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USE OF TRIANGULAR MODELS OF NON-STATIONARY PROCESSES IN MODELING VARIABILITY OF HEART RHYTHM

The **subject matter** is a mathematical model describing the process of heart rhythm variability, which is based on the use of triangular models of non-stationary random processes in the Hilbert space. The **goal** of the research is to develop a mathematical model of non-stationary processes of cardiac activity based on a triangular model. This research was the basis for the development of a Matlab model that implements the proposed method for analyzing heart rate variability. **Tasks** are: to give a description of the variability heart rate as a non-stationary process in Hilbert space in terms of correlation functions; to research the possibility of constructing a correlation and spectral theory of a non-stationary process using triangular models; to synthesize the mathematical model of nonstationary process on the basis of correlation theory for solving mathematical processing and forecasting tasks on the basis of ECG data. Using the proposed mathematical method, we implemented the Matlab model of a heart signal generator, which allowed us to synthesize an ECG with different variability parameters in noisy conditions. **Methods** of mathematical statistics, simulation modeling, theory of random processes and control theory are used in this work. **Results** of this research are as follows: 1) It has been shown that the new approach to the description of the HRV as a random process in the application of the triangular model in the Hilbert space made it possible to obtain expressions for the correlation function. 2) The imitation simulation showed the sensitivity of the method within the 5% error rate under the conditions of different types of influence on HRV. The qualitative assessment of the possibilities of the proposed models to generate artificial ECG provided the possibility of visual analysis by the cardiologist of the identity of the interpretation of real ECG records. The identities of modeling results were checked on time samples of electrocardiographs of 7 patients from open PhysioNet cardiographic libraries on samples with the duration T = 10 s. 3) The standard low-frequency oscillations and "white noise" barrier are clearly differentiated on the applied correlation function by the distribution of spectral density power within the frequency range of 0,15–0,3 Hz. **Conclusion**. The simulation results confirmed the correctness of the theoretical conclusions about the possibility of using models based on the representation of non-stationary processes in a triangular Hilbert space.

Keywords: heart rhythm; non-stationary random process; electrocardio signal; correlation function; triangular model; simulation modeling.

Introduction

Currently, one of the topical problems of modern medicine is the development of new methods of mathematical analysis of the aggregate of quantitative characteristics, obtained as a result of registration of certain parameters and that reflect the state of the human body. The need for development of this direction is dictated by the possibility of using the proposed mathematical approach to determine the availability and the degree of various pathological changes, to identify the early stages of the development of any disease.

The basic information on the state of the systems, that regulate the heart rhythm, is enclosed in the "functions of the spread" of the parameters of the cardio signal. One of the methods for assessing the state of the mechanisms of regulation of physiological functions in the human body is the variability of the heart rhythm (VHR) [1]. In case of heart arrhythmia of different origins the use of special mathematical methods for the restoration of the stationarity of the process under research or of specific analytical approaches is required [2–4].

The structure of the heart rhythm includes not only oscillating components in the form of respiratory and non-respiratory waves, but also non-periodic processes (so-called fractal components). The origin of these components of the heart rhythm is associated with the multilevel and non-linear nature of the processes of regulation of the heart rhythm and the presence of transient processes. Thus, the rhythm of the heart is not a strictly stationary random process with ergodic properties [5].

At this time, there is no common methodology in which it is possible to analyze the properties of a non-stationary random process of any type, using its individual implementation. Therefore, stationary random processes are used to analyze non-stationary processes [6, 7]. This highlights the need for the development of special simulation methods, which can only be used for certain classes of non-stationary processes.

Most often these random processes of heart rhythm are unique and can not be repeated under statistically similar conditions. One of the main issues is the study of non-stationary random processes that are considered as functions or sequences in the corresponding spaces, using triangular and universal models.Furthermore, the triangular models give the opportunity to construct some "elementary" non-stationary processes, and, with the help of universal models, to collect from them more general classes of non-stationary random functions or sequences [8].

The objectivity of the interpretation of the variability of the heart rhythm depends on the choice of optimal approaches to the mathematical processing of a numerical array in the form of which it appears. The urgency of the topic is determined by the need to improve the methods of studying the process of forming the heart rhythm and mathematical models that allow the synthesis of artificial electrocardiograms (ECG). The development of the method of generation of artificial electrocardiograms with variations of parameters under one or another random law. This will simulate the ECG with more informativity.

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Analysis of literary data and problem statement

The ECG signals can easily be described using three different approaches for any research such as: time domain, frequency domain, and frequency-time domain. The classic approach in electrocardiology is to use techniques for analyzing the time domain of the signal.As a rule, the ECG mathematical models are represented as a Fourier series [9], polynomial functions [10, 11] or by derivatives [12].

Such models determine the ECG value at any given time, often describing each segment or ECG deflection, but they are not always sufficient to describe all features of the ECG signal. In addition, due to a large number of variables, they are quite complex for further implementation.

A more realistic form of cardiac signal is provided by models in which individual elements are approximated by Gaussian functions. The variants of description and approximation of an ECG signal with application of Gaussian pulse and piecewise-specified function have been considered inthe work[13].

The proposed mathematical models do not take into account the effect of internal and external perturbations on the heart rhythm, that limits their scope of application.

Some authors use the ECG signal simulation in one-dimensional and two-dimensional planes. The construction of a one-dimensional signal of its phase plane is considered in [14]. The application of this approach allows to analyze both the amplitude and the speed parameters of any elements of the ECG, and to detect deviations in them in comparison with the traditional analysis of the ECS in the time domain. Thereafter, this made it possible to approximate the ECG with interpolation models described in the works [15, 16].

The similar approaches are used in a number of other works. In [17], the model based on the Fourier analysis of the phase plane obtained from two synchronized cardio signals was considered. In [18], the proposed method for describing the dynamic system of the cardio signal by constructing a three-dimensional phase space and equations describes the trajectory of the motion of points in this space.

However, such interpolation models do not take into account the physical and biological features of the pathological states of humain cardiovascular and respiratory systems.

The systematization of the results of the above-mentioned studies of the time domain suggests that the existing mathematical models allow to synthesize the cardio signal of a realistic form, considering it to be stationary.

