\lim_{z\to\zeta}\frac{\partial u}{\partial\nu}(z)=\varphi(\zeta) \tag{17}$$

along γ_ζ *for a.e.* $\zeta\in\partial D$ *with respect to the logarithmic capacity.*

Furthermore, the space of all such A-harmonic functions u has the infinite dimension for any such prescribed A, φ, ν, *and* $\{\gamma_\zeta\}_{\zeta\in\partial D}$.

Now, the following statement concerning the Neumann problem for A-harmonic functions is a special significant case of Corollary 3.

Corollary 4. *Let* $A(z)$, $z\in\mathbb{D}$, *be a matrix function of class* $\mathcal{B}\cap C^{a}$, $\alpha\in(0,1)$, *and let* $\varphi:\partial\mathbb{D}\to\mathbb{R}$ *be measurable with respect to the logarithmic capacity.*

Then there exist A-harmonic functions $u:\mathbb{D}\to\mathbb{R}$ *of the class* $C^{1+\alpha}$ *such that, for a.e. point* $\zeta\in\partial\mathbb{D}$ *with respect to the logarithmic capacity, there exist:*

1) the finite radial limit

$$u(\zeta):=\lim_{r\to 1}u(r\zeta), \tag{18}$$

2) the normal derivative

$$\frac{\partial u}{\partial n}(\zeta):=\lim_{t\to 0}\frac{u(\zeta+t\cdot n)-u(\zeta)}{t}=\varphi(\zeta), \tag{19}$$

3) the radial limit

$$\lim_{r\to 1}\frac{\partial u}{\partial n}(r\zeta)=\frac{\partial u}{\partial n}(\zeta), \tag{20}$$

where $n=n(\zeta)$ *denotes the unit interior normal to* $\partial\mathbb{D}$ *at the point* ζ.

Furthermore, the space of all such A-harmonic functions u *has the infinite dimension for any such prescribed* A *and* φ.

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В.Я. Гутлянський, В.І. Рязанов, А.С. Єфімушкін

Інститут прикладної математики і механіки НАН України, Слов'янськ
E-mail: vladimirgut@mail.ru, vl_ryazanov@mail.ru, a.yefimushkin@gmail.com

ПРО НОВИЙ ПІДХІД ДО ВИВЧЕННЯ КРАЙОВИХ ЗАДАЧ НА ПЛОЩИНІ

Наведено короткий опис нещодавніх результатів, отриманих новим методом, по крайових задачах, таких як задачі Гільберта, Діріхле, Неймана, Пуанкаре та Рімана, для рівнянь Бельтрамі і аналогів рівнянь Лапласа в анізотропних і неоднорідних середовищах. Показано, що наш підхід дає можливість вивчати багато проблем математичної фізики з довільними граничними даними, вимірними відносно логарифмічної ємності.

Ключові слова: *рівняння Бельтрамі, крайові задачі, анізотропні середовища, неоднорідні середовища.*

В.Я. Гутлянский, В.И. Рязанов, А.С. Ефимушкин

Институт прикладной математики и механики НАН Украины, Славянск
E-mail: vladimirgut@mail.ru, vl_ryazanov@mail.ru, a.yefimushkin@gmail.com

О НОВОМ ПОДХОДЕ К ИЗУЧЕНИЮ КРАЕВЫХ ЗАДАЧ НА ПЛОСКОСТИ

Приводится краткое описание наших недавних результатов, полученных новым методом, по краевым задачам, таким как задачи Гильберта, Дирихле, Неймана, Пуанкаре и Римана, для уравнений Бельтрами и аналогов уравнений Лапласа в анизотропных и неоднородных средах. Показано, что наш подход позволяет изучать многие проблемы математической физики с произвольными граничными данными, измеримыми относительно логарифмической емкости.

Ключевые слова: *уравнение Бельтрами, граничные задачи, анизотропные среды, неоднородные среды.*