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FUZZY MODELING OF VERBAL INFORMATION FOR PRODUCTION SYSTEMS

The **subject** of the article's research is the formalization of unstructured or semistructured verbal information for a fuzzy production system. The **purpose** of the work is to develop a method of constructing membership functions for fuzzy sets of terms of a linguistic variable that will allow formalizing unstructured or semistructured verbal information for fuzzy production systems. The following **tasks** are solved in the article: to develop a method of constructing the membership functions to determine the sequence of stages: the stage of modeling verbal information in the form of digraphs, the stage of constructing the order relation on the elements of this model, the step of determining a linguistic variable based on the created model, and determining the functions of fuzzy sets of linguistic terms variable. **Methods** are used: graph theory, mathematical induction, fuzzy modeling. **Results** obtained: a method for constructing the membership function of linguistic variables that formalizes unstructured or semi-structured qualitative information for fuzzy production systems is developed. For this purpose, the process of constructing the membership function has been broken down into stages. The implementation of the first stage requires the creation of a model of unstructured or semistructured verbal information. Three models of information based on oriented trees are considered with increasing complexity. A model based on an acyclic oriented graph is considered as a generalization. Such a model is the basis for processing information that has a structure of greater complexity. The second stage provides a theoretical basis for constructing the order relation for the elements of the models under consideration. For the implementation of the third stage, a method of identifying the order elements on the basis of the positional system is proposed. Based on the ID of each ordered element, functions of fuzzy sets of terms of a linguistic variable are constructed. Appropriate procedures have been developed to implement the steps. **Conclusions:** application of the method will allow automating the assignment of vectors of input and output information, to automate the formation of fuzzy sets of terms of the corresponding linguistic variables, will allow to build fuzzy products as a knowledge base of a fuzzy production system, and to train such a system.

Keywords: membership function; ratio of order; linguistic variable.

Introduction

Under uncertainty, when the decision-maker (DM) has less information than is appropriate for decision-making, the use of a product system can reduce the cost of resources. Uncertainty in solving problems has a different nature. This may be unreliability, lack, and inaccuracy, fundamental impossibility of obtaining additional information, insufficient qualification of the person charged with the responsibility for making a decision, the need to make a decision in the conditions of limited resources of time, money, and executors. The production system contains expert knowledge, the application of which can partially or completely eliminate uncertainty. Knowledge - these are the laws of the subject area, which are identified expertly in the course of professional practical activity and such that allow you to solve problems in a particular field. The main modules of the production system are: a database that stores known facts about the state of the subject area; a set of production rules; product interpreter.

In the context of semistructured or unstructured information, a verbal, qualitative description of the most important elements whose quantitative dependencies are difficult to identify must be used to make a decision. The use of fuzzy output systems allows combining fuzzy inputs with logic based on fuzzy production rules, in which terms and conditions are formulated in terms of fuzzy linguistic expressions. [1, 2, 3].

Analysis of recent research and publications

Formally, the production system *System* displays *System: Input* \rightarrow *Output* where, in the most general

sense, input is given to the input *Input*, which should influence the output *Output*. At the stage of system design *System*, it is necessary to decide on the model of data representation, extraction and structuring of knowledge and creation of knowledge bases that form the core of the system.

The knowledge base requires the use of expertise for direct use in logical inference to construct inferences. In general, the input information *Input* is verbal, qualitative, nonmetric, unstructured, or poorly structured. This information may contain a description of the phenomenon, object, process. It is necessary to distinguish such information about one or more attributes, properties of these phenomena, objects or processes that are significant in the sense of influencing the output *Output*. Output results *Output* contain information about possible alternative solutions, meaning this information can be considered more structured.

A fuzzy measure is a quantification of linguistic (verbal) ambiguity related to the peculiarities of human thinking [4].

When applying fuzzy mathematics methods, one of the traditional problems is to construct membership functions of fuzzy sets corresponding to a particular problem [5]. The problem is that the membership function is defined outside the theory of fuzzy sets (in metatheory), that is, the correctness of construction cannot be verified by the methods of theory, and can only be verified after solving the problem. A positive consequence of the problem is that the definition of a fuzzy set does not limit the choice of the type of membership function. The methods are currently divided into two groups, direct and indirect. Work [3] systematically outlines various approaches and methods for constructing membership

functions. Examples of constructing membership functions for metric quantities are carefully presented. The theory of fuzzy sets has a wealth of experience in constructing membership functions depending on the context of the problems. Achievements in fuzzy and fuzzy modeling and control of systems are considered in [6]. For example, self-organizing systems on metric input and output values. In work [7], problems of the use of fuzzy modeling in control of dynamic systems are considered. In study [8], a hybrid model of site selection in Vilnius was constructed. The hybrid approach has combined the multi-criteria selection problem based on a convolution of criteria with a fuzzy hierarchical synthesis of the weights of individual criteria. The membership functions were selected triangular with expertly defined parameters. Direct methods are commonly used for measurable concepts, or when there are polar meanings, but such methods have a significant share of subjectivity. The determination of the intensity of a particular property of an object is due to its nature. The difficulty lies in the inaccurate measurement of intensity, the fundamental impossibility of using a measuring instrument, due to the individual peculiarities of expert perception [9, 10]. In the case of immeasurable concepts, indirect methods are used to construct the membership functions. In this case, the original expert information is further processed. An example of such processing is the procedure for normalizing various features. In [11], a method of normalization of term sets was proposed for modeling of operational risk factors, which allowed the construction of membership functions for different input variables in uniform coordinates. In [12], a method for solving multicriteria estimation of the optimality of medical institution schedules was proposed, based on fuzzy modeling. Triangular membership functions were expertly defined and had metrics on the abscissa. Similar problems are solved by qualimetric methods in utility theory, when constructing psychological and pedagogical assessments [13, 14]. A classic method is the pairwise comparisons of Saati [15, 16]. In [17], the authors of this article constructed a system of fuzzy inference to support court decisions. For the product system, the input variable is information about the person responsible, aggravating and mitigating circumstances. The identity of the perpetrator is verbal, qualitative, non-metric, poorly structured information. The application of known methods of constructing the membership function [3] proved to be ineffective. There was a need to develop a method of modeling such information.

