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CYBER-PHYSICAL MODEL OF THE IMMUNOSENSOR SYSTEM AT THE HEXAGONAL LATTICE WITH THE USE OF DIFFERENCE EQUATIONS OF THE POPULATION DYNAMICS

The **subject** matter of the study is a model of cyber-physical immunosensory systems. The **goal** of the work is to create and to study the stability of the cyber-physical model of the immunosensory system at the hexagonal lattice using difference equations. The following **tasks** are solved in the article: development of functional scheme and cyber-physical model of immunosensory system; creation of discrete dynamics of the studied system; development of dynamic logical simulation of the cyber-physical immune system; definition of permanent states for studying the stability of a model of an immunosensor at the hexagonal lattice; the analysis of the results of numerical simulation of the cyber-physical model of the immunosensory system in the form of image of phase planes, the probability of contact of antigens with antibodies, lattice images of the probability of antibody bonds and an electron signal from the converter, that characterizes the number of fluorescing pixels. The following **methods** are used: methods of mathematical statistics and random processes, methods of the theory of optimization and operations research. The following **results** were obtained: The cyber-physical model of the immunosensory system at the hexagonal lattice using the difference equations that takes into account the presence of colonies of antigens and antibodies localized in pixels as well as the diffusion of colonies of antigens between pixels was developed. Discrete dynamics of populations in conjunction with dynamic logic is described. A class of delay time difference equations was introduced to simulate the interaction of "antigen-antibody" in the pixels of the immunosensor. The stability of the cyber-physical model of the immunosensory system with the help of the R package is researched. The results of numerical simulation in the form of phase planes image, the probability of contact of antigens with antibodies, lattice images of the probability of antibody bonds and an electron signal from the converter, that characterizes the number of fluorescing pixels, are obtained. The identical and endemic stable states of the cyber-physical model of the immunosensory system at the hexagonal lattice using differential equations of population dynamics are proposed. **Conclusions**: The numerical simulation of the developed cyber-physical model of the immunosensory system was conducted. It is established that its qualitative behavior significantly depends on the time of the immune response r. An electrical signal, modeled by the number of fluorescent immunopips, is important in the design of cyber-physiological immunosensory systems and studies of their resilience. Limit cycle or steady focus determine the appropriate form of immunosensory electrical signal. The conclusion on the stability of immunosensors is based on the grid image of the pixels that are fluorescing. The obtained experimental results allowed to perform a complete analysis of the stability of the immunosensor model, taking into account the delay in time.

Keywords: cyber-physical model; immunosensory system; biosensor; immunosensor; stability of the model; difference equations; hexagonal lattice.

Introduction

Cyber-physical system (CPS) is a physical system that implements the integration of computing and physical processes. It occurs more often in the form of embedded systems and networks for monitoring and controlling physical processes in feedback systems. In such systems, the dynamics of physical processes is the source of information of the investigated phenomenon with the ability to control and calculate the signals of control of the object [1].

Cyber-physical systems are identified with the manifestation of the fourth industrial revolution that takes place in the modern world [2]. Thus, there is also a physical opportunity to use technologies of "Internet of Things", where it is necessary to use signals from sensors and measuring devices. Thus, more and more publications [3] appear in the literature that draw attention to the modern concepts and offer the innovative solutions. A. Platzer proposed an approach based on "dynamic logic", which describes and analyzes cyber-physical systems [4], [5]. In these works, the hybrid programs (HPs) are used at the simple programming language with the simple semantics. HPs allow the programmer to refer directly to the actual values of variables that represent the real values and determine their dynamics.

Analysis of the problem and the existing methods

With the growth of the pace of life and the need for more accurate methods for monitoring various parameters, interest in cyber-physical systems and biosensors as their components is growing in science and industry. Biosensors are an alternative to well-known measurement methods that are characterized by poor selectivity, high cost, poor stability, slow response, and can often be performed only by the highly trained personnel. This is a new generation of sensors that use biological material in a design that provides very high selectivity and allows you to quickly and simply measure [6], [7].

Cell biosensors can be used to quantify the infection of an organism with certain electrochemical or optical phenomena. The article [8] describes a cellular biosensor that uses electrochemical impedance spectroscopy. This biosensor is intended for counting human CD4 + cells. The sensing area of this biosensor includes electrode pixels, each of which is compared to the size of the CD4 + cell, which is attracted to the pixels of the electrode. They are detected by observing informative changes on the pixel. The "On" or "Off" state of the electrode pixel indicates the detection of one CD4 + cell. Thus, in order to calculate CD4 + cells, it is necessary to summarize electrode pixels in the "On" state.

This general approach to quantitative detection of cells is used to simulate an immunosensory system based

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on the fluorescence phenomenon. Immunosensors [9] are a subgroup of biosensors, in which the immunochemical reaction is associated with the transducer. The principle of all immunosensors is the specific molecular recognition of antigens by antibodies to form a stable complex.

An important stage in the design of cyber-physical immunosensory systems is the development and research of their mathematical models that adequately reflect the important aspects of the spatial structure of immune-pixels important in terms of the research tasks. After all, the quality of the immunosensor model determines the effectiveness of its processing methods in measuring systems. The design of cyber-physical immunosensory systems involves the selection of parameters that would ensure their operational stability. Such a task, in particular, arises in the development of an immunosensor, which includes a three-dimensional array of immune pixels, and which consists in finding appropriate parameters describing immunological and diffusion processes. This problem can be solved by developing and studying the stability of the corresponding cyber-physical model of the immunosensory system on a hexagonal lattice using difference equations.

Purpose of the work. To develop a cyber-physical model of the immunosensory system on a hexagonal lattice using the differential equations of population dynamics with the possibility of studying its stability. The article addresses the following tasks:

- development of functional scheme and cyber-physical model of immunosensory system;
- creation of discrete dynamics of the studied system;
- development of dynamic logical modeling of the cyber-physical immune system;
- definition of permanent states for studying the stability of a model of an immunosensor on a hexagonal lattice;
- analysis of the results of numerical simulation of the cyber-physical model of the immunosensory system in the form of image of phase planes, the probability of contact of antigens with antibodies, lattice images of the probability of antibody bonds and an electron signal from the converter, which characterizes the number of fluorescing pixels.

Solving the problem

Cyber-physical Immunosensory System (CPISS). The definition of the term "Cyber-physical sensory system (CPSS)" is given in [10]. This definition was introduced for the industrial use of sensors. The general definition of the CPSS involves "a higher degree of combination, system sharing, the ability to use embedded systems in the field of automation and compliance with existing standards." The considered approach is used for the characterization of CPISS, the functional scheme of which is presented in fig. 1 and allows to perform numerical simulation of the system under study.