For cases of non-stationary signals, which is a cardio signal, two approaches are used. First, the local Fourier transformation, which is a traditional one, it results in a frequency spectrum of the signal. In this case, the non-stationary signal, as with the stationary, is pre-divided into segments (windows) whose statistics do not change over time. The disadvantage of the Fourier transformation is that frequency components can not be localized over time, that is why it is sometimes impossible to get exhaustive signal information.The second approach is a wavelet transformation. In this case, the non-stationary signal is analyzed by decomposition of the basic functions obtained from a certain prototype by compression, stretching and shifts [19].

In cases where the heart rhythm is significantly non-stationary, that may be due to random effects, the most appropriate is the use of correlation analysis methods [4].

In this article we propose to use another approach, which is to consider the random process as a curve in a special Hilbert space. [6, 7]. This approach was used in the simulation modeling of non-stationary random processes in the technique [20].

The study of random scalar processes as mathematical objects of a rather complicated nature essentially reduces to the application of already regular functions. This allows the use of a well-designed functional analysis apparatus, in particular the theory of functional analysis and triangular models [21], for constructing a correlation and spectral theory of non-stationary random processes. The essence of the proposed transformation is the variability of cardiointervals as a random variable. With this approach, the correlation function of the dynamic series of cardiointervals for the ideal ECG and the one that is changed by this or that random process is determined.

Given that the cardio signals include deterministic, stochastic and chaotic components, this approach can be used for the analysis and modeling of heart activity processes.

The changing of the heart activity state in functional samples can be monitored visually or automatically using appropriate algorithms that take into account the non-stationarity and the non-linearity of the process. The ECG analysis can have an independent diagnostic and prognostic value. In practice, the repetition of samples is not always possible. Therefore, there is reason to believe that it is important to build a mathematical model of non-stationary process of heartactivity, it will allow the simulation modeling of behavior of the heart in a variety of influences, leading to significant variations in its monitored parameter.

Purpose and objectives of the research

The purpose of the work is to develop a mathematical model of non-stationary processes of heart activity based on a triangular model.

To achieve this goal the following tasks were solved:

- give the description of the variability of the heart rhythm as a non-stationary process in the Hilbert space in terms of correlation functions;
- to study the possibility of constructing a correlation and spectral theory of a non-stationary process using triangular models for this;
- to construct a mathematical model of a non-stationary process based on the correlation theory for solving mathematical processing and prediction tasks based on ECG data.

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Materials and methods for studying the variability of the heart rhythm. Study of the random process in the Hilbert space

The methods of mathematical statistics, simulation modeling, theory of random processes and control theory are used in this work.

The proposed method of using a triangular model may be used in cases when on the background of monotony of the rhythm there are sudden violations due to recording defects, to the appearance of various noises or to the occurrence of various types of arrhythmias, that can be represented as random processes.

Since the correlation function $K(t, s)$ is the kernel, which determines the random process $\xi(t)$ as a curve in the Hilbert space, the characteristic properties $\xi(t)$ are found in the properties of the function $K(t, s)$. The stochastic process $\xi(t)$ generated by the Cauchy problem is considered in this paper:

$$\begin{cases} \dfrac{d\xi(t)}{dt} = A(t)\xi(t) \\ \xi(0) = z_0 \end{cases}. \quad (1)$$

With certain restrictions on operator $A(t)$, which is convenient to formulate in terms of $K(t, s)$, it is possible to analyze the random process $\xi(t)$. In applied problems they arrive, more often, to partial differential equations for the correlation function $K(t, s)$. In this connection, the classes of non-stationary evolutionarily depicted random processes generated by the equations for correlation functions are of interest, while non-linear evolution operator equations are obtained for the operator $A(t)$. The solution of these equations explicitly gives the possibility to obtain new spectral solutions of some classes of non-stationary random curves.

Since the random process $\xi(t)$ in the Hilbert space H_ξ, then

$$\xi(t) = e^{iAt}\xi(0), \quad (2)$$

so any stationary curve in this space has a represantation (2), where A is a self-connected unbounded operator [6, 22].

Suppose $K(t, s)$ satisfies the equation

$$\left(\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial s^2}\right)K(t,s) = 0. \quad (3)$$

For the ermitian integral of the function $K(t, s)$ we assume that the operator has form $A(t) = iA$. From the definition for the correlation function $K(t, s)$ [6, 22] and expression (2) for the random process $\xi(t)$ we have:

$$\frac{\partial^2 K}{\partial t^2} = \frac{\partial\left(\dfrac{\partial\langle\xi(t),\xi(s)\rangle}{\partial t}\right)}{\partial t} = \left\langle\left(\frac{dA(t)}{dt} + A^2(t)\right)\xi(t),\xi(s)\right\rangle, \quad (4)$$

$$\frac{\partial^2 K}{\partial s^2} = \left\langle\xi(t),\left(\frac{dA(s)}{ds} + A^2(s)\right)\xi(s)\right\rangle. \quad (5)$$

Then we get the ratio

$$\left\langle\left(\frac{dA}{dt} + A^2(t)\right)\xi(t),\xi(s)\right\rangle = \left\langle\xi(t),\left(\frac{dA}{ds} + A^2(s)\right)\xi(s)\right\rangle. \quad (6)$$

Or

$$\langle B(t)\xi(t),\xi(s)\rangle = \langle\xi(t),B(s)\xi(s)\rangle,$$

where $B(t) = \dfrac{dA}{dt} + A^2(t)$, which does not depend on t, from the ratio (6) we get self-adjoint operator $B = B^*$, and for $A(t)$ Riccati operator equation [23]:

$$\frac{dA}{dt} + A^2 = B. \quad (7)$$

Taking into account the previous assumptions, for the random process $\xi(t)$ we have a differential equation of second order with a constant operator coefficient:

$$\xi''(t) = \frac{d\left(\xi'(t)\right)}{dt} = \frac{d\left(A(t)\xi(t)\right)}{dt} = \left(\frac{dA(t)}{dt} + A^2(t)\right)\xi(t) = B\xi(t). \quad (8)$$