For ordering methods in theory (in particular in [18]), general rules for constructing membership functions are formulated. One of them is the definition of a universal set on the principle of natural ordering.

So, when developing new methods for constructing membership functions, we pursue two goals: analytic representation of functions and minimizing the cost of computing resources.

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Highlighting previously unresolved parts of a common problem

When creating a method of plotting a membership function $\mu(x)$ for nonmetric information *Input*, it is necessary to solve the problem with the abscissa scale and the general appearance of the function. The abscissa shows the measurement scale $x \in X$ to be constructed according to the internal nature of the information *Input*. The choice of the general appearance of the membership function $\mu(x)$ and the method of its construction must also be determined according to the requirements of continuity, differentiation, monotony of the function.

The **purpose** of the study is to develop a method of constructing membership functions for fuzzy sets of terms of a linguistic variable that will allow formalizing unstructured or semistructured verbal information for fuzzy production systems.

Results of the studies and their discussion

In the case of a fuzzy output production system *System*, the variables describing the input *Input* must be represented as linguistic variables [2, 3]. The source information *Output* can look like both a clear and a linguistic variable. Traditionally, linguistic and fuzzy variables have the following definitions [4].

Definition 1. A linguistic variable α is a tuple $\langle \alpha, T, X, G, S \rangle$, where α – the name of a linguistic variable, T – a term set $T = \{t_z\}, z = \overline{1, |T|}$ where each term t_z is a fuzzy variable, X – a universal set of basic values (an area in which the values of a linguistic variable are defined), G – a syntactic procedure for the formation of new values of a linguistic variable, S – a procedure that allows the content of new linguistic variable values that are formed by the procedure G to be content by constructing a fuzzy set.

Definition 2. A fuzzy variable t is a tuple v , where t – the name of a fuzzy variable, X_t – the scope of a fuzzy variable, $(x, \mu(x))$ – a fuzzy set at X_t .

In order to build a measurement scale $x \in X$, it is necessary to determine the internal nature of the *Input* information and its processing methods. To reflect the internal structure of *Input* information, we build its model.

Assume that the information *Input* is verbal and contains a description of a specific feature α characterized by a plurality of features P .

Let us consider in detail the steps of the method of constructing membership functions for fuzzy sets of terms of a linguistic variable.

The first stage is the modeling of verbal information in the form of digraphs.

The apparatus of graph theory was chosen for modeling as having the highest clarity [19]. A digraph $D = (V, E)$ is a set of ordered pairs $E = \{(v_i, v_j) | v_i, v_j \in V, i, j = 1, |V|\}$. Oriented Tree (Ortree) is an acyclic digraph H in which only one vertex has zero half-degree of input, and all other vertices have half-degree of input one. The zero-entry vertex is the root of the tree; the zero-exit vertex is the leaves. The path between root and leaf is a branch. The length of the largest branch of the order is its height. The distance from the root to the top determines its level. The root has a level of zero. Tops of one level form a tier of a tree.

Consider cases where the input information *Input* can be represented by an Ortree H .

In the first variant, consider the model "Ortree H_1 with height one".

Assume that an object is characterized by a non-metric attribute that can take values. In this case, the model of qualitative information is the Ortree H_1 (fig. 1 a)). The first tier is formed by the leaves of the Ortree H_1 , which are variants of the values p_{ij} of the attribute P_i .

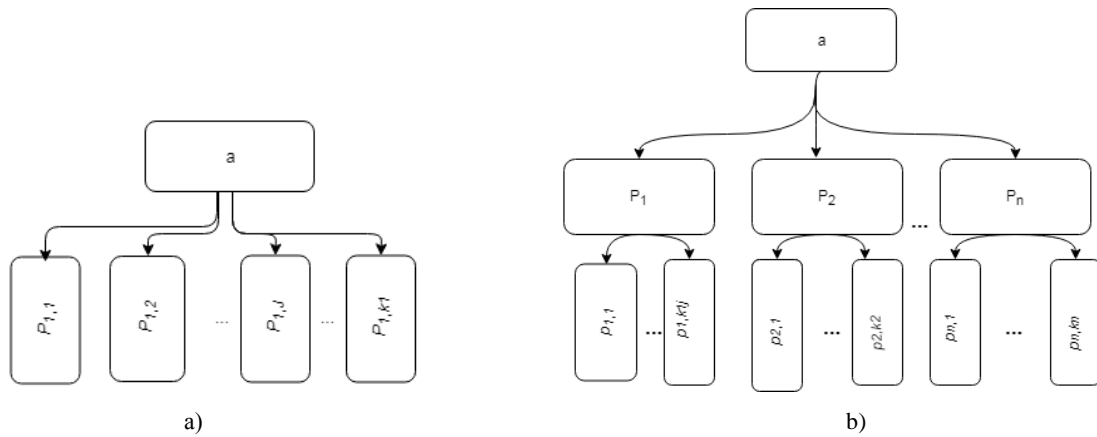


Fig. 1. Input data model of *Input* type a) "Ortree with height one", b) "Ortree with height two"

In the second variant we will consider the model "Ortree with height two" (fig. 1b).