According to [10], the definitions and schemes for CPIS are used to define the CPS. CPISS converts physically measured immunological parameters into the digital information, which enables them to process signals in time using certain algorithms. There is also an interaction with their own capabilities, requirements, internal data and internal tasks in terms of distribution to the same or higher level of the hierarchy.

The concept of CPS at the basis of the CPISS (the external rectangle in fig. 1) , with the account of the features of intellectual imaging sensors is used. With the additional skills (dotted line in fig. 1), the sensor extends to CPIS, which allows to receive more diagnostic information about the object being studied.

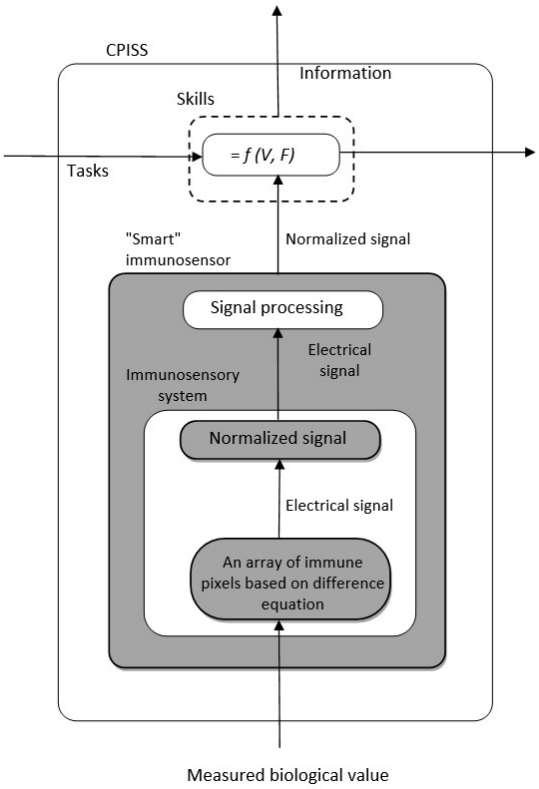

Fig. 1. Functional scheme of CPISS

Four main types of detection are used in immunosensory devices: electrochemical (potentiometric, amperometric or conductivity (capacitive), optical and thermometric [9]). All types of sensors can be used as direct (not marked) or as indirect (marked) immunosensors. Direct sensors are able to detect physical changes during the formation of the immune complex, while indirect use different levels of the generated signal that enable more sensible and universal detection in measuring systems.

CPISS refers to the high-intelligence information systems. They use an affordable set of interfaces that allow you to receive fast and accurate information of the status and internal system data that should be available to other CPSs. According to [10] CPISS as the self-organizing system requires comprehensive knowledge of its own dynamic structure and infrastructure of the general system. In order to make this, it is necessary to determine the types of immunosensory devices, taking into account their functional application. For example, immunosensors can be used to assess critical states in cardiovascular diseases, insulin values when measuring glucose levels in

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blood and to identify quantitative parameters in some pharmaceutical formulations.

In the article [10] the general structure of CPSS is proposed. While applying this scheme, in the case of immunosensors, three directions can be singled out: general information about the immunosensor; measurements of immunological parameters and skills in relation to unit conversion and calibration; interaction with other immunosensors. In this way, the certain methods are described that allow the immunosensor to be described. In the study of CPISS, the programming language R was used. Despite the great variety of programming languages used in the development of CPS (Assembly, C, C ++, D, Java, JavaScript, Python, Ada, etc. [11]), the language R is widely used in Nowadays, in many industries involved in machine learning and visualization of data.

Discrete Dynamics CPISS. For the CPISS dynamics we use the mathematical description with the help of nonlinear difference equations with delay.

The model of the immunosensor on the basis of a hexagonal lattice is considered. In this case, for the numbering of immune pixels (i, j, k), $i, j, k = \overline{-N, N}$, $i + j + k = 0$ the cubic coordinate system is used [11].

Let $V_{i,j,k}(t)$ is the concentration of antigens, $F_{i,j,k}(t)$ is the concentration of antibodies in the immunopixel (i, j, k); $i, j, k = \overline{-N, N}$, $i + j + k = 0$.

The model is based on such biological assumptions for an arbitrary immunopile (i, j, k).

1. Antigens are detected, bind, and finally neutralized by antibodies with some probability velocity $\gamma > 0$.

2. It is assumed that when colonies of antibodies are absent, colonies of antigens are regulated by a logistic equation with a delay:

$$V_{i,j,k}(n+1) = (1 + \beta - \delta_\upsilon V_{i,j,k}(n-r))V_{i,j,k}(n), \qquad (1)$$

where β and δ_υ – positive numbers, and $r \geq 0$ mean latency of the negative responce of the antigens' colonies.

3. The fertility rate $\beta > 0$ for the antigen population is introduced.

4. Antigens are neutralized by antibodies at a certain probability rate $\gamma > 0$.

5. The population of antigens tries to reach a certain limit of saturation with a spee $\delta_\upsilon > 0$.

6. The diffusion of antigens from six adjacent pixels is considered $(i+1, j, k-1)$, $(i+1, j-1, k)$, $(i, j-1, k+1)$, $(i-1, j, k+1)$, $(i-1, j+1, k)$ i $(i, j+1, k-1)$ (fig. 1) with diffusion speed $D\Delta^{-2}$, where $D > 0$ – coefficient of diffusion; $\Delta > 0$ – distance between two adjacent pixels.

7. The constant mortality of antibodies $\mu_f > 0$ is introduced.

8. As a result of the immune response the antibody density increases with a probabilistic velocity $\eta\gamma$.

9. The antibody population is approaching a certain level of saturation with a speed $\delta_f > 0$.

10. The immune response occurs with some constant delay in a time $r > 0$.

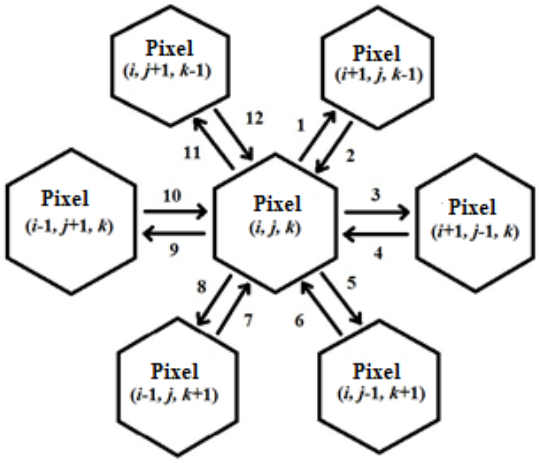

Fig. 2. Hexagonal latice, which binds six neighboring pixels in the model of the immunopixel using the cubic coordinates:

$$1, 3, 5, 8, 9, 11 - \left(\frac{D}{\Delta^2}V_{i,j,k}(t)\right); \; 2 - \left(\frac{D}{\Delta^2}V_{i+1,j,k-1}(t)\right);$$

$$4 - \left(\frac{D}{\Delta^2}V_{i+1,j-1,k}(t)\right); \; 6 - \left(\frac{D}{\Delta^2}V_{i,j-1,k+1}(t)\right);$$

$$7 - \left(\frac{D}{\Delta^2}V_{i-1,j,k+1}(t)\right); \; 10 - \left(\frac{D}{\Delta^2}V_{i-1,j+1,k}(t)\right);$$

$$12 - \left(\frac{D}{\Delta^2}V_{i,j+1,k-1}(t)\right).$$

On the basis of the above information, we will write the mathematical model of late-antigen-antibody interaction for a hexagonal array of immunopicles based on the well-known Marchuk model [12-14] and uses the spatial operator $\hat{S}$ proposed in [15] (additional information is on page 10.

$$\begin{aligned} V_{i,j,k}(n+1) &= V_{i,j,k}(n)\exp\left\{\beta - \gamma F_{i,j,k}(n-r) - \delta_\upsilon V_{i,j,k}(n-r)\right\} + \hat{S}\left\{x_{i,j,k}(n)\right\} \\ F_{i,j,k}(n+1) &= F_{i,j,k}(n)\exp\left\{-\mu_f + \eta\gamma V_{i,j,k}(n-r) - \delta_f F_{i,j,k}(n)\right\} \end{aligned}, \qquad (2)$$

where $\hat{S}\left\{V_{i,j,k}\right\}$ is a discrete diffusion for a spatial operator $\hat{S}$.

$$\hat{S}\left\{V_{i,j,k}\right\} = \begin{cases} D\Delta^{-2}\left[V_{i+1,j,k-1} + V_{i+1,j-1,k} + V_{i,j-1,k+1} + V_{i-1,j,k+1} + V_{i-1,j+1,k} + V_{i,j+1,k-1} - 6nV_{i,j,k}\right] \\ i, j, k \in \overline{-N+1, N-1}, \quad i + j + k = 0. \end{cases} \qquad (3)$$

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Dynamic logical simulation of CPISS. In order to simulate the dynamic logic of CPISS, we use the syntax proposed by A. Platser for the general CPS [4]. The CPS uses the Hybrid Programming Language (HP), which has more features than differential equations. The first level of HP is a dynamic program that is defined by the following grammar

$$a ::= V_{i,j,k}(n+1) = V_{i,j,k}(n)\exp\left\{\beta - \gamma F_{i,j,k}(n-r) - \delta_x V_{i,j,k}(n-r)\right\} + \hat{S}\left\{V_{i,j,k}(n)\right\},$$
$$F_{i,j,k}(n+1) = F_{i,j,k}(n)\exp\left\{-\mu_y + \eta\gamma V_{i,j,k}(n-r) - \delta_y F_{i,j,k}(n)\right\} \& \Phi_t, \quad (4)$$

where Φ_t is an evolutionary domain constraint in the form of a formula for the logic of the first order of real arithmetic

$$\Phi_t \overset{def}{\equiv} V^{\min} \le V_{i,j,k}(n) \le V^{\max},$$
$$\wedge F^{\min} \le F_{i,j,k}(n) \le F^{\max} \wedge i,j,k = \overline{-N,N} \wedge n > 0, i+j+k = 0. \quad (5)$$

The functioning of the immunopixel (i,j,k) is determined by two states, with respect to fluorescence. Namely, s_{fl} is a state of fluorescence and $s_{non\,fl}$ is one of the non-fluorescence states. The use of the first order of semantics of logic and the satisfaction ratio $s \models L$ for the first-order formula L of real arithmetic and state s can be determined for some pixels $(i,j,k); i,j,k = \overline{-N,N}$, $i+j+k=0$ states s_{fl} and $s_{non\,fl}$ as

$$s_{fl} \models k_{fl} V_{i,j,k}(n) F_{i,j,k}(n) \ge \theta_{fl},$$
$$s_{nonfl} \models k_{fl} V_{i,j,k}(n) F_{i,j,k}(n) < \theta_{fl}. \quad (6)$$

Discrete changes occur in computer programs when they accept new values for variables. This situation occurs when a fluorescence phenomenon occurs in a pixel (i,j,k); $i,j,k = \overline{-N,N}$, $i+j+k=0$. The state $s_{fl,i,j,k} := 1$ is assigned a value of 1 to the variable $s_{fl,i,j,k}$. This leads to a discrete, jump-like change, as the value $s_{fl,i,j,k}$ does not change smoothly, but rapidly when it suddenly changes from 1 to $s_{fl,i,j,k}$, causing a discrete jump of values $s_{fl,i,j,k}$. In this way, we obtain a discrete model of change $s_{fl,i,j,k} := 1$, except for the model of change (6).

Investigation of stability of immunosensor model on hexagonal lattice. Constant states.

In general, the state of equilibrium $\varepsilon_{i,j,k} \equiv \left(V_{i,j,k}, F_{i,j,k}\right)$, $i,j,k = \overline{-N,N}$, $i+j+k=0$ for the system (2) can be found as a solution of an algebraic system:

$$V_{i,j,k} = V_{i,j,k}\exp\left\{\beta - \gamma F_{i,j,k} - \delta_v V_{i,j,k}\right\} + \hat{S}\left\{V_{i,j,k}\right\}$$
$$F_{i,j,k} = F_{i,j,k}\exp\left\{-\mu_f + \eta\gamma x_{i,j,k} - \delta_f F_{i,j,k}\right\}. \quad (7)$$

Considering $\left(V_{i,j,k}, F_{i,j,k}\right)$, $i,j,k = \overline{-N,N}$, $i+j+k=0$, we have the following cases.

Stable state without antigens and antibodies $\varepsilon^{0,0}_{i,j,k} \equiv \varepsilon^{0,0} = (0,0)$, $i,j,k = \overline{-N,N}$, $i+j+k=0$.

Stable state without antibodies $\varepsilon^{*,0}_{i,j,k} \equiv \varepsilon^{*,0} = \left(\frac{\beta}{\delta_v}, 0\right)$, $i,j,k = \overline{-N,N}$, $i+j+k=0$.