If you use the spectral decomposition and look for a solution of equation (7) in the form of

$$A = \int_{-\infty}^{+\infty}\phi(t,\lambda)dE_\lambda, \quad (9)$$

then for the function φ we obtain Riccati scalar equation, the solution of which is a function which depend from the spectrum of the operator the following form $\phi(t,\lambda) = \sqrt{\lambda}\,\mathrm{th}\sqrt{\lambda}t$ and, so

$$A(t) = \int_{-\infty}^{+\infty}\sqrt{\lambda}\,\mathrm{th}\sqrt{\lambda}t\,dE_\lambda, \quad (10)$$

and for the random process $\xi(t)$ from the equation (8) we get the spectral expression

$$\xi(t) = \int_{-\infty}^{+\infty}\mathrm{ch}\sqrt{\lambda}t\,d\zeta(\lambda), \quad (11)$$

where $d\varsigma(\lambda) = dE_\lambda\xi_0$, that is $\zeta(t)$ – the standard curve in space H with orthogonal increments.

From the ratio (9) we have representation for the correlation function $K(t, s)$:

If the operator $B(t) \geq 0$ in equation (8), then $\lambda \in [0, +\infty)$

$$K(t,s) = \frac{1}{2}\int_0^{+\infty}\left[\mathrm{ch}\sqrt{\lambda}(t-s) + \mathrm{ch}\sqrt{\lambda}(t+s)\right]dF(\lambda). \quad (12)$$

If the operator $B(t) \leq 0$, then $\lambda \in (-\infty, 0]$

$$K(t,s) = \frac{1}{2}\int_0^{+\infty}\left[\cos\sqrt{\lambda}(t-s) + \cos\sqrt{\lambda}(t+s)\right]d\tilde{F}(\lambda), \quad (13)$$

where $\Delta F(\lambda) = \langle\zeta(\lambda+\Delta\lambda) - \zeta(\lambda), \zeta(\lambda+\Delta\lambda) - \zeta(\lambda)\rangle$; $\tilde{F}(\lambda) = -F(-\lambda)$.

It should be determined that from the ratio (11), the random process $\xi(t)$ can be represented as the sum of two

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orthogonal random processes where $\xi(t)=\xi^{(1)}(t)+\xi^{(2)}(t)$, $\xi^{(1)}(t)=\int_0^{+\infty}\mathrm{ch}\sqrt{\lambda}td\zeta_1(\lambda)$, $\xi^{(2)}(t)=\int_0^{+\infty}\mathrm{ch}\sqrt{\lambda}td\zeta_2(\lambda)$.

Then, the correlation functions $K(j)(t, s)$, $(j = 1, 2)$ are as follows (12) and (13) and

$$\left\langle\zeta_1(\lambda),\zeta_2(\lambda)\right\rangle=0\text{, де }\lambda\in[0,+\infty).$$

As an example, the case was considered when the operator has the following form:

$$A=\begin{pmatrix}0 & i\omega_1 & i\omega_1\\ -i\omega_1 & 0 & i\omega_2\\ -i\omega_1 & -i\omega_2 & 0\end{pmatrix},\qquad(14)$$

where ω_1, ω_2 – some constant values, and $\xi(t)=\{f_1(t),\ f_2(t),\ f_3(t)\}$ – three-dimensional vector in $spacel^2$. The coordinates $f_j(t),(j=1,2,3)$ satisfy the following system of equations:

$$\begin{cases}\dfrac{df_1}{dt}=-\omega_1(f_1+f_3)\\ \dfrac{df_2}{dt}=\omega_1 f_1-\omega_2 f_3\\ \dfrac{df_3}{dt}=\omega_1 f_1+\omega_2 f_2\end{cases}.\qquad(15)$$

Then we have a matrix representation of the expression $-i\dfrac{d\xi}{dt}=A\cdot\xi$, the solution of which is a random process of appearance (2).

Since the operator $A(t)$ does not depend on t, then for the operator $B(t)$ we obtain the following form:

$$B(t)=\begin{pmatrix}2\omega_1^2 & \omega_1\omega_2 & -\omega_1\omega_2\\ \omega_1\omega_2 & \omega_1^2+\omega_2^2 & \omega_1^2\\ -\omega_1\omega_2 & \omega_1^2 & \omega_1^2+\omega_2^2\end{pmatrix}.\qquad(16)$$

The random process $\xi(t)$ set by the system (15) and the operator $B(t)$ satisfy the equation (8).

If you enter coordinates $f_1(t)=\dot y(t)$, $f_2(t)=y(t)$, $f_3(t)=\nu(t)$, where $\nu(t)$ – "White noise", that is, oscillation with any frequencies and phases [24], and give the following value to the constants $\omega_1=\sqrt{\dfrac{T_2}{2T_1}}$, $\omega_2=\sqrt{\dfrac{2}{T_1^2T_2}}$, where T_1, T_2 – some time constants of the dynamic link, then the differentiation of the first line and the substitution in it from the second and third lines of the system (15) gives the equation

$$\ddot y(t)=-\frac{T_2}{2T_1^2}\dot y(t)+\sqrt{\frac{T_2}{2T_1^2}}\cdot\sqrt{\frac{2}{T_1^2T_2}}\nu(t)-\frac{T_2}{2T_1^2}\dot y(t)-\sqrt{\frac{T_2}{2T_1^2}}\cdot\sqrt{\frac{2}{T_1^2T_2}}y(t).$$

$$T_1^2\ddot y(t)+T_2\dot y(t)+y(t)=\nu(t),$$

which defines the differential equation of the dynamic link of the second order, the correlation function of the random process y (t) is given by the formula (13).

In this case, for the operator A in the expression (2) of the random process $\xi(t)$, the condition holds $I-A^*A=\langle\bullet,g\rangle g$, whereg – channel operator vector *A*.