Assume that the variable α is characterized by n nonmetric signs P_i , $i = \overline{1, n}$ forming a set P , $|P| = n$. Each trait P_i can take k_i values p_{ij} , $j = \overline{1, k_i}$. Each branch of the Ortree H_2 defines a specific sign P_i , $i = \overline{1, n}$. The peaks corresponding to the features P_i , $i = \overline{1, n}$ form the first tier. The second tier consists of the leaves of the

Ortree H_2 , which are possible values p_{ij} of the trait P_i (fig. 1b).

The third variant is an Ortree in which the leaves are on different tiers (fig. 2). Let's call this model "general Ortree H_3 ". The Ortree is a subset of acyclic graphs [20]. Therefore, the fourth consider the option of presenting information with an acyclic graph H_4 .

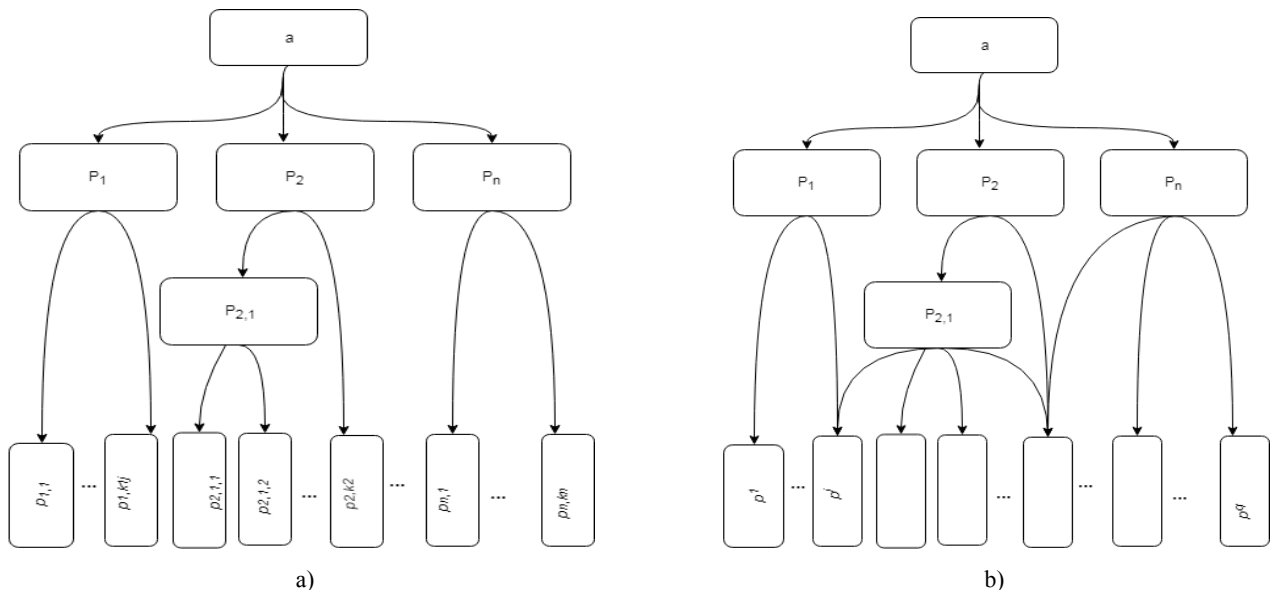


Fig. 2. Model of input data *Input* a) "general Ortree", b) acyclic graph H_4

The second step of the method is to construct the relation of the order on the set X .

Without losing the generality, let's say that information *Input* can be formalized by a single linguistic variable α that is fed to the input of a fuzzy production system *System*. Let's agree that a linguistic variable is named after an object α , and, by **definition 1**, we fix a term set $T = \{t_z, z = \overline{1, |T|}\}$ in the tuple $\langle \alpha, T, X, G, S \rangle$, where each term t_z is a fuzzy variable with a scope X_z . Assume as a universal set of basic values the set $X = X_1 = \dots = X_z$. The most common variant of term-set formation T is the introduction of terms that are ordered in some relation of order $\rho : < T, < >$ [21]. The term of the plurality T is a fuzzy variable t_z for which a membership function $\mu_z(x)$ must be constructed in the definition area X . The membership function is a cross-section of the order ρ of X .

We take as the domain of definition of fuzzy variables of term-set T the set X defined by Cartesian product of nonmetric features

$$X = P_1 \times P_2 \times \dots \times P_n, |X| = \prod_{i=1}^n k_i :$$

$X = \{x_s | x_s = (p_{1j_1}, p_{2j_2}, \dots, p_{nj_n}), 1 \leq j_i \leq k_i, i = \overline{1, n}\}$. The set X is discrete. To display it on the abscissa it is natural to offer a nominal scale. The nominal scale permits one-to-one transformations that maintain the equivalence relation and the arbitrary arrangement of discrete elements $x_s \in X$ on the abscissa axis. The arbitrary positioning of the value x_s on the abscissa does not allow the construction of membership functions $\mu(x)$ with preset properties.

To plot the membership functions $\mu_z(x_s)$, $z = \overline{1, |T|}$ it is necessary to define a mapping $X \rightarrow M$ where $M = \{\mu_z\}$ is the set of term t_z membership functions. This mapping must meet two requirements: 1) correct reproduction of the binary order relation $\rho : < T, < >$ and 2) fulfillment of conditions of monotonicity of the membership functions $\mu_z(x_s)$. The binary order relation is antireflective and transitive.