Identical endemic steady state. In the case if $V_{i,j,k} \equiv V^{ident} > 0$, $i,j,k = \overline{-N,N}$, $i+j+k=0$, $\left(\hat{S}\left\{V_{i,j,k}\right\} \equiv 0\right)$, we receive the stable state $\varepsilon_{i,j,k} \equiv \varepsilon^{ident} = \left(V^{ident}, F^{ident}\right)$, where

$$V^{ident} = \frac{\beta\delta_f + \gamma\mu_f}{\eta\gamma^2 + \delta_v\delta_f}, \quad F^{ident} = \frac{-\mu_f\delta_v + \eta\gamma\beta}{\eta\gamma^2 + \delta_v\delta_f}.$$

So if $-\mu_f\delta_v + \eta\gamma\beta > 0$, then ε^{ident} is an endemic state.

Non-identical endemic steady state. In the general case, we need to solve the algebraic system (7) and find an endemic stable state, which will be called non-identical stationary state $\varepsilon^{non-ident} = (V^{non-ident}_{i,j,k}, F^{non-ident}_{i,j,k})$, $i,j,k = \overline{-N,N}$, $i+j+k=0$. In case all $(V^{non-ident}_{i,j,k}, F^{non-ident}_{i,j,k}) > 0$, then $\varepsilon^{non-ident}$ is an endemic state. Values V^{ident} and F^{ident} can be used as the initial approximations for numerical methods for solving a nonlinear algebraic system (7).

Numerical modeling.

Model (2) is considered at $h = 0.01^2$; $\beta = 2h$; $\gamma = 2h$; $\mu_f = h$; $\eta = 0.01184/\gamma$; $\delta_v = 0.5h$; $\delta_f = 0.5h$; $D/\Delta^2 = 2.22\sqrt{h}$; $N = 4$.

Similar to the model based on the differential equations [16, 17], in a system with the discrete time when the delay time value is changed r we observe the qualitative changes in the behavior of immune pixels and the model under study as a whole. Numerical modeling is performed at the values of the parameters given above. In this case, the long-term behavior of the system (2), which describes a hexagonal array of immunopixels at $N = 4$ for $r = 5$; $r = 17$; $r = 22$. Phase diagrams of antibody and antigen populations for pixel and adjacent pixels at different values are shown in fig. 3–5.

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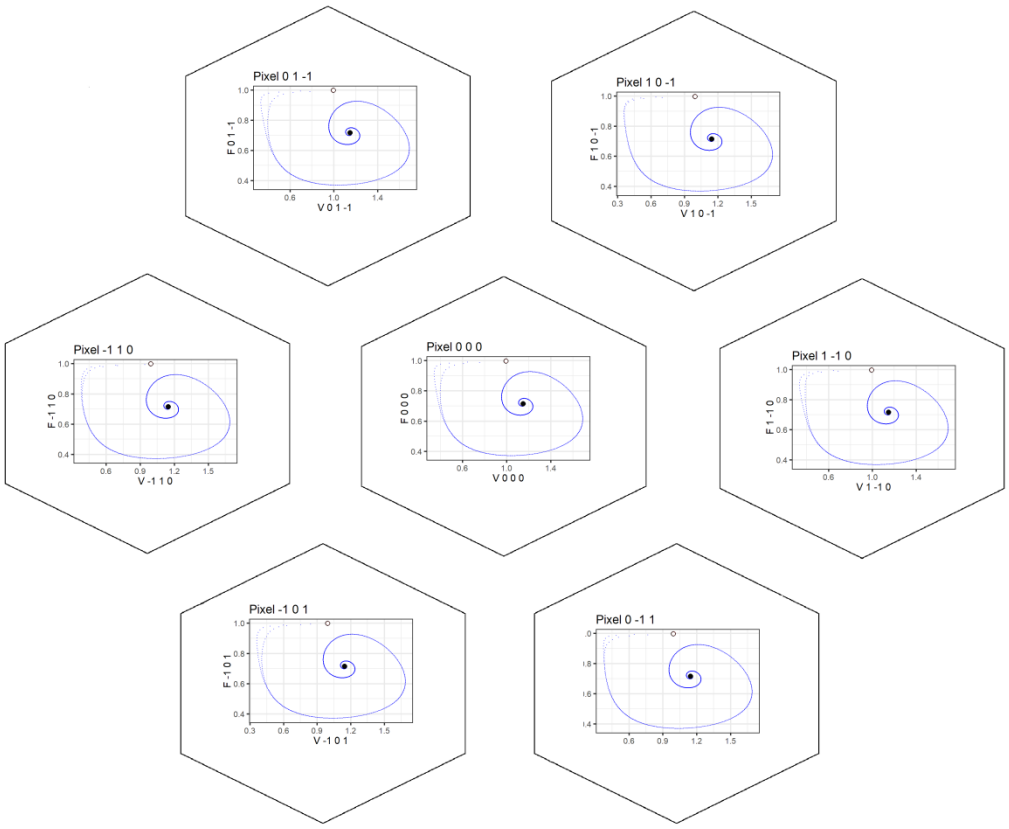

Fig. 3. Numerical modeling of the system (2) at $r = 5$. The image of the phase planes in coordinates $\left(V_{i,j,k}, F_{i,j,k}\right)$ for the pixel $(0,0,0)$ and its six neighboring pixels. Designation: ○ – identical stable state, ● – non-identical steady state