After taking elemental calculations, we obtain that

$$\frac{\partial^2}{\partial t\partial s}K(t,s)-K(t,s)=-\left\langle\left(I-A^*A\right)\xi(t),\xi(s)\right\rangle.\qquad(17)$$

Since operator *A* is a quasionary operator of rank $r = 1$, then for formula (17) there is a representation $\dfrac{\partial^2}{\partial t\partial s}K(t,s)-K(t,s)=-\phi(t)\overline{\phi(s)}$, where $\phi(t)=\left\langle e^{itA}\xi_0,g\right\rangle$.

Using the expression of the matrix exponent of the linear bounded operator A [6, 22], we obtain that

$$\phi(t)=-\frac{1}{2\pi i}\oint_\Gamma e^{itA}\left\langle\xi(0),f(\bar\lambda)\right\rangle d\lambda,\qquad(18)$$

$$f(\bar\lambda)=\left(A^*-\bar\lambda I\right)^{-1}g,$$

where Γ – an arbitrary closed loop that covers the whole spectrum of the operator *A*.

If the model space is implemented as a Hilbert space l2, the operator is given as follows:

$$(\overset{\circ}{A}f)(k)=\lambda_k f(k)+i\sum_{j=k+1}^{N}f(j)\beta_j\beta_k\ \left(k=\overline{1,N}\right).\qquad(19)$$

Thus, the matrix expression for the operator has the form

$$\overset{\circ}{A}=\begin{Vmatrix}\lambda_1 & i\beta_1\beta_2 & \dots & i\beta_1\beta_k & \dots\\ 0 & \dots & \dots & i\beta_2\beta_k & \dots\\ \dots & \dots & \dots & \dots & \dots\\ \dots & \dots & \dots & \lambda_k & \dots\\ 0 & 0 & \dots & \dots & \dots\end{Vmatrix}.\qquad(20)$$

If you choose in this space the basis $\{h_k\}\Big|_{k=1}^{\infty}$:

$h_1=\{1,0,0,...\}$, $h_2=\{0,1,0,...\},..\ h_2=\{0,0,...0,10,...\},...$

Then the function $\varphi(t)$ can be represented in the following form:

$$\varphi(t)=\sum_{k=1}^{\infty}f_0(k)\left\{-\frac{1}{2\pi i}\oint_c e^{i\lambda t}\overline{\dot g_k(\lambda)}d\lambda\right\},\qquad(21)$$

where $f_0(k)=\langle f_0,h_k\rangle$, $\dot g_k(\lambda)=\left\langle\left(\overset{\circ}{A}^*-\bar\lambda I\right)^{-1}g,h_k\right\rangle$.

For further calculations, an expression for the resolvent of the operator is found $\overset{\circ}{A}^*$. Considering $R_{\underset{A^*}{\circ}}(\lambda)=\left(\overset{\circ}{A}^*-\lambda I\right)^{-1}\overset{\circ}{g}=f$, we got

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$$R_{\underset{A^*}{\circ}}(\lambda)=\frac{\beta_k}{\overline{\lambda_k}-\lambda}\prod_{j=1}^{k-1}\frac{\lambda_j-\lambda}{\overline{\lambda_j}-\lambda}. \quad (22)$$

For the function $\overline{\dot{g}_k(\lambda)}$ the following formula is used (22):

$$\overline{\dot{g}_k(\lambda)}=\frac{\beta_k}{\lambda_k-\lambda}\prod_{j=1}^{k-1}\frac{\overline{\lambda_j}-\lambda}{\lambda_j-\lambda}=\frac{\sqrt{2\mathrm{Im}\lambda_k}}{\lambda_k-\lambda}\prod_{j=1}^{k-1}\frac{\overline{\lambda_j}-\lambda}{\lambda_j-\lambda}. \quad (23)$$

Having added

$$\Lambda_k(t)=-\frac{1}{2\pi i}\sqrt{2\mathrm{Im}\lambda_k}\oint_c e^{i\lambda t}\frac{1}{\lambda_k-\lambda}\prod_{j=1}^{k-1}\frac{\overline{\lambda_j}-\lambda}{\lambda_j-\lambda}d\lambda, \quad (24)$$

we got

$$\phi(t)=\sum_{k=1}^{\infty}\xi_0(k)\Lambda_k(t). \quad (25)$$

The formula (24) shows that the function $\Lambda k\ (t)$ is uniquely constructed on the spectrum of the operator *A*, and $\Lambda k\ (t)$ can be represented as

$$\Lambda_k(t)=\sum_{j=1}^{k}P_{k,j}(t)e^{i\lambda jt}, \quad (26)$$

where $P_{k,j}(t)$ – a polynomial,the degree of which is one point less than the multiplicity of the eigenvalue λ_j.

Determination of the correlation function of the random process

If the random process has the following look:

$$\xi(t)=\xi e^{i\lambda_0 t}, \quad (27)$$

where the average value of ξ is zero, and λ_0 is a real constant $\left(\lambda_0=\overline{\lambda_0}\right)$.

Such process describes the periodic oscillation of the circular frequency λ_0 with a random amplitude and a random phase. In this case

$$\xi(t)=\xi\left(\xi_1\cos\lambda_0 t+\xi_2\sin\lambda_0 t\right)+i\left(\xi_1\sin\lambda_0 t-\xi_2\cos\lambda_0 t\right), \quad (28)$$

where $\xi=\xi_1-i\cdot\xi_2$.

The actual part of each implementation can be represented by a sinusoidal of the following form $\eta(t)=a\sin(\lambda_0 t+\theta)$, where $a=\sqrt{\xi_1^2+\xi_2^2}$, $\sin\theta=\frac{\xi_1}{\sqrt{\xi_1^2+\xi_2^2}}$, $\cos\theta=\frac{\xi_2}{\sqrt{\xi_1^2+\xi_2^2}}$, with a and θ varying from implementation to implementation.

For this case let's consider the model space l^2, and the operator $\overset{\circ}{A}$ in the form of: $\overset{\circ}{A}t=\lambda_0 t$. This is a case for an operator with a discrete spectrum and, by the expression (26), the Λ-function has the formof $\Lambda(t)=p_0e^{i\lambda 0t}$, where p_0 – constant value. Then for $\varphi(t)$ we will get the following expression $\phi(t)=c_0p_0e^{i\lambda 0t}=\zeta_0e^{i\lambda 0t}$.