To fulfill the first requirement, let us turn to the isotonic mapping, since it has the property of maintaining the order relation [21]. Determine the isotonic mapping $F : T \rightarrow X$. It is known that an inverse mapping to an order mapping is also an order [21]. That is, display $F^{-1} : X \rightarrow T$ is an order.

To fulfill the second requirement, we define an isotonic mapping $\Psi : T \rightarrow M$ with which you can set the required properties of the membership function. The result is an order on many membership functions $< M, < >$.

It is known that the composition of two orders is also an order. We build a composition of two orders $\Psi \circ F^{-1} : X \rightarrow M$.

So, in order to plot the membership function of terms of a linguistic variable α , it is necessary to order the set X , for which mappings F and F^{-1} is defined. Next, arrange the set M , which determine the mapping Ψ . After that, perform the composition $\Psi \circ F^{-1}$, that is, to build graphs $\mu_z(x_s), z = \overline{1, |T|}$.

Consider the theoretical basis of the ordering of the set X .

It is necessary to decide on such representation of the set of values of the signs of the set P , which are fixed by the vector x_s , which would allow us to construct isotonic mappings F and F^{-1} , that is, to determine the order on the set $X : < X, < >$.

Here is information about ordered sets.

Definition 3. A strict ordering graph L is an acyclic, transitive digraph with single arcs [20].

For the set of elements that are the leaves of the Ortree T , we construct a graph $L = (V_L, E_L)$. The intersection of graphs $T \cap L = (V_L, \emptyset)$ is an edgeless graph. The arcs of the set E_L of graph L denote the ratio of the dominance of the set of vertices V_L .

We construct a graph $L = (V_L, E_L)$ for three models "ortree H_1 with height one", "ortree H_2 with height two", "ortree H_3 of general type".

For the model "ortree H_1 with height one" (fig. 1) we construct a graph $L^1 = (\{p_{1,j}\}, E_{L^1}^1)$, $j = \overline{1, k_1}$. At graph $L^1 = (\{p_{1,j}\}, E_{L^1}^1)$, $j = \overline{1, k_1}$ the set of vertices is the set of vertices of the first tier of the Ortree H_1 . We define the order by ratio ρ on the set of attribute values $< \{p_{1,j}\}, < >$, $j = \overline{1, k_1}$ (fig.3 a)). The set of arcs reproduces the order on the set $\{p_{1,j}\}, j = \overline{1, k_1}$ according to the ratio $< \{p_{1,j}\}, < >$. We place identifiers of values of a sign $\{p_{1,j}\}, j = \overline{1, k_1}$ on the abscissa axis of the membership function graph $\mu(x)$ according to a certain ratio of order $< \{p_{1,j}\}, < >$.

For the model "ortree H_2 with height two" (fig.1b), first, let us define the order by ratio ρ on the set of signs $< \{P_i\}, < >$, $i = \overline{1, n}$. Strict order graph $L^{2,1} = (\{P_i\}, E_{L^{2,1}}^1)$, $i = \overline{1, n}$ for this model is shown in fig. 3 b). Second, we define the order by the ratio ρ of the multiple values for each trait $< \{p_{i,j}\}, < >$, where $i = \overline{1, n}$, $j = \overline{1, k_i}$. We construct n graphs $L_i^{2,2} = (\{p_{i,j}\}, E_{L_i^{2,2}}^2)$, $i = \overline{1, n}$, $j = \overline{1, k_i}$ (fig. 3 c)). Fig. 3 c) shows incomparability in the ratio ρ to the values of features P_x and $P_z, \forall x, z = \overline{1, n}, x \neq z$. Third, we perform an operation on the orders of "sum" $\sum_{i=1}^n L_i^{2,2}$ and to determine the relation of the order ρ formulate.