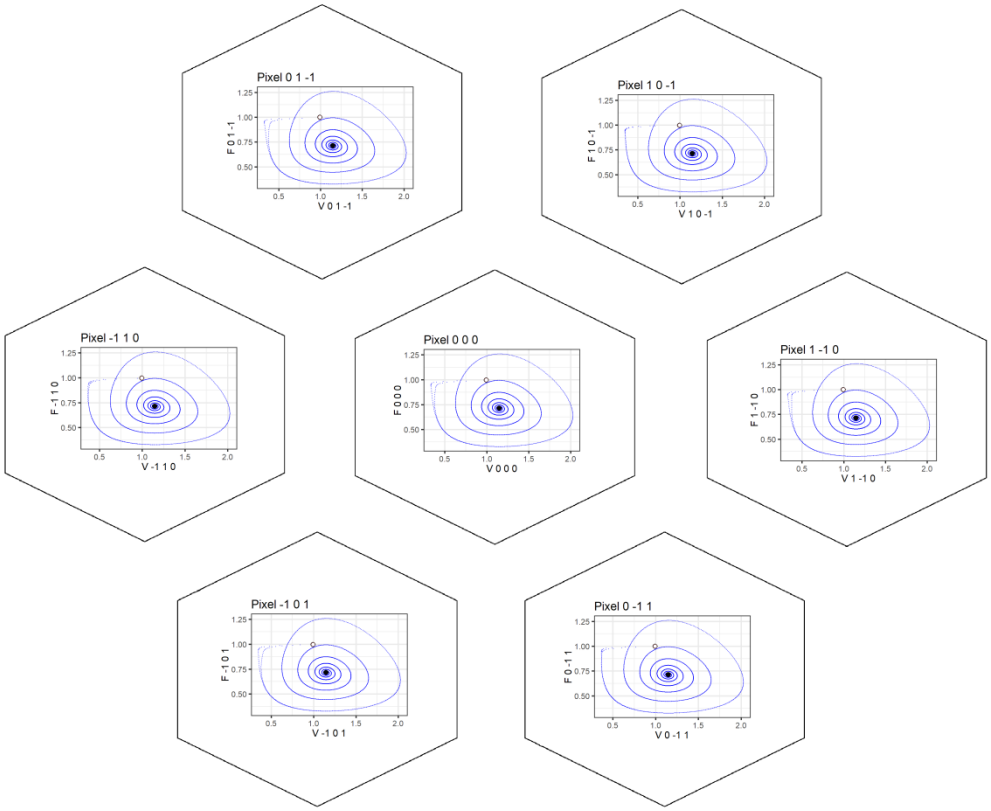

Fig. 4. Numerical modeling of the system (2) at $r = 17$. The image of the phase planes in the coordinates $\left(V_{i,j,k}, F_{i,j,k}\right)$ for a pixel $(0,0,0)$ and its six neighboring pixels. Symbols: ○ – identical steady state, ● – non-identical steady state

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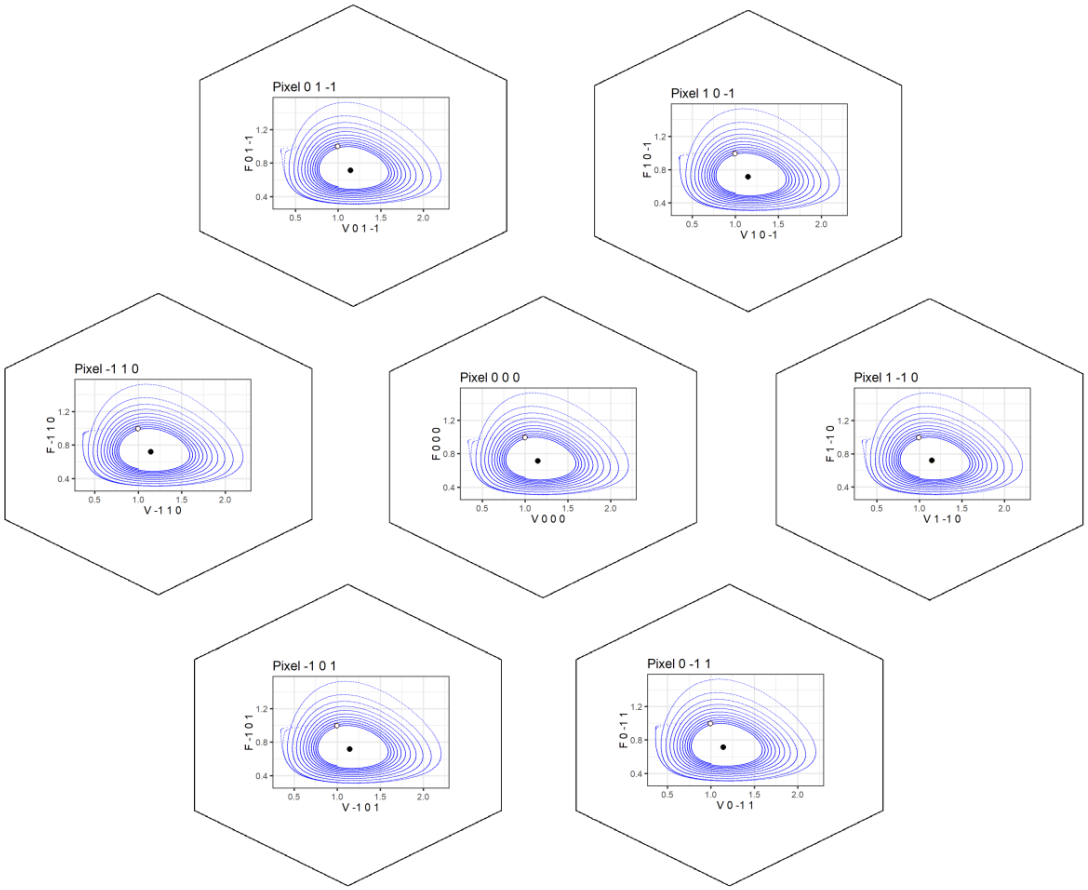
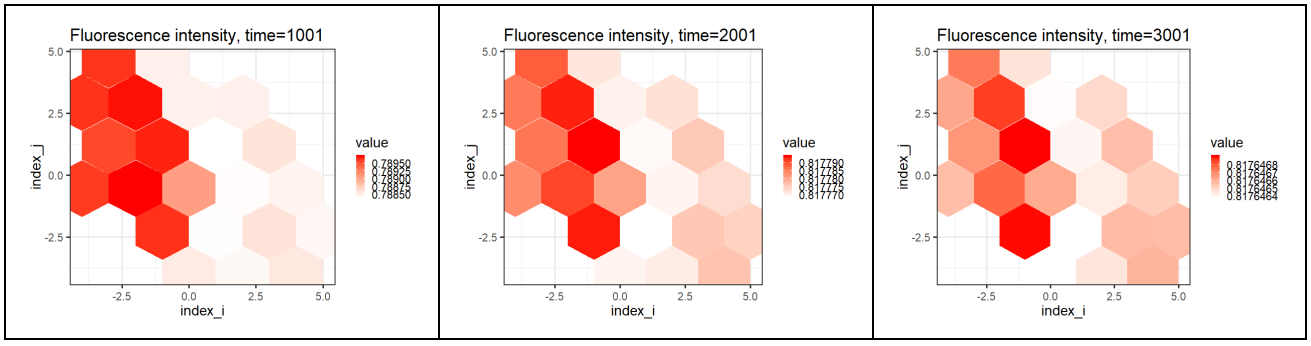


Fig. 5. Numerical modeling of the system (2) at $r = 22$. The image of the phase planes in the coordinates $\left(V_{i,j,k}, F_{i,j,k}\right)$ for a pixel $(0,0,0)$ and its six neighboring pixels. Symbols: ○ – identical steady state, ● – non-identical steady state.

Thus at $r \le 16$ there are trajectories that correspond to a stable focus for all pixels (fig. 3). At a value $r = 17$ Hopf bifurcation occurs – the following trajectories correspond to stable ellipsoidal boundary cycles for all pixels (fig. 4). The results of numerical modeling are consistent with the theoretical results on the basis of the theorem on the Hopf bifurcation [18], which confirms the appearance of small invariant cycles of the radius $O(\sqrt{h})$. Fig. 5 for $r = 22$ shows the phase diagrams, which are the limit cycles with two extremums (one local maximum and one local minimum).