Since $\mathrm{Im}\overset{\circ}{A}=0$ and the operator $\overset{\circ}{A}$ has completeness, in this case it is a dissipative asymptotically fading process, whose correlation function can be found by the following formula

$$K(t,s)=be^{i(t-s)\lambda 0}, \quad (29)$$

where *b* is the mathematical expectation of the square of the amplitude, proportional to the mean energy of the oscillation per unit time.

It should be noted that the correlation function does not depend on the statistical characteristics of the phase of oscillation. It is obviously possible to show that any stationary process can be obtained as a boundary of the sequence of processes with a discrete spectrum. Thus, each random process $\xi(t)$ can be arbitrarily well approximated by a linear combination of harmonic oscillations. The random stationary processes of general form are obtained by considering the linear combinations of the processes of the type (27).

Having considered the processes that are the superposition of *n*-random periodic oscillations with different frequencies

$$\xi(t)=\sum_{k=1}^{n}\xi_k e^{i\lambda_k t}, \quad (30)$$

where $M\xi1=M\xi2=\ldots=M\xi n=0$, taking into account that $M\xi(t)=0$, we have

$$M\xi(t+\tau)\overline{\xi(t)}=M\sum_{k=1}^{n}\left|\xi_k\right|^2e^{i\lambda_k\tau}+$$
$$+M\sum_{\substack{k,m=1\\(k\neq m)}}^{n}\xi_k\overline{\xi_m}\left(e^{i(\lambda k-\lambda m)t+i\lambda_k\tau}+e^{-i(\lambda k-\lambda m)t+i\lambda_k\tau}\right). \quad (31)$$

In order for the random process (30) to be stationary in the narrow sense, it is necessary that the last expression (31) does not depend on *t*. As the functions $e^{i(\lambda k-\lambda m)t}$ i $e^{-i(\lambda k-\lambda m)t}$ $(k\neq m)$ are linearly independent, there fore, this expression does not depend on *t*, in case if $M\xi_k\overline{\xi_m}=0$, with $k\neq m$.

Thus, the random process (30) will be stationary, if $\xi_k\left(k=\overline{1,n}\right)$ – are uncorrelated random variables with value of zero. Consequently, $\xi(t)$ describes the superposition *n* of non-correlated (in particular, independent) oscillations with different frequencies and random amplitudes and phases.

Since the correlation function of the sum of uncorrelated random processes is equal to the sum of the correlation functions of these processes, then by virtue of expression (29) for the correlation function of the stationary random process (30) we have

$$K(t-s)=\sum_{k=1}^{n}b_k e^{i\lambda_k(t-s)}, \quad (32)$$

where the coefficients $b_k > 0$ determine the average energy of the individual harmonic oscillations included in the expression (30), and λ – multitude $\{\lambda_k\}_{k=1}^{n}$ in this case, is

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the spectrum of a random process. Considering in equality (32) $t - s = 0$, we have

$$K(0) = \sum_{k=1}^{n} b_k \ . \quad (33)$$

Thus, in the case of superpositions of the periodic oscillations with uncorrelated amplitudes the middle energy of the cardio pulse equals the sum of the energies of certain periodic components.

In order for the process (30) to be valid, the number n is to be pair (equal $2n$) and $2j$ summands (30), must fall into j pairs of complex additions. In this case the random process $\xi(t)$ can be represented in the following form:

$$\xi(t) = \sum_{l=1}^{j} a_l \left(\sin \lambda_l t + \theta_l\right), \quad (34)$$

where taking into account (28) $M\eta_k\zeta = 0$ at all index valuesk, land $M\eta_k\eta_l = M\zeta_k\zeta_l = 0$ a $t_k \neq l$, a $M\eta_l^2 = M\zeta_l^2 = b_l$, $\sin\theta_l = \frac{\eta_l}{\sqrt{\eta_l^2 + \zeta_l^2}}$, $\cos\theta_l = \frac{\zeta_l}{\sqrt{\eta_l^2 + \zeta_l^2}}$, $a_l = \sqrt{\eta_l^2 + \zeta_l^2}$, $\left(l = \overline{1,n}\right)$.

The correlation function (32) taking into account the variable $\tau = t - s$ can be represented as:

$$K(\tau) = \sum_{l=1}^{j} b_l \cos \lambda_l \tau \ . \quad (35)$$

The obtained theoretical conclusions provide the basis for further simulation modeling.

Results of mathematical modeling of heart rhythm variability

A computer simulation modeling method for heart rhythm variability has been developed on the basis of the mathematical model of the correlation function of the non-stationary random process with a discrete spectrum. A series of simulation experiments has been conducted to confirm and statistically substantiate the adequacy of the developed model. The heart rhythm system is a dynamic system. The dynamics of heart rhythm changes is constantly influenced by the central nervous system (CNS) and the vegetative-vascular system (VVS), respiratory oscillations, blood oxygen saturation and other characteristics [25]. The HRV signal can be represented as a periodic curve formed by a common overlay of high-frequency and low-frequency oscillations.

If we assume that the input of the system simulating the cardio signal is "white noise", then there is an occasional process at the output reflecting the variability of the heart rhythm.

Each implementationXv(t) of the random process $\xi(t)$ can be represented as follows:

$$X_v(t) = Y_1^v(t) + Y_2^v(t), \quad (36)$$

$$Y_1^v(t) = a_0^v, \ Y_2^v(t) = \sum_{\omega=1}^{10} a_w^v \sin\left(\omega t + \theta_\omega^v\right), \quad (37)$$

where $Y_1^v(t)$ – an occasional process that is a particular case of a random stochastic process with the value $t = 0$, $Y_2^v(t)$ – random process, and θ_ω^v – initial phases. It is proved that the correlation function of the stochastic process $X_v(t)$ has the following form:

$$K_{X_V(t)}(\tau) = \left|a_0^v\right|^2 + \sum_{\omega=1}^{10} 2\pi \left|a_0^v\right|^2 \cos\omega t \ . \quad (38)$$

In [8] the connection between the correlation function $KX(\tau)$ of the random process $\xi(t)$ and its spectral density is considered $SX(\omega)$. It is shown that the spectral density $SX(\omega)$ is equal to the square of the amplitude-frequency characteristic of the link or system.