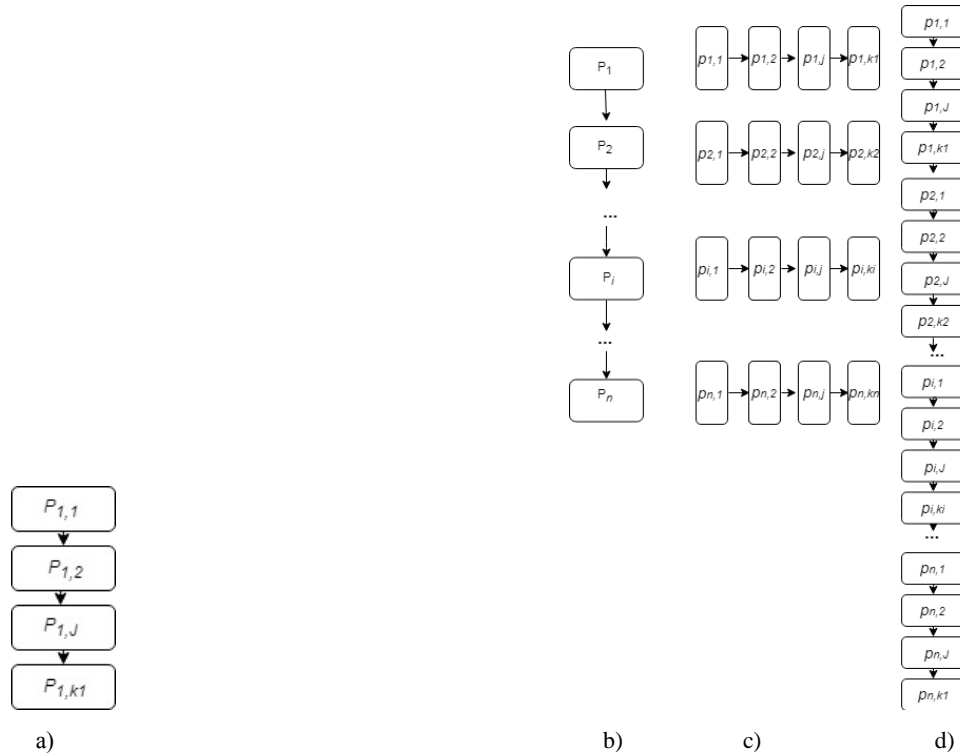


Fig. 3. a) Strict order graph L^1 , b) Strict order graph $L^{2,1}$, c) n Strict order graphs $L_q^{2,2}$, $i = \overline{1, n}$, d) Strict order graph L^2

Rule 1: we will assume that $p_{i,j} \prec p_{x,y}$, so $p_{x,y}$ dominates $p_{i,j}$, if 1) or according to the ratio $\langle \{p_{i,j}\}, \prec \rangle$, where $i = \overline{1, n}$, $j = \overline{1, k_i}$ for $i = x$ values $p_{i,y}$ $p_{i,j} : p_{i,j} \prec p_{i,y}$ dominate; 2) or according to the ratio $\langle \{P_i\}, \prec \rangle$, $i = \overline{1, n}$ attribute P_x dominates attribute P_i , and a certain values $p_{i,j}$ and $p_{x,y}$ of these features are incomparable: $p_{i,j} \parallel p_{x,y}$.

Applying Rule 1, we construct the graph of fig. 3 c. The method of construction is obvious.

Statement 1. Constructed by rule 1, the set of feature values $\{p_{i,j}\}$, $i = \overline{1, n}$, $j = \overline{1, k_i}$ is a strict order $\langle \{p_{i,j}\}, \prec \rangle$, $i = \overline{1, n}$, $j = \overline{1, k_i}$. A constructed graph $L^2 = (V_L^2, E_L^2)$ is a graph of strict order $\langle \{p_{i,j}\}, \prec \rangle$, $i = \overline{1, n}$, $j = \overline{1, k_i}$.

Statement 2. Cartesian product of orders $\langle \{p_{1,j}\}, \prec \rangle \times \langle \{p_{2,j}\}, \prec \rangle \times \dots \times \langle \{p_{i,j}\}, \prec \rangle \times \dots \times \langle \{p_{n,j}\}, \prec \rangle$, $i = \overline{1, n}$, $j = \overline{1, k_i}$ when applying rule 1 (paragraph 2), forms on the plural $X = \{x_s \mid x_s = (p_{1j_{s1}}, p_{2j_{s2}}, \dots, p_{nj_{sn}}), 1 \leq j_{si} \leq k_i, i = \overline{1, n}\}$ the strict order ratio $\langle X, \prec \rangle$.

Proof

1. In case $n = 1$ vector $x_s = (p_{1j_{s1}})$ is one-component. On the set of values of the trait

$\langle \{p_{1,j}\}, \prec \rangle$, $j = \overline{1, k_1}$ an order according the ratio ρ is determined, that is, statement 2 is fulfilled.

2. In case $n = 2$ vector $x_s = (p_{1j_{s1}}, p_{2j_{s2}})$ is two-component. The binary properties of the order relation are satisfied at $p_{1k} \prec p_{1q}$, $p_{2l} \prec p_{2y}$, then $(p_{1k}, p_{2l}) \prec (p_{1q}, p_{2y})$. If $p_{1k} \prec p_{1q}$ and $p_{2l} \succ p_{2y}$ or $p_{1k} \succ p_{1q}$ and $p_{2l} \prec p_{2y}$, thus for cases $(p_{1k}, p_{2l}) \parallel (p_{1q}, p_{2y})$, rule 1 (paragraph 2) defines the order. In case one of the pairs is not comparable $(p_{1k} \parallel p_{1q} \text{ and } p_{2l} \prec p_{2y})$ or $(p_{1k} \prec p_{1q} \text{ and } p_{2l} \parallel p_{2y})$ is not implemented on initial terms.

We assume that the order relation is defined for k -component vectors $x_s = (p_{1j_{s1}}, p_{2j_{s2}}, \dots, p_{kj_{sk}})$. Let us prove that for the $k+1$ component vectors the order relation $x'_s = (p_{1j_{s1}}, p_{2j_{s2}}, \dots, p_{kj_{sk}}, p_{(k+1)j_{s(k+1)}})$ is also defined. Let's divide the components of the vector x'_s into two tuples $x'_s = ((p_{1j_{s1}}, p_{2j_{s2}}, \dots, p_{kj_{sk}}), p_{(k+1)j_{s(k+1)}})$. By the condition the order in the first tuple $(p_{1j_{s1}}, p_{2j_{s2}}, \dots, p_{kj_{sk}})$ is defined, the second tuple consists of one component $p_{(k+1)j_{s(k+1)}}$. Further proof is made as in a case $n = 2$. That is, statement 2 is proved inductively.

To construct the graph $L^3 = (\{p\}, E_L^3)$ of the model "ortree H_3 of general type" and the graph $L^4 = (\{p\}, E_L^4)$ of the model "acyclic graph H_4 " (fig. 2) rule 1 takes a more generalized form.