Lattice graphs were used for numerical modeling of the cyber-physical model of the immunosensor. Firstly, the corresponding graphs were constructed, where the probability of antigen-antibody contact was given for each pixel, and as $V_{i,j,k} \times F_{i,j,k}$ at $r = 5$, $r = 17$, $r = 22$, are shown at fig. 6 (a-c).

At the second stage of numerical simulation of CPISS, lattice graphs of fluorescing pixels were obtained based on the condition (6) shown at fig. 7.



a)

Fig. 6. Lattice images of the probability of antibody bonds with antibodies in pixels of the system (2) at $r = 5$ (a)

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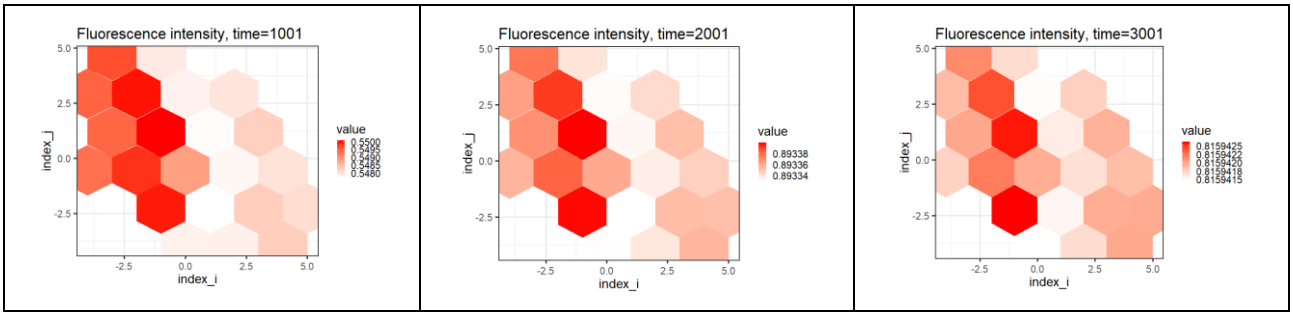
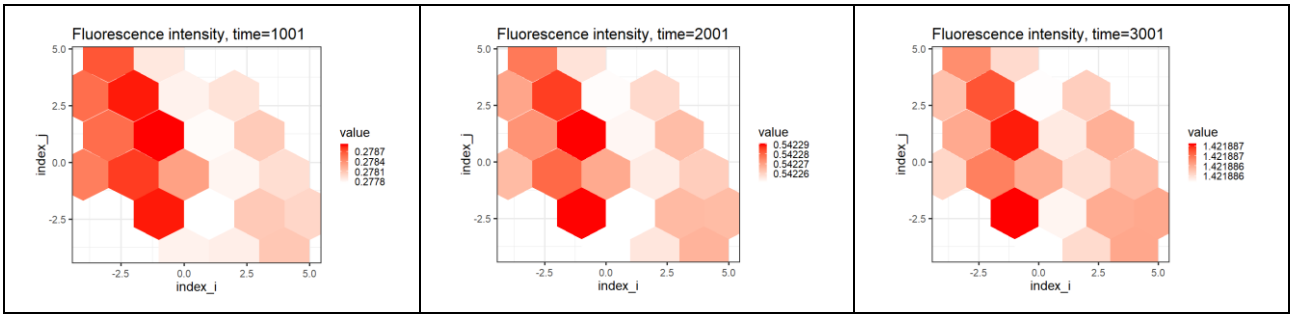
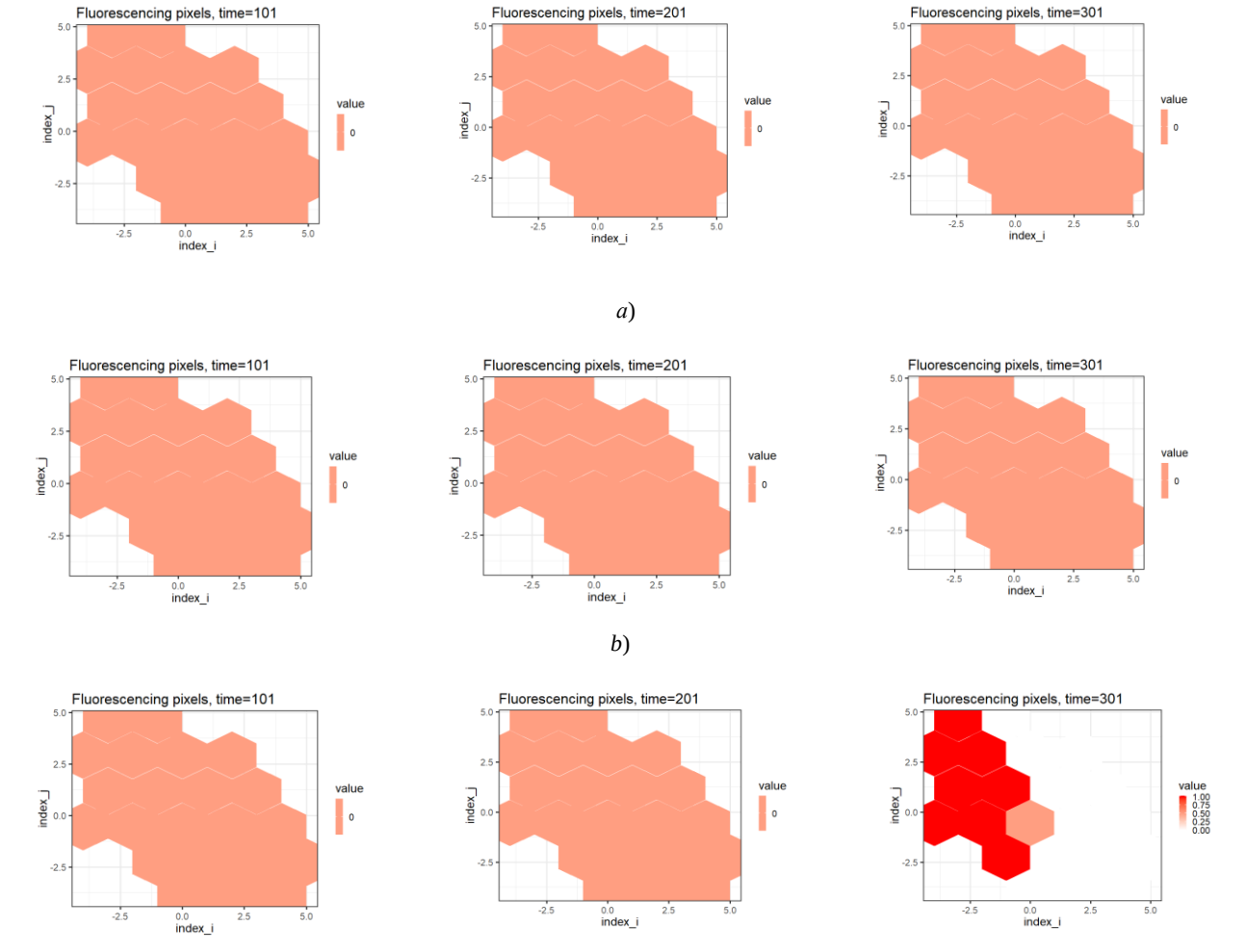


b)



c)