Using the mathematical model of heart rhythm variability, presented in [25], with slight simplifications, when simulating you should take into account the low-frequency oscillations of the central nervous system to the sinus node and oscillatory behavior of blood pressure.

Thus, the transfer function of the forming dynamic system of cardiac pressure, which transforms the "white noise" $v(t)$ into a random process $\xi(t)$, can be represented

$$W(p) = \frac{k}{\left(T_{11}^2 p^2 + T_{12} + 1\right)\left(T_{21}^2 p^2 + T_{22} + 1\right)}, \quad (39)$$

the spectral density has an analytical form

$$S_X(p) = \frac{k^2}{\left[\left(1 - \omega^2 T_{11}^2\right)^2 + \omega^2 T_{12}^2\right]\left[\left(1 - \omega^2 T_{21}^2\right)^2 + \omega^2 T_{22}^2\right]}. \quad (40)$$

The result of the simulation of the influence of random processes on the character of the VHR is presented as a series of two vibrational dynamic links with transmitting functions $W_1(p) = \frac{k}{\left(T_{11}^2 p^2 + T_{12} + 1\right)}$; $W_2(p) = \frac{1}{\left(T_{21}^2 p^2 + T_{22} + 1\right)}$, equations of which are written in the form of:

$$\begin{cases} T_{11}^2 \ddot{y}(t) + T_{12}\dot{y}(t) + y(t) = kv(t); \\ T_{21}^2 \ddot{X}(t) + T_{22}\dot{X}(t) + X(t) = y(t). \end{cases} \quad (41)$$

In the normal form the Cauchy system (41) looks like:

$$\begin{cases} \dot{x}_1(t) = x_2(t); \\ \dot{x}_2(t) = -\frac{1}{T_{11}^2} x_1(t) - \frac{T_{12}}{T_{11}^2} x_2(t) + \frac{k}{T_{11}^2} v(t); \\ \dot{x}_3(t) = x_4(t); \\ \dot{x}_4(t) = -\frac{1}{T_{21}^2} x_3(t) - \frac{T_{22}}{T_{21}^2} x_4(t) + \frac{k}{T_{21}^2} x_1(t) \end{cases}, \quad (42)$$

where $x_1(t) = y(t)$, $x_2(t) = \dot{y}(t)$, $x_3(t) = X(t)$, $x_4(t) = \dot{X}(t)$.

The solution of the system (42) uses the Runge-Kutta method [18, 22]. This method has a high degree of

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accuracy and, despite its complexity, is the most suitable for algorithmization. In addition, the important advantage of this method is the possibility of applying an "alternate step".

The simulation of heart rhythm variability was performed in Simulink / Matlab software using Runge-Kutta 4th order method with variable step of integration.

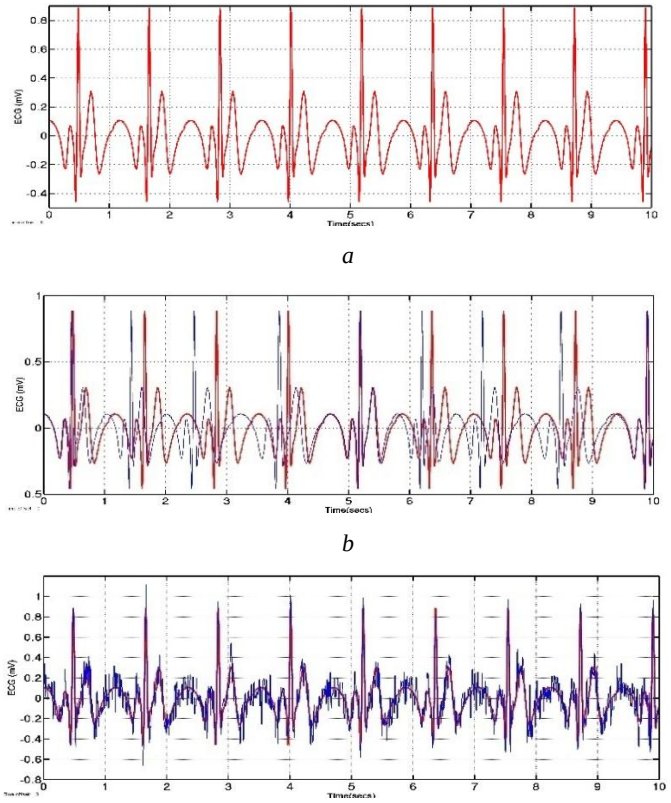
The initial data were obtained by digitizing the real ECG of seven patients from open PhysioNet cardiographic libraries [26]. The simulation was carried out on samples of duration T = 10 s.

The temporary indices of artificial ECG series, generated using the developed mathematical model [27], are comparable to those of real ECG series with the same spectral characteristics, therefore the obtained models on this criterion can be considered substantial and adequate.

The fig. 1 shows the synthesized ECG signals and signal-noise functions: red lines – basic ECG signal, blue lines - random signal.

According to fig. 1 and taking into account the conclusions of Section 4, the correlation functions obtained (fig. 2) and the spectral density power distribution functions (fig. 3).

Correlation functions for ECG signals, corresponding fig. 1, pictured on fig. 2.