Rule 1 (generalization): let's assume that it is necessary to order elements having a vector of indexes of length q . Let's assume, that $p_{i,...,j} \prec p_{x,...,y}$, thus $p_{x,...,y}$ so $p_{i,...,j}$ dominates, if the index vectors coincide with $(q-1)$ -th index, and for q -th index a statement $p_{i,...,j} \prec p_{x,...,y}$ is valid. In case of incomparability $p_{i,...,j} \parallel p_{x,...,y}$ of signs by the q -th index of the order is determined by the previous index $(q-l)$ by which the order relation is determined. This makes it possible to determine the order of the index $(q-l+1)$ and so on by going to the q -th index.

For an acyclic graph H_4 , the direct application of rule 1 (generalization) requires appropriate leaf indexing. The most natural one is 1) to repeat a vertex with more than one half-west; 2) align the lengths of the index vectors, denoting missing levels, such as zeros.

The result of the second step of the method is an ordered set of values $\langle \{p\}, \prec \rangle$ of the signs of the set P . In terms of construction, statement 2, which is based on generalized rule 1, is also valid. This enables the ordered arrangement of the abscissa of the vectors of the set $X = \{x_s | x_s = (p_{1js1}, p_{2js2}, \dots, p_{njsn}), 1 \leq j_{si} \leq k_i, i = \overline{1, n}\}$.

That is, the mapping is constructed $F: T \rightarrow X$.

The third stage of the method is to determine the linguistic variable based on the model created and to define the membership functions for fuzzy sets of terms of the linguistic variable.

Consider the implementation of mapping $F^{-1}: X \rightarrow T$, mapping $\Psi: T \rightarrow M$, and their composition $\Psi \circ F^{-1}: X \rightarrow M$ to build graphs $\mu_z(x_s)$, $z = 1, |T|$.

To determine the identifier x_s , we apply a positional numbering system in which the digit 10^{n-i+1} is determined by the vertex number describing the sign P_i in the strict order column and the digit j_{si} by the ordinal number of the value $p_{ij_{si}}$ for the sign P_i that was implemented in the vector x_s . We denote the absence of a value for a particular characteristic as "0". We place the vectors of the values of the signs P on the abscissa axis according to the defined order of the order $\langle \{p\}, \prec \rangle$.

Constructing a set $F^{-1}: X \rightarrow T$ mapping is a fuzzy linguistic simulation. For each fuzzy variable t_z , it is necessary to construct a fuzzy set on $X: (x, \mu_z(x))$. That is, each tuple of implemented values $x_s \in X$ acquires a specific value of the membership function $\mu_z(x_s)$. The constructed membership functions $\mu_z(x_s)$ are a component of some fuzzy production system. For training of this system, it is necessary to construct vectors of input data $x_s = (p_{1js1}, p_{2js2}, \dots, p_{njsn}), 1 \leq j_{si} \leq k_i, i = \overline{1, n}$ on which there will be an adjustment of values of output vectors.

The kind of functional dependence $\mu(x_s)$ should reproduce intuitively expected estimation of the object α , that is, correspond to the order relation ρ on the term-set T , and provide the General requirements of monotonic decrease/increase of the membership function.

For large values $k_i, i = \overline{1, n}$ the n number of vectors is too large to determine the membership function expertly. Therefore, it is advisable to automatically build membership functions that will continue to be used to train a fuzzy production system.

We offer three types of linear formulas for determining the membership function $\mu_z(x_s)$, in which $\beta_r, r = \overline{1, 7}$ – parameters:

$$\mu(x_s) = 1 - \beta_1 * \sum_{i=1}^n j_{si} * 10^{-i}, 0 \leq j_{si} \leq k_i; \quad (1)$$

$$\mu(x_s) = \begin{cases} \beta_2 + \beta_3 * \sum_{i=1}^n j_{si} * 10^{-i} \\ \beta_4 - \beta_5 * \sum_{i=1}^n j_{si} * 10^{-i} \end{cases}; \quad (2)$$

$$\mu(x_s) = \beta_6 + \beta_7 * \sum_{i=1}^n j_{si} * 10^{-i}, 0 \leq j_{si} \leq k_i. \quad (3)$$

The method of presentation described has its own characteristics, which must be taken into account in practical application. The first feature is that such fuzzy sets are unnormalized. To normalize the membership functions (1) – (3), we introduce a proportionality factor inverted to the height of the corresponding fuzzy set. This method provides a global scale when not all the attributes of a qualitative variable α have their characteristics. The second peculiarity of is that the membership function is discrete. The third one is that for the accepted method, not all values of the abscissa axis are realized in vectors x_s . Entering an identifier that corresponds to the number of possible values $k_i, i = \overline{1, n}$ for each trait will eliminate redundancy. The fourth one is that with many features there are limitations in the accuracy of the calculations. In this case, it would be appropriate to break the set of features into subsets, with their subsequent implementation in separate product rules.

Based on the theoretical conclusions, we define the method of plotting the membership function of terms of a linguistic variable α as a sequence of stages. At the first stage, a model of verbal information is constructed in the form of an acyclic graph (fig. 1, 2). The second step is to build an ordered set X . The next third step is the procedure for determining the linguistic variable and the definition of membership functions for fuzzy sets of terms of the linguistic variable by the formulas (1)–(3).

Let us consider how the developed method works on the example of fuzzy modeling of the linguistic variable of court decisions *Personality*, characterizing the identity of the perpetrator, which was determined in detail by the authors in the work [17].