Tne end of the **fig. 6.** Lattice images of the probability of antibody bonds with antibodies in pixels of the system (2) at $r = 17$ (b), $r = 22$ (c).



a)

b)

c)

Fig. 7. Image of the fluorescence pixels of the system (2) at $r = 5$ (a), $r = 17$ (b), $r = 22$ (c)

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As an example of the final stage of numerical simulation of the CPISS, the form of an electrical signal of the converter is obtained, which characterizes the number of fluorescing pixels, depending on the different value of the delay in time r. Fig. 8 (a, b) shows the result of the numerical simulation of the system (2) at $r=5$ and $r=17$ that corresponds to a stable focus. At $r=22$ there is a running wave of fluorescence pixels that is represented in fig. 8 (c).

For the numerical simulation of CPISS, a threshold value for fluorescence was used $\Theta_{fl}=1,5$.

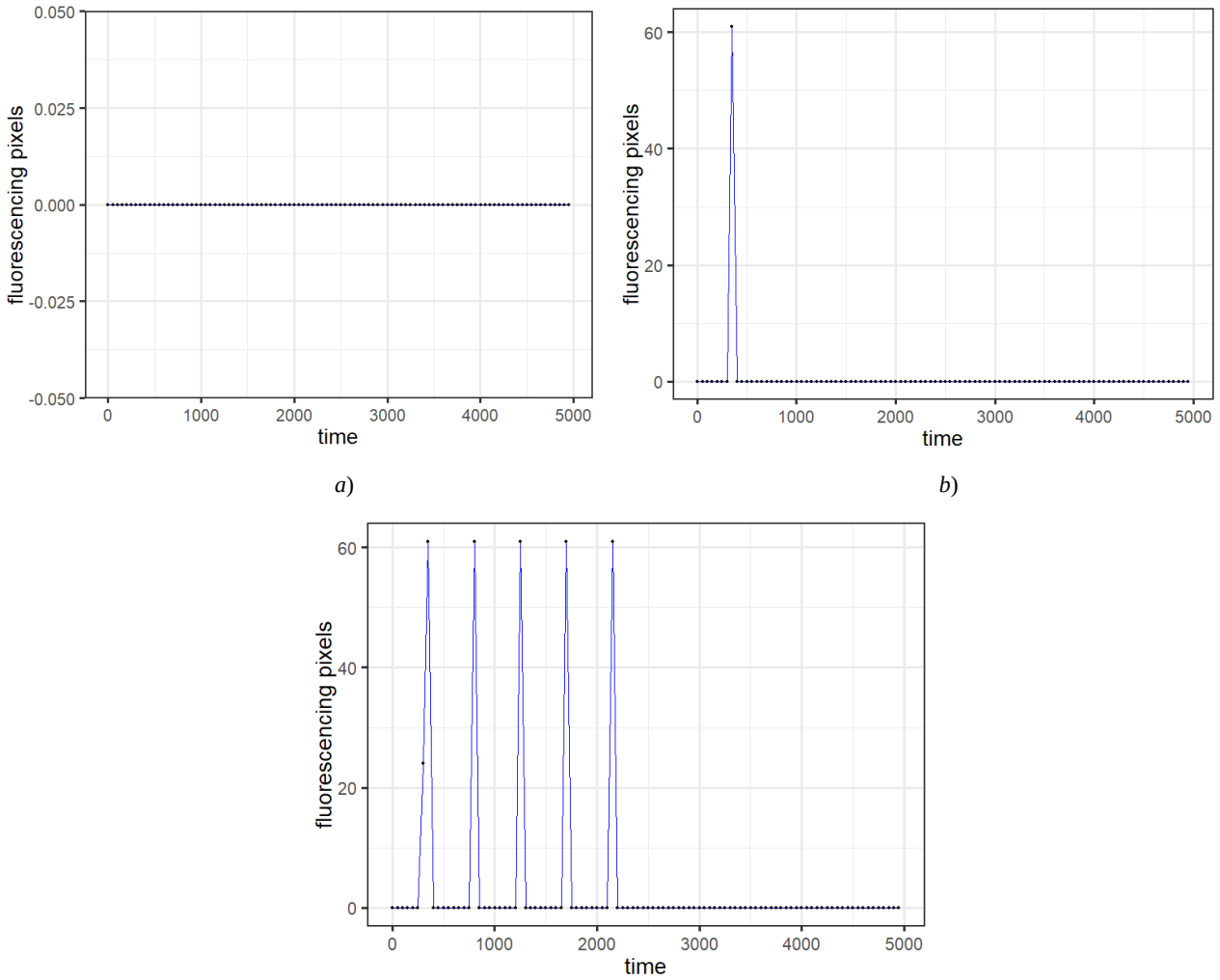

Fig. 8. An electrically transmitted signal from a transducer that characterizes the number of fluorescing pixels at $r=5$ (a), $r=17$ (b), $r=22$ (c)

As it was shown by the numerical analysis fluorescing states in immunopixels are changed according to the laws of discrete dynamics. Analyzing the obtained results, it was concluded that when changing the values of r, the behavior of pixels and CPISS changes qualitatively.

Conclusions

In this work a model of cyber-physical immunosensory system on a hexagonal lattice with the use of difference equations is developed and its stability is investigated. The presence of colonies of antigens and antibodies that are localized in pixels, as well as the diffusion of colonies of antigens between pixels was taken into account. The mathematical description of CPISS contains a discrete population dynamics, which is combined with the dynamic logic used for discrete events.

The continuous dynamics of the immunological response, each immunopixel is considered as CPISS was studied. The electrical signal, modeled by the number of fluorescent immuno pixels, is important in the design of CPISS and the study of their sustainability. Sustained focus or limiting cycle determines the appearance of an immunosensory electrical signal. The conclusion on the stability of immunosensors can be made on the basis of the lattice image of the pixels that are fluorescing. The experimental results obtained allow us to perform a complete analysis of the stability of the immunosensor model, taking into account the delay in time.