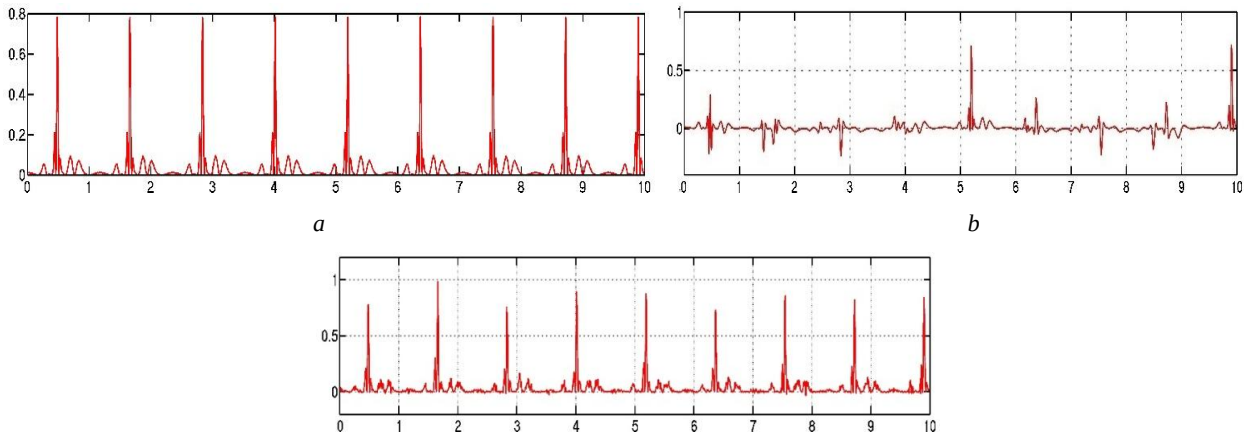


a

b

c

Fig. 1. Synthesized ECG signals and signal-noise functions: *a* – basic ECG signal excluding HRV and noise; *b* – ECG signal with a change in cardiac rhythm in random law; *c* – ECG signal with the addition of perturbation of the type of "white noise"



a

b

c

Fig. 2. Correlation functions for ECG signals

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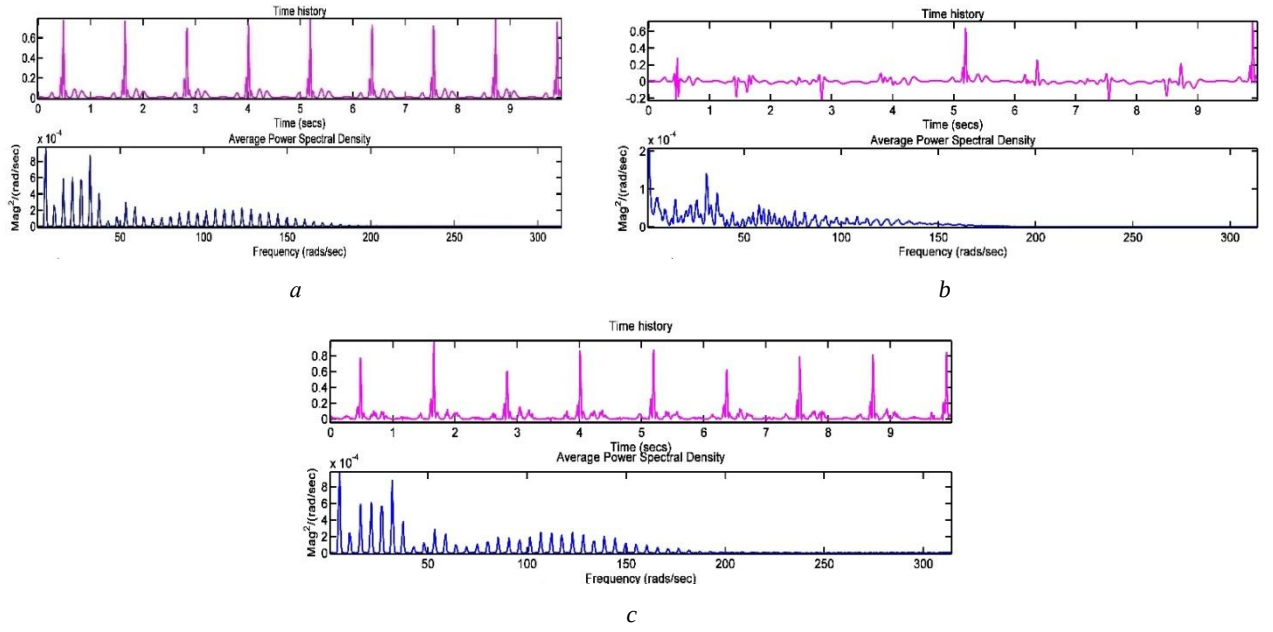

Fig. 3. Distribution of spectral density power

The developed simulation model allows to consider and to simulate a random process as a superposition of elementary random variables for confirmation of theoretical assumptions.

Discussion of the results of the study of the influence of random processes on the variability of the heart rhythm.

The obtained results of mathematical modeling indicate that the proposed method for studying non-stationary random processes can be used to simulate random processes in cardio signals.

As a result, correlation functions have been obtained for various random processes. Indeed, from the fig. 2, a the correlation function is periodic with the period of R-R interval, this allows to conclude that the correlation coefficients approach the maximum value at the moments of the R-deflections of the ECG.

With the appearance of the variability of the heart rhythm, one can observe that the correlation function of fig. 2, b has both positive and negative peaks with oscillatory character of influence. This indicates the presence of components with frequencies of 0.17-0.3 Hz, reflecting the influence of sympathetic and parasympathetic activity of the central nervous system.

The analysis of the correlation function, obtained with the help of the triangular model, showed a high sensitivity, even at noisy areas of the ECG, and allowed to accurately identify the influence of both external and internal factors on the humain heart rhythm.

It should be noted that during the modeling of the effect of "white noise" (10-4 W) it was found that it does not affect the distribution of power of spectral density. The obtained result can be explained by the fact that its spectral density is a constant throughout the frequency range (fig. 3, b). The proposed method can be used when the spectrum of the analyzed signal has clearly expressed peaks. Indeed, in fig. 3, it is possible to observe the presence of both low-frequency and high-frequency sections in the distribution curve of the spectral density of the simulated signal. This indicates the sensitivity of the method in the analysis of the effects of both the sympathetic and parasympathetic CNS branches on the HRV. These parts correspond to the previous assumed availability of oscillation circuits with different steady time.

The direction of further research may relate to a more detailed identification of the effects of random processes on HRV, as well as methods of hardware implementation of the determination of the parameters of the ECG [28].

The proposed method opens up additional possibilities for refinement and improvement of the model, bringing it to the level of the maximum full quantitative description of the experimental data.