By definition 1, a linguistic variable *Personality* is a tuple $\langle \text{Personality}, \text{Personality-Term}, X, G, S \rangle$ where $\text{Personality-Term} = \{\text{Negative}, \text{Norm}, \text{Positive}\}$. Each of the fuzzy variables *Negative*, *Norm*, *Positive* is a fuzzy set X with a corresponding membership function $\mu_{\text{Negative}}(x_s)$, $\mu_{\text{Norm}}(x_s)$, $\mu_{\text{Positive}}(x_s)$. There are such order relations on the term-set *Personality-Term*:

"Display negative feature": $\text{Negative} \succ \text{Norm} \succ \text{Positive}$,

"Display positive feature": $\text{Positive} \succ \text{Norm} \succ \text{Negative}$ and partial order

"Display neutral characteristic" $\text{Norm} \succ \text{Negative}$ and $\text{Norm} \succ \text{Positive}$.

Let us define the domain of definition X of fuzzy variables *Negative*, *Norm*, *Positive*. For this purpose, the model of qualitative variable "Person of the guilty" is constructed.

A set of features $P = \{\text{employment}, \text{family_state}, \text{residence registration}, \text{characteristic}, \text{logging}, \text{relations}, \text{criminal liability}, \text{conviction}\}$, $|P| = 9$. Therefore, the domain of definition X of fuzzy variables of the term-set *Personality-Term* is determined by the Cartesian product of signs $X = P_1 \times P_2 \times \dots \times P_9$ whose power is $|X| = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 2 = 1728$ (fig. 4).

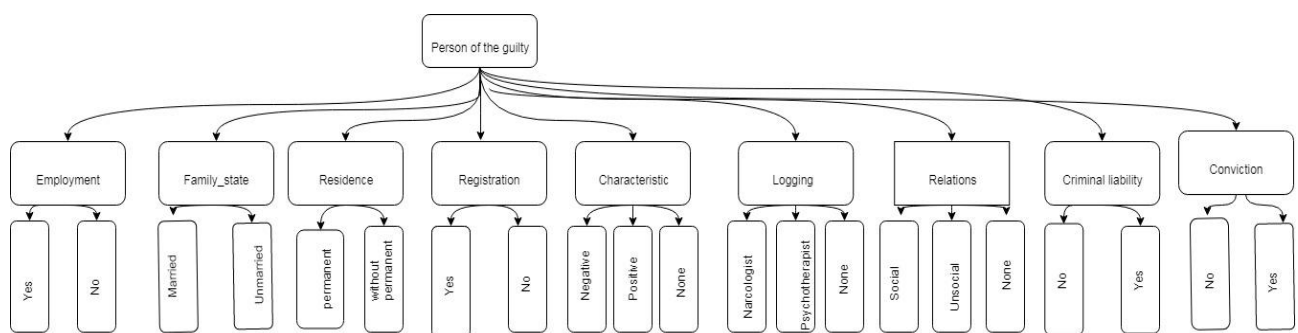


Fig. 4. Model of qualitative data "Person of the guilty"

For each fuzzy variable *Negative*, *Norm*, *Positive*, we construct the membership functions $\mu_{\text{Negative}}(x_s)$, $\mu_{\text{Norm}}(x_s)$, $\mu_{\text{Positive}}(x_s)$. In formulas (1) – (3)

we assume that $\beta_k = 1, k = \overline{1, 7}$. The functions of belonging to a linguistic variable *Personality* are presented in fig. 5.

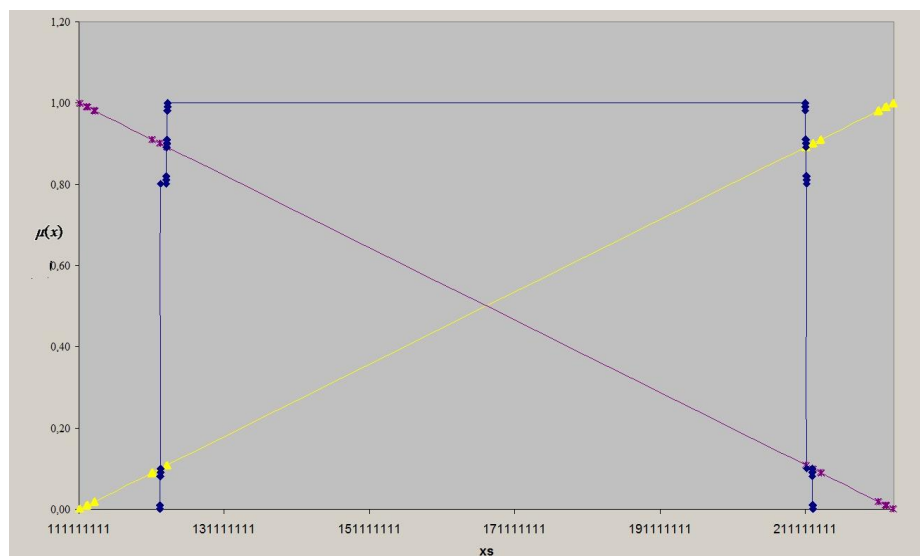


Fig. 5. Functions of belonging to a linguistic variable *Personality*

Conclusions and prospects for further development

The article proposes a method of fuzzy modeling of verbal information, which consists of a sequence of three stages: modeling of verbal information in the form of digraphs; building order relations on the elements of this model; definition of a linguistic variable on the basis of

the created model and definition of membership functions for fuzzy sets of terms of a linguistic variable. The feasibility of all stages is theoretically substantiated in the form of substantiated statements. The first stage considers information models that underlie acyclic digraphs, which, according to the authors, are the basis for developing formalizations of more complex information models. The

second stage of the method is to build order relations for unstructured or poorly structured verbal information for fuzzy production systems. In the third stage, formulas are proposed to automatically determine the membership function, which will allow training to adjust the values of the output vectors with the input of the fuzzy production system in any subject area.