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КІБЕР-ФІЗИЧНА МОДЕЛЬ ІМУНОСЕНСОРНОЇ СИСТЕМИ НА ГЕКСАГОНАЛЬНІЙ РЕШІТЦІ З ВИКОРИСТАННЯМ РІЗНИЦЕВИХ РІВНЯНЬ ПОПУЛЯЦІЙНОЇ ДИНАМІКИ

Предметом дослідження є моделі кібер-фізичних імуносенсорних систем. **Мета** роботи – створення та дослідження стійкості кібер-фізичної моделі імуносенсорної системи на гексагональній решітці з використанням різницевих рівнянь. В статті вирішуються наступні **завдання**: розробка функціональної схеми та кібер-фізичної моделі імуносенсорної системи; створення дискретної динаміки досліджуваної системи; розробка динамічного логічного моделювання кібер-фізичної імуносенсорної системи; визначення постійних станів для дослідження стійкості моделі імуносенсора на гексагональній решітці; аналіз результатів чисельного моделювання кібер-фізичної моделі імуносенсорної системи у вигляді зображення фазових площин, ймовірності контакту антигенів з антитілами, решітчастих зображень ймовірності зв'язків антигенів з антитілами та елекричного сигналу з перетворювача, який характеризує кількість флуоресціюючих пікселів. Використовуються такі **методи**: методи математичної статистики та випадкових процесів, методи теорії оптимізації та дослідження операцій. Отримано наступні **результати**: Розроблено кібер-фізичну модель імуносенсорної системи на гексагональній решітці з використанням різницевих рівнянь, яка враховує наявність колоній антигенів та антитіл, що локалізовані у пікселях, а також дифузію колоній антигенів між пікселями. Описано дискретну динаміку популяцій у поєднанні з динамічною логікою. Введено клас різницевих рівнянь із запізненням в часі для моделювання взаємодії "антиген-антитіло" в пікселях імуносенсора. Досліджено стійкість кібер-фізичної моделі імуносенсорної системи за допомогою пакету R. Отримано результати чисельного моделювання у вигляді зображення фазових площин, ймовірності контакту антигенів з антитілами, решітчастих зображень ймовірності зв'язків антигенів з антитілами та елекричного сигналу з перетворювача, який характеризує кількість флуоресціюючих пікселів. Запропоновано ідентичний та ендемічний стійкі стани кібер-фізичної моделі імуносенсорної системи на гексагональній решітці з використанням різницевих рівнянь популяційної динаміки. **Висновки**: Проведено чисельне моделювання розробленої кібер-фізичної моделі імуносенсорної системи. Встановлено що її якісна поведінка суттєво залежить від часу імунної відповіді r. Електричний сигнал, що моделюється кількістю імунопікселів, які флуоресціюють, є важливим при проектуванні кібер-фізичних імуносенсорних систем та дослідженнях їх стійкості. Граничний цикл або стійкий фокус визначають відповідний вигляд імуносенсорного електричного сигналу. Висновок про стійкість імуносенсорів зроблено на основі решітчастого зображення пікселів, що флуоресціюють. Отримані експериментальні результати дозволили виконати повний аналіз стійкості моделі іммуносенсора з врахуванням запізнення в часі.

Ключові слова: кібер-фізична модель; імуносенсорна система; біосенсор; імуносенсор; стійкість моделі; різницеві рівняння; гексагональна решітка.

КИБЕР-ФИЗИЧЕСКАЯ МОДЕЛЬ ИМУНОСЕНСОРНОЙ СИСТЕМЫ НА ГЕКСАГОНАЛЬНОЙ РЕШЁТКЕ С ИСПОЛЬЗОВАНИЕМ РАЗНОСТНЫХ УРАВНЕНИЙ ПОПУЛЯЦИОННОЙ ДИНАМИКИ

Предметом исследования являются модели кибер-физических имуносенсорних систем. **Цель** работы – создание и исследование устойчивости кибер-физической модели имуносенсорной системы на гексагональной решётке с использованием разностных уравнений. В статье решаются следующие **задачи**: разработка функциональной схемы и кибер-физической модели имуносенсорной системы; создание дискретной динамики исследуемой системы; разработка динамического логического моделирования кибер-физической имуносенсорной системы; определение постоянных состояний для исследования устойчивости модели иммуносенсора на гексагональной решётке; анализ результатов численного моделирования кибер-физической модели имуносенсорной системы в виде изображения фазовых плоскостей, вероятности контакта антигенов с антителами, решётчатых изображений вероятности связей антигенов с антителами и элекрического сигнала с преобразователя, который есть характеристикой количества флуоресцирующих пикселей. Используются следующие **методы**: методы математической статистики и случайных процессов, методы теории оптимизации и исследования операций. Получены следующие **результаты**: Разработано кибер-физическую модель имуносенсорной системы на гексагональной решётке с использованием разностных уравнений, учитывающей наличие колоний антигенов и антител, которые локализованы в пикселях, а также диффузию колоний антигенов между пикселями. Описанная дискретная динамика популяций в сочетании с динамической логикой. Введено класс разностных уравнений с запаздыванием во времени для моделирования взаимодействия "антиген-антитело" в пикселях иммуносенсора. Исследована устойчивость кибер-физической модели имуносенсорной системы с помощью пакета *R*. Полученные результаты численного моделирования в виде изображения фазовых плоскостей, вероятности контакта антигенов с антителами, решетчатых изображений вероятности связей антигенов с антителами и элекрического сигнала с преобразователя, характеризующий количество флуоресцирующих пикселей. Предложено идентическое и эндемическое устойчивые состояния кибер-физической модели имуносенсорной системы на гексагональной решетке с использованием разностных уравнений популяционной динамики. **Выводы**: Проведено численное моделирование разработанной кибер-физической модели имуносенсорной системы. Установлено, что ее качественное поведение существенно зависит от времени иммунного ответа r. Электрический сигнал, моделируется количеством имунопикселей, которые флуоресцируют, является важным при проэктировании кибер-физических имуносенсорных систем и исследования их устойчивости. Предельный цикл или устойчивый фокус определяют соответствующий вид имуносенсорного электрического сигнала. Вывод об устойчивости иммуносенсора сделано на основе решетчастого изображения пикселей, что флуоресцируют. Полученные экспериментальные результаты позволили выполнить полный анализ устойчивости модели иммуносенсора с учетом опоздания во времени.

Ключевые слова: кибер-физическая модель; имуносенсорна система; биосенсор; иммуносенсор; устойчивость модели; разностные уравнения; гексагональная решётка.