Conclusions

1. It has been shown that the new approach to the description of the HRV as a random process in the application of the triangular model in the Hilbert space made it possible to obtain expressions for the correlation function.

2. The imitation simulation showed the sensitivity of the method within the 5 % error rate under the conditions of different types of influence on HRV. The qualitative assessment of the possibilities of the proposed models to generate artificial ECG provided the possibility of visual analysis by the cardiologist of the identity of the interpretation of real ECG records. The identities of modeling results were checked on time samples of electrocardiographs of 7 patients from open PhysioNet cardiographic libraries on samples with the duration T = 10 s.

3. The standard low-frequency oscillations and "white noise" barrier are clearly differentiated on the applied correlation function by the distribution of spectral density power within the frequency range of 0,15–0,3 Hz.

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ВИКОРИСТАННЯ ТРИКУТНИХ МОДЕЛЕЙ НЕСТАЦІОНАРНИХ ПРОЦЕСІВ ПРИ МОДЕЛЮВАННІ ВАРІАБЕЛЬНОСТІ СЕРЦЕВОГО РИТМУ

Предмет дослідження є математична модель, яка описує процес варіабельності серцевого ритму, в основі якої лежить використання трикутних моделей нестаціонарних випадкових процесів в гільбертовому просторі. **Мета** дослідження – розробка математичної моделі нестаціонарних процесів серцевої діяльності на основі трикутної моделі. Це дослідження стало основою для розробки моделі Matlab, що реалізує запропонований метод для аналізу варіабельності серцевого ритму. **Завдання**: дати опис варіабельності серцевого ритму як нестаціонарного процесу в гільбертовому просторі в термінах кореляційних функцій; дослідити можливість побудови кореляційної та спектральної теорії нестаціонарного процесу з використанням трикутних моделей; синтезувати математичну модель нестаціонарного процесу на основі кореляційної теорії для розв'язання задач математичної обробки і прогнозування на основі даних ЕКГ. За допомогою запропонованого математичного методу була реалізована модель Matlab генератора серцевого сигналу, що дозволило синтезувати ЕКГ з різними параметрами мінливості в умовах шуму. У роботі використані **методи** математичної статистики, імітаційного моделювання, теорії випадкових процесів і теорії управління. **Результати** цього дослідження такі: 1) Було показано, що новий підхід до опису ВСР як випадкового процесу при застосуванні трикутної моделі в гільбертовому просторі дозволив отримати вирази для кореляційної функції. 2) Імітаційне моделювання показало чутливість методу в межах 5% помилок в умовах різних типів впливу на ВСР. Якісна оцінка можливостей пропонованих моделей для генерації штучної ЕКГ надала можливість візуального аналізу кардіологом ідентичності інтерпретації реальних записів ЕКГ. Ідентичність результатів моделювання була перевірена на тимчасових вибірках електрокардіографів 7 пацієнтів з відкритих кардіографічних бібліотек PhysioNet на вибірках тривалістю T = 10 с. 3) Стандартні низькочастотні коливання і бар'єр "білого шуму" чітко диференціюються за застосовуваною кореляційної функції з розподілу потужності спектральної щільності в діапазоні частот 0,15-0,3 Гц. **Висновок**. Результати моделювання підтвердили правильність теоретичних висновків про можливість використання моделей, заснованих на уявленні нестаціонарних процесів в трикутному гільбертовому просторі.

Ключові слова: серцевий ритм; нестаціонарний випадковий процес; електрокардіосигнал; кореляційна функція; трикутна модель; імітаційне моделювання.

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ИСПОЛЬЗОВАНИЕ ТРЕУГОЛЬНЫХ МОДЕЛЕЙ НЕСТАЦИОНАРНЫХ ПРОЦЕССОВ ПРИ МОДЕЛИРОВАНИИ ВАРИАБЕЛЬНОСТИ СЕРДЕЧНОГО РИТМА

Предмет исследования представляет собой математическую модель, описывающую процесс вариабельности сердечного ритма, в основе которой лежит использование треугольных моделей нестационарных случайных процессов в гильбертовом пространстве. **Цель** исследования – разработка математической модели нестационарных процессов сердечной деятельности на основе треугольной модели. Это исследование стало основой для разработки Matlab модели, реализующей предложенный метод для анализа вариабельности сердечного ритма. **Задачи**: дать описание вариабельности сердечного ритма как нестационарного процесса в гильбертовом пространстве в терминах корреляционных функций; исследовать возможность построения корреляционной и спектральной теории нестационарного процесса с использованием треугольных моделей; синтезировать математическую модель нестационарного процесса на основе корреляционной теории для решения задач математической обработки и прогнозирования на основе данных ЭКГ. С помощью предложенного математического метода была реализована модель Matlab генератора сердечного сигнала, что позволило синтезировать ЭКГ с различными параметрами изменчивости в условиях шума. В работе использованы **методы** математической статистики, имитационного моделирования, теории случайных процессов и теории управления. **Результаты** этого исследования следующие: 1) Было показано, что новый подход к описанию ВСР как случайного процесса при применении треугольной модели в гильбертовом пространстве позволил получить выражения для корреляционной функции. 2) Имитационное моделирование показало чувствительность метода в пределах 5% ошибок в условиях различных типов влияния на ВСР. Качественная оценка возможностей предлагаемых моделей для генерации искусственной ЭКГ предоставила возможность визуального анализа кардиологом идентичности интерпретации реальных записей ЭКГ. Идентичность результатов моделирования была проверена на временных выборках электрокардиографов 7 пациентов из открытых кардиографических библиотек PhysioNet на выборках длительностью T = 10 с. 3) Стандартные низкочастотные колебания и барьер "белого шума" четко дифференцируются по применяемой корреляционной функции по распределению мощности спектральной плотности в диапазоне частот 0,15–0,3 Гц. **Заключение**. Результаты моделирования подтвердили правильность теоретических выводов о возможности использования моделей, основанных на представлении нестационарных процессов в треугольном гильбертовом пространстве.

Ключевые слова: сердечный ритм; нестационарный случайный процесс; электрокардиосигнал; корреляционная функция; треугольная модель; имитационное моделирования.