Practical application of the method developed by the authors may require solving the problems related to the limitation in the accuracy of calculations, the discretion, the need for normalization, and the excess of identification. These challenges require further research.

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НЕЧІТКЕ МОДЕЛЮВАННЯ ВЕРБАЛЬНОЇ ІНФОРМАЦІЇ ДЛЯ ПРОДУКЦІЙНИХ СИСТЕМ

Предметом дослідження статті є формалізація неструктурованої або слабоструктурованої вербальної інформації для нечіткої продукційної системи. **Мета** роботи – розробка методу побудови функцій належності для нечітких множин термів лінгвістичної змінної, який дозволить формалізувати неструктуровану або слабоструктуровану вербальну інформацію для нечітких продукційних систем. В статті вирішуються наступні **завдання**: для розробки методу побудови функцій належності визначити послідовність етапів: етап моделювання вербальної інформації у вигляді орграфів, етап побудови відношення порядку на елементах цієї моделі, етап визначення лінгвістичної змінної на базі створеної моделі та визначення функцій належності для нечітких множин термів лінгвістичної змінної. Використовуються **методи**: теорії графів, математичної індукції, нечіткого моделювання. Отримано **результати**: розроблено метод побудови функції належності лінгвістичних змінних, які формалізують неструктуровану або слабоструктуровану якісну інформацію для нечітких продукційних систем. Для цього процес побудови функції належності було розбито на етапи. Суттю окремих етапів є формалізація певних відображень. Реалізація першого етапу потребує створення моделі неструктурованої або слабоструктурованої вербальної інформації. Розглянуто три моделі інформації на базі орієнтованих дерев за зростаючою складністю. Як узагальнення розглянуто модель на базі ациклічного орієнтованого графу. Така модель є основою для обробки інформації, що має структуру більшої складності. На другому етапі надано теоретичне підґрунтя побудови відношення порядку для елементів розглянутих моделей. Для реалізації третього етапу запропоновано спосіб ідентифікації елементів, що знаходяться у відношенні порядку, на базі позиційної системи. На основі ідентифікатора кожного упорядкованого елемента побудовано функції належності нечітких множин термів лінгвістичної змінної. Для реалізації етапів розроблено відповідні процедури. **Висновки**: застосування методу дозволить автоматизувати надання значень векторам вхідної та вихідної інформації, автоматизувати формування нечітких множин термів відповідних лінгвістичних змінних, дозволить будувати нечіткі продукції як базу знань нечіткої продукційної системи, а також проводити навчання такої системи.

Ключові слова: функція належності; відношення порядку; лінгвістична змінна.

НЕЧЕТКОЕ МОДЕЛИРОВАНИЕ ВЕРБАЛЬНОЙ ИНФОРМАЦИИ ДЛЯ ПРОДУКЦИОННЫХ СИСТЕМ

Предметом исследования статьи является метод построения функций принадлежности для нечетких множеств термов лингвистической переменной, которая формализует неструктурированную или слабоструктурированную вербальную информацию для нечетких продукционных систем. **Цель** работы – создание метода формализации неструктурированной или слабоструктурированной вербальной информации для нечеткой продукционной системы. В статье решаются следующие **задачи**: для создания метода построения функций принадлежности необходимо определить последовательность этапов. Разработке подлежат формирование модели вербальной информации, построение отношения порядка на элементах этой модели, определение лингвистической переменной на базе созданной модели и определение функций принадлежности для нечетких множеств термов одной лингвистической переменной. Используются следующие **методы**: методы теории графов, метод математической индукции, методы нечеткого моделирования. Получены следующие **результаты**: разработан метод построения функции принадлежности для лингвистических переменных, которые формализуют неструктурированную или слабоструктурированную качественную информацию для нечетких продукционных систем. Для этого процесс построения функции принадлежности был разбит на этапы. Сутью отдельных этапов является формализация определенных отображений. Реализация первого этапа требует создания модели неструктурированной или слабоструктурированной вербальной информации. Рассмотрены три модели информации на базе ориентированных деревьев с возрастающей сложностью. Как обобщение рассмотрена модель на базе ациклического ориентированного графа. Методы работы с этими моделями могут быть основой для обработки информации, имеющей структуру большей сложности. В статье дано теоретическое обоснование построения отношения порядка для элементов рассмотренных моделей. Для реализации второго и третьего этапов предложен метод идентификации элементов, находящихся в отношении порядка, на базе позиционной системы. Предложен метод предоставления значений функций принадлежности нечетких множеств различных термов одной лингвистической переменной на базе идентификатора каждого упорядоченного элемента. Метод построения функции принадлежности состоит из последовательности следующих этапов: формирование модели вербальной информации, построение отношения порядка на элементах этой модели, определение лингвистической переменной на базе созданной модели и определение функций принадлежности для нечетких множеств термов одной лингвистической переменной. **Выводы**: применение метода позволит автоматизировать определение значений векторов входящей и исходящей информации, автоматизировать формирование нечетких множеств термов соответствующих лингвистических переменных, позволит строить нечеткие продукты как базу знаний нечеткой продукционной системы, а также проводить обучение нечеткой продукционной системы.

Ключевые слова: функция принадлежности; отношение порядка; лингвистическая переменная.

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